Approximate theories of bending of thin elastic plates have been around since the middle of the nineteenth century. The reason for their existence is twofold: on the one hand, they reduce the full three-dimensional model to a simpler one in only two independent variables; on the other hand, they give prominence to the main characteristics of bending, neglecting other effects that are of lesser interest in the study of this physical process.

In spite of their good agreement with experiments and their wide use by engineers in practical applications, such theories never acquire true legitimacy until they have been validated by rigorous mathematical analysis. The study of the classical (Kirchhoff) model (Kirchhoff 1850) is almost complete (see, for example Ciarlet and Destuynder 1979; Gilbert and Hsiao 1983). In this book, we turn our attention to plates with transverse shear deformation, which include the Reissner (1944, 1945, 1947, 1976, 1985) and Mindlin (1951) models, discussing the existence, uniqueness, and approximation of their regular solutions by means of the boundary integral equation and stress function methods in the equilibrium (static) case.

With the exception of a few results of functional analysis, which are quoted from other sources, the presentation is self-contained and includes all the necessary details, from basic notation to the full-blown proofs of the lemmas and theorems.

Chapter 1 concentrates on the geometric/analytic groundwork for the investigation of the behavior of functions expressed by means of integrals with singular kernels, in the neighborhood of the boundary of the domain where they are defined.

In Chap. 2, we introduce potential-type functions and determine their mapping properties in terms of both real and complex variables, and discuss the solvability of singular integral equations.

Next, in Chap. 3, we describe the two-dimensional model of bending of elastic plates with transverse shear deformation, derive a matrix of fundamental solutions for the governing system, state the main boundary value problems, and comment on the uniqueness of their regular solutions.

All the references cited here can be found at the end of the book.
The layer and Newtonian plate potentials are introduced, respectively, in Chaps. 4 and 5, where we investigate their Hölder continuity and differentiability.

In Chap. 6, we prove the existence of regular solutions for the interior and exterior displacement, traction, and Robin boundary value problems by means of single-layer and double-layer potentials, and discuss the smoothness of the integrable solutions of these problems.

Chapter 7 is devoted to the construction of the complete integral of the system of equilibrium equations in terms of complex analytic potentials, and the clarification of the physical meaning of certain analytic constraints imposed earlier on the asymptotic behavior of the solutions.

In Chap. 8, we explain how the method of generalized Fourier series can be adapted to provide approximate solutions for the Dirichlet and Neumann problems.

Some of the results incorporated in this book have been published in Constanda (1985, 1986a, b, 1987, 1988a, b, 1989a, b, 1990a, b, 1991, 1994, 1996a, b, 1997a, b; Schiavone 1996; Thomson and Constanda 1998, 2008); additionally, Constanda (1990) is an earlier—incomplete—version compiled as research notes. Chapter 5 is based on material included in Thomson and Constanda (2011a). The technique developed in Chaps. 2–4 and 6 was later extended to the case of bending of micropolar plates in Constanda (1974), Schiavone and Constanda (1989), and Constanda (1989).

A comprehensive view and comparison of direct and indirect boundary integral equation methods for elliptic two-dimensional problems in Cartesian coordinates and Hölder spaces can be found in Constanda (1999).

Potential methods go hand in hand with variational techniques when the data functions lack smoothness. The distributional solutions of equilibrium problems with a variety of boundary conditions have been constructed by this combination of analytic procedures in Chudinovich and Constanda (1997, 1998, 1999a, b, 2000a, b, c, d, e, 2001a, b). The harmonic oscillations of plates with transverse shear deformation form the object of study in Constanda (1998), Schiavone and Constanda (1993, 1994), Thomson and Constanda (1998, 1999, 2009a, b, c, 2010, 2011a, b, 2012a, b, c, 2013), and the case that includes thermal effects has been developed in Chudinovich and Constanda (2005a, b, 2006, 2008a, b, c, 2009, 2010a, b, c, 2007).

Finally, a number of problems that impinge on the solution of this mathematical model are discussed in Chudinovich and Constanda (2000f, 2006), Constanda (1978a, b), Constanda et al. (1995), Mitric and Constanda (2005), and Constanda (2006).

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