Parameterized complexity/multivariate complexity algorithmics is an exciting field of modern algorithm design and analysis, with a broad range of theoretical and practical aspects that answers the vital need for efficient algorithms by almost every facet of modern society. The last decade and a half has seen remarkable progress. There are now whole conferences devoted even to subareas of the field. There has been an exhilarating development of techniques where we see an extended discourse with a combinatorial problem via parameters as articulated in the original monograph, Downey and Fellows [247]. We have seen the development of deep techniques for systematically attacking the tractability of problems, as well as techniques for showing that the techniques are nearly optimal. This “yin and yang” of parameterized complexity is a stunning endorsement of the methodology, although many excellent questions remain, not the least of which is why things like SAT-solvers work so well, although the parameterized framework is providing answers through work by Gaspers and Szeider [340] and others.

In the preface to our original book, we described how the project began with some concrete puzzlements that seemed elemental. In particular, the graph minors project of Robertson and Seymour had suggested the notion, central to this field, of fixed-parameter tractability.

At that time, we knew that what is seemingly the most important problem in combinatorial optimization, the VERTEX COVER problem, is fixed-parameter tractable, parameterized in the natural way, by solution size. We knew that the similarly defined graph optimization problems INDEPENDENT SET and DOMINATING SET seemed to resist the power of well-quasiordering. And we knew a reduction carrying the issue of fixed-parameter tractability from the INDEPENDENT SET problem to the DOMINATING SET problem, but not vice versa. It seemed like an interesting theory should be possible! We had no clear idea if it would be useful. Fired by a common love of surfing and wine, we embarked!

After legendary adventures, numerous papers, and the sacrificing of many graduate students to what was long considered a cult, especially in North America (a reception to our ideas that we did not anticipate), we amassed enough interesting results to offer the first book, conceived in 1990 and finally published in 1999. The first book was full of bugs (we hope this one has fewer), but that does not seem to have mattered so much, as it had fresh ideas and conceptual approaches, and a nice list of challenges at the end. So does this one.
The field of parameterized/multivariate algorithms and complexity is now firmly, and permanently, a vibrant part of theoretical computer science. There is a thriving conference series, hundreds of papers, two further books,\(^1\) and any number of new fields of applications. The subject is evolving very rapidly as brilliant young authors remake it in their own image. In writing this book we were especially keen to showcase the wealth of recent techniques for proving parameterized tractability and to showcase the powerful new lower bound techniques.

The multivariate perspective has proved useful, even arguably essential, to modern science, starting with bioinformatics as a charter area for applications. Two volumes of the *Computer Journal* [250] essay just some of the areas of applications of multivariate algorithms. As we mention in this book, this work remains hand-in-hand with algorithm engineering.

It has become clear that, far from being an accidental incitement to the central notion of fixed-parameter tractability, deep mathematical structure theory, such as represented by the graph minors project, is a fundamental companion of the central complexity notion.

When this book was being prepared, the first author gave a series of tutorials at *a Coruna* for the conference LATA 2012. Someone in the audience asked whether mastery of this (upcoming) book would suffice to understand complexity issues in their own area of research. The answer is definitely “Yes”. Mastering a reasonable fragment of this book will enable a researcher to use the positive and negative toolkits in their own research. The multivariate framework allows an extended dialog with a problem, which we describe in this book.

This book is targeted at the beginning graduate student, and accessible to an advanced senior (i.e. final year undergraduate student). It is also aimed at the general computer scientist, and the mathematically aware scientist seeking tools for their research. We have tried to make the material as self-contained as possible.

For many problems we have given a large number of methods for algorithmic solutions for the *same* problem. Similarly for the same problem we have often given a series of lower bounds based on ever stronger complexity hypotheses, showing how the methods and ideas have evolved over the past 25 years.

We have enjoyed teaching many young researchers to surf, and trotting the ideas all over the globe. We welcome you to enjoy the far-reaching ideas of multivariate algorithmics, the heartbeat of algorithms and complexity.

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\(^1\)Namely Flum and Grohe [312] and Niedermeier [546].
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