

Contents

1	Basic Measure Theory	1
1.1	Classes of Sets	1
1.2	Set Functions	11
1.3	The Measure Extension Theorem	16
1.4	Measurable Maps	32
1.5	Random Variables	41
2	Independence	47
2.1	Independence of Events	47
2.2	Independent Random Variables	54
2.3	Kolmogorov's 0–1 Law	61
2.4	Example: Percolation	65
3	Generating Functions	77
3.1	Definition and Examples	77
3.2	Poisson Approximation	80
3.3	Branching Processes	82
4	The Integral	85
4.1	Construction and Simple Properties	85
4.2	Monotone Convergence and Fatou's Lemma	93
4.3	Lebesgue Integral Versus Riemann Integral	95
5	Moments and Laws of Large Numbers	101
5.1	Moments	101
5.2	Weak Law of Large Numbers	108
5.3	Strong Law of Large Numbers	111
5.4	Speed of Convergence in the Strong LLN	120
5.5	The Poisson Process	124
6	Convergence Theorems	131
6.1	Almost Sure and Measure Convergence	131
6.2	Uniform Integrability	136
6.3	Exchanging Integral and Differentiation	142

7	L^p-Spaces and the Radon–Nikodym Theorem	145
7.1	Definitions	145
7.2	Inequalities and the Fischer–Riesz Theorem	147
7.3	Hilbert Spaces	153
7.4	Lebesgue’s Decomposition Theorem	156
7.5	Supplement: Signed Measures	159
7.6	Supplement: Dual Spaces	166
8	Conditional Expectations	169
8.1	Elementary Conditional Probabilities	169
8.2	Conditional Expectations	172
8.3	Regular Conditional Distribution	180
9	Martingales	189
9.1	Processes, Filtrations, Stopping Times	189
9.2	Martingales	194
9.3	Discrete Stochastic Integral	198
9.4	Discrete Martingale Representation Theorem and the CRR Model	200
10	Optional Sampling Theorems	205
10.1	Doob Decomposition and Square Variation	205
10.2	Optional Sampling and Optional Stopping	209
10.3	Uniform Integrability and Optional Sampling	214
11	Martingale Convergence Theorems and Their Applications	217
11.1	Doob’s Inequality	217
11.2	Martingale Convergence Theorems	219
11.3	Example: Branching Process	229
12	Backwards Martingales and Exchangeability	231
12.1	Exchangeable Families of Random Variables	231
12.2	Backwards Martingales	236
12.3	De Finetti’s Theorem	238
13	Convergence of Measures	245
13.1	A Topology Primer	245
13.2	Weak and Vague Convergence	252
13.3	Prohorov’s Theorem	260
13.4	Application: A Fresh Look at de Finetti’s Theorem	269
14	Probability Measures on Product Spaces	273
14.1	Product Spaces	273
14.2	Finite Products and Transition Kernels	277
14.3	Kolmogorov’s Extension Theorem	285
14.4	Markov Semigroups	289
15	Characteristic Functions and the Central Limit Theorem	295
15.1	Separating Classes of Functions	295
15.2	Characteristic Functions: Examples	302

- 15.3 Lévy’s Continuity Theorem 309
- 15.4 Characteristic Functions and Moments 314
- 15.5 The Central Limit Theorem 320
- 15.6 Multidimensional Central Limit Theorem 328
- 16 Infinitely Divisible Distributions 331**
 - 16.1 Lévy–Khinchin Formula 331
 - 16.2 Stable Distributions 342
- 17 Markov Chains 351**
 - 17.1 Definitions and Construction 351
 - 17.2 Discrete Markov Chains: Examples 358
 - 17.3 Discrete Markov Processes in Continuous Time 362
 - 17.4 Discrete Markov Chains: Recurrence and Transience 367
 - 17.5 Application: Recurrence and Transience of Random Walks 371
 - 17.6 Invariant Distributions 377
 - 17.7 Stochastic Ordering and Coupling 384
- 18 Convergence of Markov Chains 389**
 - 18.1 Periodicity of Markov Chains 389
 - 18.2 Coupling and Convergence Theorem 393
 - 18.3 Markov Chain Monte Carlo Method 398
 - 18.4 Speed of Convergence 405
- 19 Markov Chains and Electrical Networks 411**
 - 19.1 Harmonic Functions 411
 - 19.2 Reversible Markov Chains 415
 - 19.3 Finite Electrical Networks 416
 - 19.4 Recurrence and Transience 422
 - 19.5 Network Reduction 427
 - 19.6 Random Walk in a Random Environment 435
- 20 Ergodic Theory 439**
 - 20.1 Definitions 439
 - 20.2 Ergodic Theorems 443
 - 20.3 Examples 446
 - 20.4 Application: Recurrence of Random Walks 447
 - 20.5 Mixing 450
 - 20.6 Entropy 453
- 21 Brownian Motion 457**
 - 21.1 Continuous Versions 457
 - 21.2 Construction and Path Properties 463
 - 21.3 Strong Markov Property 469
 - 21.4 Supplement: Feller Processes 472
 - 21.5 Construction via L^2 -Approximation 475
 - 21.6 The Space $C([0, \infty))$ 482
 - 21.7 Convergence of Probability Measures on $C([0, \infty))$ 484

21.8 Donsker’s Theorem 487

21.9 Pathwise Convergence of Branching Processes* 491

21.10 Square Variation and Local Martingales 497

22 Law of the Iterated Logarithm 509

22.1 Iterated Logarithm for the Brownian Motion 509

22.2 Skorohod’s Embedding Theorem 512

22.3 Hartman–Wintner Theorem 517

23 Large Deviations 521

23.1 Cramér’s Theorem 522

23.2 Large Deviations Principle 526

23.3 Sanov’s Theorem 531

23.4 Varadhan’s Lemma and Free Energy 535

24 The Poisson Point Process 543

24.1 Random Measures 543

24.2 Properties of the Poisson Point Process 548

24.3 The Poisson–Dirichlet Distribution* 555

25 The Itô Integral 563

25.1 Itô Integral with Respect to Brownian Motion 563

25.2 Itô Integral with Respect to Diffusions 571

25.3 The Itô Formula 575

25.4 Dirichlet Problem and Brownian Motion 583

25.5 Recurrence and Transience of Brownian Motion 585

26 Stochastic Differential Equations 589

26.1 Strong Solutions 589

26.2 Weak Solutions and the Martingale Problem 598

26.3 Weak Uniqueness via Duality 605

Notation Index 613

References 617

Name Index 625

Subject Index 629



<http://www.springer.com/978-1-4471-5360-3>

Probability Theory

A Comprehensive Course

Klenke, A.

2014, XII, 638 p. 46 illus., 20 illus. in color., Softcover

ISBN: 978-1-4471-5360-3