Introduction

This book is an introduction to the theory of automorphic forms. Starting with classical modular forms, it leads to representation theory of the adelic GL(2) and corresponding L-functions. Classical modular forms, which are introduced in the beginning of the book, will serve as the principal example until the very end, where it is verified that the classical and representation-theoretic approaches lead to the same L-functions. Modular forms are defined as holomorphic functions on the upper half plane, satisfying a particular transformation law under linear fractional maps with integer coefficients. We then lift functions on the upper half plane to the group $SL_2(\mathbb{R})$, a step that allows the introduction of representation-theoretic methods to the theory of automorphic forms. Finally, ground rings are extended to adelic rings, which means that number-theoretical questions are built into the structure and can be treated by means of analysis and representation theory.

For this book, readers should have some knowledge of algebra and complex analysis. They should be acquainted with group actions and the basic theory of rings. Further, they should be able to apply the residue theorem in complex analysis. Additionally, knowledge of measure and integration theory is useful but not necessary. One needs basic notions of this theory, like that of a σ-algebra and measure and some key results like the theorem of dominated convergence or the completeness of L^p-spaces. For the convenience of the reader, we have collected these facts in an appendix.

The present book focuses on the interrelation between automorphic forms and L-functions. To increase accessibility, we have tried to obtain the central results with a minimum of theory. This has the side effect that the presentation is not of the utmost generality; therefore the interested reader is given a guide to the literature.

In Chap. 1 the classical approach to modular forms via doubly periodic functions is presented. The Weierstrass $\wp$-function leads to Eisenstein series and thus to modular forms. The modular group and its modular forms are the themes of Chap. 2, which concludes with the presentation of L-functions. According to Dieudonné, there have been two revolutions in the theory of automorphic forms: the *intervention of Lie groups* and the *intervention of adeles*. Lie groups intervene in Chap. 3, and adeles in the rest of the book. We try to maintain continuity of presentation by continually referring back to the example of classical modular forms. Chapters 4 and 5 pave the way for Tate's thesis, which is introduced in Chap. 6. We present it in a simplified form over the rationals instead of an arbitrary number field. This is more than adequate for our purposes, as it brings out the central ideas better. In Chap. 7 automorphic forms on the group of invertible 2×2 matrices with adelic entries are investigated, and Chap. 8 we transfer the ideas of Tate's thesis to this setting and perform the analytic continuation of L-functions. For classical cusp forms we finally show that the classical and representation-theoretic approaches give the same L-functions.

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