

Contents

1	Introduction	1
1.1	The Origins of Mathematical Logic	1
1.2	Propositional Logic	2
1.3	First-Order Logic	3
1.4	Modal and Temporal Logics	4
1.5	Program Verification	5
1.6	Summary	5
1.7	Further Reading	6
1.8	Exercise	6
	References	6
2	Propositional Logic: Formulas, Models, Tableaux	7
2.1	Propositional Formulas	7
2.2	Interpretations	16
2.3	Logical Equivalence	21
2.4	Sets of Boolean Operators *	26
2.5	Satisfiability, Validity and Consequence	29
2.6	Semantic Tableaux	33
2.7	Soundness and Completeness	39
2.8	Summary	44
2.9	Further Reading	45
2.10	Exercises	45
	References	47
3	Propositional Logic: Deductive Systems	49
3.1	Why Deductive Proofs?	49
3.2	Gentzen System \mathcal{G}	51
3.3	Hilbert System \mathcal{H}	55
3.4	Derived Rules in \mathcal{H}	58
3.5	Theorems for Other Operators	62
3.6	Soundness and Completeness of \mathcal{H}	64

3.7	Consistency	66
3.8	Strong Completeness and Compactness *	67
3.9	Variant Forms of the Deductive Systems *	68
3.10	Summary	71
3.11	Further Reading	71
3.12	Exercises	72
	References	73
4	Propositional Logic: Resolution	75
4.1	Conjunctive Normal Form	75
4.2	Clausal Form	77
4.3	Resolution Rule	80
4.4	Soundness and Completeness of Resolution *	82
4.5	Hard Examples for Resolution *	88
4.6	Summary	92
4.7	Further Reading	92
4.8	Exercises	92
	References	93
5	Propositional Logic: Binary Decision Diagrams	95
5.1	Motivation Through Truth Tables	95
5.2	Definition of Binary Decision Diagrams	97
5.3	Reduced Binary Decision Diagrams	98
5.4	Ordered Binary Decision Diagrams	102
5.5	Applying Operators to BDDs	104
5.6	Restriction and Quantification *	107
5.7	Summary	109
5.8	Further Reading	110
5.9	Exercises	110
	References	110
6	Propositional Logic: SAT Solvers	111
6.1	Properties of Clausal Form	111
6.2	Davis-Putnam Algorithm	115
6.3	DPLL Algorithm	116
6.4	An Extended Example of the DPLL Algorithm	117
6.5	Improving the DPLL Algorithm	122
6.6	Stochastic Algorithms	125
6.7	Complexity of SAT *	126
6.8	Summary	128
6.9	Further Reading	128
6.10	Exercises	128
	References	129
7	First-Order Logic: Formulas, Models, Tableaux	131
7.1	Relations and Predicates	131
7.2	Formulas in First-Order Logic	133

7.3	Interpretations	136
7.4	Logical Equivalence	140
7.5	Semantic Tableaux	143
7.6	Soundness and Completion of Semantic Tableaux	150
7.7	Summary	153
7.8	Further Reading	153
7.9	Exercises	153
	References	154
8	First-Order Logic: Deductive Systems	155
8.1	Gentzen System \mathcal{G}	155
8.2	Hilbert System \mathcal{H}	158
8.3	Equivalence of \mathcal{H} and \mathcal{G}	160
8.4	Proofs of Theorems in \mathcal{H}	161
8.5	The C-Rule *	163
8.6	Summary	165
8.7	Further Reading	165
8.8	Exercises	165
	References	166
9	First-Order Logic: Terms and Normal Forms	167
9.1	First-Order Logic with Functions	167
9.2	PCNF and Clausal Form	172
9.3	Herbrand Models	177
9.4	Herbrand's Theorem *	180
9.5	Summary	182
9.6	Further Reading	182
9.7	Exercises	182
	References	183
10	First-Order Logic: Resolution	185
10.1	Ground Resolution	185
10.2	Substitution	187
10.3	Unification	189
10.4	General Resolution	195
10.5	Soundness and Completeness of General Resolution *	198
10.6	Summary	202
10.7	Further Reading	202
10.8	Exercises	202
	References	203
11	First-Order Logic: Logic Programming	205
11.1	From Formulas in Logic to Logic Programming	205
11.2	Horn Clauses and SLD-Resolution	209
11.3	Search Rules in SLD-Resolution	213
11.4	Prolog	216
11.5	Summary	220

11.6	Further Reading	221
11.7	Exercises	221
	References	222
12	First-Order Logic: Undecidability and Model Theory *	223
12.1	Undecidability of First-Order Logic	223
12.2	Decidable Cases of First-Order Logic	226
12.3	Finite and Infinite Models	227
12.4	Complete and Incomplete Theories	228
12.5	Summary	229
12.6	Further Reading	229
12.7	Exercises	230
	References	230
13	Temporal Logic: Formulas, Models, Tableaux	231
13.1	Introduction	231
13.2	Syntax and Semantics	233
13.3	Models of Time	237
13.4	Linear Temporal Logic	240
13.5	Semantic Tableaux	244
13.6	Binary Temporal Operators *	258
13.7	Summary	260
13.8	Further Reading	261
13.9	Exercises	261
	References	262
14	Temporal Logic: A Deductive System	263
14.1	Deductive System \mathcal{L}	263
14.2	Theorems of \mathcal{L}	264
14.3	Soundness and Completeness of \mathcal{L} *	269
14.4	Axioms for the Binary Temporal Operators *	271
14.5	Summary	271
14.6	Further Reading	272
14.7	Exercises	272
	References	272
15	Verification of Sequential Programs	273
15.1	Correctness Formulas	274
15.2	Deductive System \mathcal{HL}	275
15.3	Program Verification	277
15.4	Program Synthesis	279
15.5	Formal Semantics of Programs *	283
15.6	Soundness and Completeness of \mathcal{HL} *	289
15.7	Summary	293
15.8	Further Reading	293
15.9	Exercises	293
	References	295

- 16 Verification of Concurrent Programs** 297
 - 16.1 Definition of Concurrent Programs 298
 - 16.2 Formalization of Correctness 300
 - 16.3 Deductive Verification of Concurrent Programs 303
 - 16.4 Programs as Automata 307
 - 16.5 Model Checking of Invariance Properties 311
 - 16.6 Model Checking of Liveness Properties 314
 - 16.7 Expressing an LTL Formula as an Automaton 315
 - 16.8 Model Checking Using the Synchronous Automaton 317
 - 16.9 Branching-Time Temporal Logic * 319
 - 16.10 Symbolic Model Checking * 322
 - 16.11 Summary 323
 - 16.12 Further Reading 324
 - 16.13 Exercises 324
 - References 325
- Appendix Set Theory** 327
 - A.1 Finite and Infinite Sets 327
 - A.2 Set Operators 328
 - A.3 Sequences 330
 - A.4 Relations and Functions 331
 - A.5 Cardinality 333
 - A.6 Proving Properties of Sets 335
 - References 336
- Index of Symbols** 337
- Name Index** 339
- Subject Index** 341



<http://www.springer.com/978-1-4471-4128-0>

Mathematical Logic for Computer Science

Ben-Ari, M.

2012, XV, 346 p. 79 illus., Softcover

ISBN: 978-1-4471-4128-0