Preface

Fractional calculus is now being more widely accepted. The (constant) order of differentiation and/or integration can be an arbitrary real number including integers as special cases. For example, a low pass filter (LPF) with a fractional order pole can be written as $H(s) = 1/(\tau s^{\alpha_0} + 1)$, where $\alpha_0 > 0$ is a constant. Its corresponding governing differential equation is

$$\tau \frac{d^{\alpha_0}}{dt^{\alpha_0}} y(t) + y(t) = u(t),$$

where $u(t)$ and $y(t)$ are input and output signals, respectively; $\frac{d^{\alpha_0}}{d_t^{\alpha_0}} y(t)$ or $y^{(\alpha_0)}(t)$ is the notation of fractional order derivative of $y(t)$. It is mathematically immediate to generalize this constant-order LPF in distributed-order sense as

$$H_{do}(s) = \frac{1}{b-a} \int_a^b \frac{1}{\tau s^{\alpha} + 1} d\alpha,$$

where a and b are given constants and the term $\frac{1}{b-a}$ is for scaling the DC gain to be 0 dB. The distributed-order dynamics can be characterized by the following distributed-order differential equation

$$\int_a^b w(\alpha) \frac{d^{\alpha}}{dt^{\alpha}} y(t) d\alpha + y(t) = u(t),$$

where $w(\alpha)$ can be regarded as order-dependent time constant or "order weight/ distribution function."

Note that, the above constant-order model is in the same form of the famous classic Cole–Cole relaxation model, which can be recovered from the distributed-order model by setting the order distribution function $w(\alpha) = \delta(\alpha - \alpha_0)$, where $\delta(\cdot)$ is the well known Dirac Delta function. So, it is natural to believe that distributed-order Cole–Cole model $H_{do}(s)$ may be in a better position to characterize the *complex* material properties when the distribution function $w(\alpha)$ is properly chosen. The wisdom in modeling "*All models are wrong but some are useful*" and

"*All models are wrong but some are dangerous*", in fact, encourages us to explore the distributed-order generalization since we believe this notion is helpful, at least partially, as demonstrated in this Brief, with no harm.

With the above in mind, this Brief presents a general approach of distributed-order operator which can and will find its use for real world applications, as being observed from recent literature in many fields of science and engineering. It is devoted to provide an introduction of the latest research results about distributed-order dynamic system and control as well as distributed-order signal processing, which are based on the distributed-order differential/integral equations, to serve the control and signal processing community as a guide to understanding and using distributed-order differential/integral equations in order to enlarge the application domains of its disciplines, and to improve and generalize well established (constant-order) fractional-order control methods and strategies.

A major goal of this Brief is to present a concise and insightful view of the relevant knowledge by emphasizing fundamental methods and tools to understand why distributed-order concept is useful in control and signal processing, to understand its terminology, and to illuminate the key points of its applicability. The Brief is suitable for science and engineering community for broadening their toolbox in modeling, analysis, control, filtering tasks, with a hope that, transformative progress can be made in their respective research projects.

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