Preface

In ancient times, instead of fractions, the natural numbers were used. However, integers cannot always result when measuring or equally dividing things. As time went by, fractions and then non-integers were gradually understood and applied. With the introduction of fractions and more generally nonintegers, people were able to have a closer look at the beauty of nature around them. For example, people long ago realized that a rectangle of the 'golden ratio' $1.618:1$ is most pleasing. The natural exponential $e = 2.71828\ldots$, and the ratio of any circle's circumference to its diameter, $\pi = 3.14159\ldots$ are widely used in mathematics and engineering. The 'beauty' of the fraction was recognized and people came to use the 'fractional view' to observe the world, to use 'fractional thinking' to understand the natural phenomena, and to use 'fractional techniques' to solve the problems at hand.

The term 'fractal' was introduced by Mandelbrot in 1975 [192]. Fractal refers to the self-similar geometric shape, that is, a shape in which is almost identical to the entire shape except in size [91, 102]. Many objects manifest themselves in fractal shape, such as clouds, coastlines and snow flakes. In fractal theory, the fractal dimension was used to characterize the state of nature. Different from the conventional integer dimension, the fractal dimension can be fractional or any non-integer number. Based on the fractal theory, the traditional concept of three-dimensional space can be extended to the fractal (fractional) dimension (FD) which can be applied to characterize complex objects.

Likewise, (integer-order) calculus can be extended to fractional or noninteger order calculus. It should be remarked at this point that due to historical reasons, the term 'fractional' we use here and throughout this monograph should actually be understood as 'non-integer' or 'arbitrary real number' to be precise. Fractional calculus, i.e., fractional-order differentiation and integration, is a part of mathematics dealing with derivatives of arbitrary order [139, 203, 209, 218, 237]. Leibniz raised the question about the possibility of generalizing the operation of differentiation to non-integer-orders in 1695 [237]. Fractional calculus, developed from the field of pure mathematics, has been studied increasingly in various fields [64, 142, 311, 315, 323]. Nowadays, fractional calculus is being applied to many fields of science, engineering, and mathematics [49, 74, 78, 135, 290]. Fractional calculus provides a better description of various natural phenomena, such as viscoelasticity and anomalous diffusion, than integer-order calculus can provide. The most fundamental reason for the superiority of the use is that fractional calculus based models can capture the memory and the heredity of the process. It is safe to say that fractional calculus provides a particularly useful and effective tool for revealing phenomena in nature because nature has memory.

This "fractionalization" idea can go on. Taking the Fourier transform as an example, we can naturally talk about the fractional Fourier transform (FrFT), which is a linear transformation generalizing the Fourier transform, first introduced by Victor Namias in 1980 [213]. FrFT can transform a signal from time domain into a domain between the time and frequency domains. So, the FrFT differs from the conventional Fourier transform by the rotation of fractional times of the $\pi/2$ angle in the time-frequency plane. FrFT is widely used in filter design, signal detection and image recovery [32].

Another important "fractionalization" is fractional low-order moments (FLOM). FLOM is based on the non-Gaussian α-stable distribution, a powerful tool for impulsive random signals. The density functions of α-stable distribution will decay in the tails less rapidly than the Gaussian density function does. So α-stable distribution can be used to characterize signals which exhibit sharp spikes or occasional bursts of outlying observations more frequently than normally distributed signals do. The α-stable distribution based techniques have been applied to describe many natural or man-made phenomena in various fields, such as physics, hydrology, biology, financial and network traffic [43, 70, 253, 270]. The α-stable distribution has a characteristic exponent parameter $\alpha(0 < \alpha \leq 2)$, which controls the heaviness of its tails. For a non-Gaussian stable distribution with characteristic exponent $\alpha < 2$, its second-order moment does not exist. α-stable distribution has only finite moments of order less than α. So the FLOM plays an important role in impulsive processes like the role of mean and variance in the Gaussian processes.

In this monograph, we will introduce some complex random signals which are characterized by the presence of heavy-tailed distribution or non-negligible dependence between distant observations, from the 'fractional' point of view. Furthermore, the analysis techniques for these fractional processes are investigated using 'fractional thinking'. The term 'fractional process' in this monograph refers to some random signals which manifest themselves by heavy-tailed distribution, long range dependence (LRD)/long memory, or local memory. Fractional processes are widely found in science, technology and engineering systems. Typical heavy-tailed distributed signals include underwater acoustic signals, low-frequency atmospheric noises, many types of man-made noises, and so on. Typical LRD/long memory processes and local memory processes can be observed in financial data, communications networks data and biological data. These properties, i.e., heavy-tailed distribution, LRD/long memory, and local memory, always lead to difficulty in correctly obtaining the statistical characteristics and extracting the desired information from these fractional processes. These properties cannot be neglected in time series analysis, because the tail thickness of the distribution, LRD, or local memory properties of the time series are critical in characterizing the essence of the resulting natural or man-made phenomena of the signals. Therefore, some valuable fractional-order signal processing (FOSP) techniques were provided to analyze these fractional processes. FOSP techniques, which are based on the fractional calculus, FLOM and FrFT, include simulation of fractional processes, fractional-order system modeling, fractional-order filtering, realization of fractional systems, etc. So, random signals which exhibit evident 'fractional' properties should be investigated using FOSP techniques to obtain better analysis results.

This monograph includes four parts. The first part is the overview of fractional processes and FOSP techniques. The second part presents fractional processes, which are studied as the output of the fractional order differential systems, including constant-order fractional processes and variable-order fractional processes. The third part introduces the FOSP techniques from the 'fractional signals and fractional systems' point of view. In the last part of the monograph, some application examples of FOSP techniques are presented to help readers understand and appreciate the fractional processes and fractional techniques. We sincerely wish that this monograph will give our readers a novel insight into the complex random signals characterized by 'fractional' properties, and some powerful tools to characterize those signals.

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