Preface to the German Edition

This textbook is based on lecture notes for courses on mathematical logic and model theory that I have been giving in recent years at the University of Konstanz. A goal of the model theory course was to give a thorough and self-contained presentation of the model theoretic aspects of a series of algebraic theories. This was motivated especially by the intent to write a book using the lecture notes that would allow interested mathematicians not trained in this special field to get to know and understand the best known model theoretic results in algebra common at the time. As probably the most interesting example, let us mention only J. Ax and S. Kochen's treatment of "Artin's conjecture" on p-adic number fields.

Since the character of model theoretic results and constructions differs quite profoundly from that commonly found in algebra, because of the treatment of formulae as objects, it seems to me indispensable, for a deeper understanding, to become familiar with the problems and methods of mathematical logic. I have therefore preceded the treatment of model theory with an introduction to mathematical logic. From this results a distinct partition of the book into three parts: mathematical logic (Chapter 1), model theory (Chapters 2 and 3) and the model theoretic treatment of several algebraic theories (Chapter 4).

Because of the special goal of this book, I have not aspired to completeness in either the logical or the model theoretic part – this book makes no claim to present either of these two fields as completely as is nowadays common. Instead, I have tried to be as complete and detailed as possible while pursuing the goal stated above. (This explains, among other things, the limited number of exercises.) For further reading I refer the reader to [Shoenfield, 1967] and [Chang–Keisler, 1973–90].

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Konstanz,
March 1986

Alexander Prestel

Preface to the English Edition

The original purpose of the German edition, written in 1986, was to give an introduction to basic model theory and to the model theoretic treatment of several algebraic theories – in particular, the theory of Henselian valued fields, presenting J. Ax and S. Kochen's result on the "Artin conjecture" on p-adic number fields. It was considered indispensable, for a deeper understanding, to base this introduction to model theory on a presentation of formal logic.

The purpose of this new book is to make this kind of introduction available to a larger audience of mathematicians (as German is sometimes considered to be a difficult language). The English version of the 1986 German book is essentially just a translation. It was not our intention to include subsequent developments, since we did not want to add a new book to the many good books that concentrate mainly on model theory, such as [Hodges, 1997], [Marker, 2002] and [Tent–Ziegler, 2011].

Compared with the German version, there are only two major changes: we have considerably increased the number of exercises, and we have added a second appendix, in which we try to explain to the reader the difference between first- and second-order logic.

There are several possible ways to use this book in a course, either for first- or second-year graduate students or for advanced undergraduates. For a course in logic alone (with no model theory), one could cover Chapter 1 and Appendix A (and possibly also Appendix B); for such a course, the student would need little prior knowledge of specific areas of mathematics, but only some mathematical maturity, i.e. some experience in proving theorems. For a course in model theory alone, one can skip Chapter 1 with the exception of the definitions of a formula (Section 1.2), a structure and the satisfaction of a formula by a structure (Section 1.5).[1] For a short course in model theory, Chapters 2 and 3 will suffice (here some familiarity with cardinal and ordinal numbers is required). For a longer course, one could include

[1] In skipping most of Chapter 1 (i.e. formal proofs), the proof of Theorem 2.1.1 will have to rely, instead, on Theorems 2.6.4 and 2.3.3.

Chapter 4 as well; in Sections 4.3–4.6, the student should have some prior acquaintance with valuation theory.

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