Chapter 2
Calculation of Water and Sediment Discharge
Using an Integral Calculus Method

Abstract  In order to improve the accuracy of the calculation of the amount of water discharge, a new integration equation method was developed and applied. In this study, field observation of runoff with a weir was carried out at the outlet of the Shangshe catchments and sub-catchments of four types of land use within the catchment in Yuexi prefecture, Anhui Province. Each weir was equipped with a float-type, water level recorder. In this research, the water level hydrograph of each rainfall–runoff event was divided into periods at 10-min intervals. In each period, a water level versus time linear function was established. Then, the water level versus flux relationship was transferred into a function between flux and time in each period, which could be integrated over time. Thus a new integral equation method was developed to calculate the amount of water discharge in sub-catchments and catchments. The result showed that the new integral equation method can improve the accuracy of annual water and sediment discharge by 8.7–17.1% at five monitoring sites using the field observation data in the year 2000. Compared to the instant flux coefficient method, it improves accuracy greatly and is suitable for the calculation of the amount of water and sediment discharge at weirs under various conditions of water level variation.

2.1 Introduction

During the Yangtze River flood in 1998, the Yangtze’s flow peaked at less than 2 million ft$^3$/s, a rate that it had surpassed 23 times since 1949. Zhang et al. (1999) pointed out that a flow that had caused a small flood in the basin in the past had come to be catastrophic. What would happen if a bigger flow were to occur? Despite the potential hazards to the long-term productivity of arable land due to erosion, which has also led to dam and river bed silting and low soil water retention, very limited information is available about water and soil loss processes from different types of land use. Furthermore, no extensive studies of the problem and its quantification are available. It is easy to calculate the water storage capacity decrease due to roads and construction, and by the shrinking of lakes and reservoirs, but until now we have been unable to quantify the management effects of various types of land use in the mountainous areas on the flood and sediment discharges. Although much research has been carried out concerning water and soil loss rates derived from plot and catchment studies in the Yangtze River basin (Xu, 1999), the results do not provide sufficient information for quantification of the effects of land use on floods in...
this basin. After the 1998 flood of the Yangtze River, recognition of the great importance of paying attention to providing cover for the bare land or very thin forests and shrub-covered land in the mountainous area has become common sense to the environmental experts (Wei and Fu, 1999; Zhang et al., 1999) and the Chinese Forestry Ministry. In 1999, an environmental monitoring project of the Yangtze River protection forest in 11 provinces of the Yangtze River basin was undertaken by the Yangtze River Protection Forest Department of the Chinese Forestry Ministry. That project in the Dabie Mountains of Anhui Province, China, is a main monitoring station.

2.2 Study Area

This study was carried out in the Shangshe catchment (30°12′20″N, 116°26′42″E) of the Dabie Mountains in Yuexi prefecture of Anhui Province, China (Fig. 2.1). The Shangshe catchment comprises an area of 510 ha and has a monsoon climate with a mean annual temperature of 14.6°C and a mean annual precipitation level of 1200 mm. Its non-frost season is 212 days. Concerning soils, according to the Chinese Soil Taxonomy system, the dominant type is mountainous yellow-purple soil derived from weathering granite. This soil has a coarse texture and a high mineral content, which is suitable for cultivation and susceptible to erosion. In this catchment, land types fall mainly into five groups: cultivated land, pine forest, Chinese fir forest, tea garden, and paddy field. The cultivated land types include bare land and roads, the pine forest includes shrubs and bamboo, and the tea garden includes China gooseberry bushes. Within the catchment, the valley and the lowland have been mainly developed into paddy fields. Land with steep slopes is occupied by pine forest and Chinese fir forest. Most cultivated land can be found at the foot of the mountains or hills near the valley. Land use information, obtained from a 1:10,000 scale land use map (made by the Forestry Department of Yuexi County, Anhui Province), indicates the following distribution (Fig. 2.2): pine forest comprises an area of 293.8 ha, which takes up 55% of the Shangshe catchment; paddy field is the second largest, comprising an area of 78.4 ha taking up 14.8% of the total.
2.3 Materials and Methods

2.3.1 Water Runoff Observation

A rectangular weir made of concrete and a 90° sharp-crested V-notch weir equipped with a float-type water level recorder were built at the outlet of the Shangshe catchment (Fig. 2.2). In the Shangshe catchment, four small catchments of different land types were selected as experiment sites (Fig. 2.3, Photo 2.1).
Four 45° sharp-crested, V-notch weirs were set up in the catchments of a pine forest (0.86 ha), a Chinese fir forest (0.89 ha), cultivated land (0.74 ha), and a tea garden (0.59 ha) (Photo 2.2). The main human activity in cultivated land is the clear-cutting of wheat in June, the cultivation of land in early July for corn planting, and
2.3 Materials and Methods

Photo 2.2 Sub-catchments of various types of land use with monitoring gauges

fertilization in mid-August. Clear-cutting of grass in March and August was carried out in the tea garden, and the remains of the grass were left on the terraces as strip covers. Selected characteristics of the five monitoring sites are listed in Table 2.1. Every morning at 8 o’clock, the recorder paper of each water gauge was changed

<table>
<thead>
<tr>
<th>Monitoring site</th>
<th>Area (ha)</th>
<th>Soil depth (cm)</th>
<th>Cover</th>
<th>$H$ (m)</th>
<th>$D_{1.3}$ (cm)</th>
<th>Density of individual (N ha$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chinese fir forest</td>
<td>0.89</td>
<td>35</td>
<td>Fir tree and shrub</td>
<td>8.3</td>
<td>12.1</td>
<td>1725</td>
</tr>
<tr>
<td>Pine forest</td>
<td>0.86</td>
<td>45</td>
<td>Pine tree and shrub</td>
<td>6.2</td>
<td>10.3</td>
<td>1250</td>
</tr>
<tr>
<td>Cultivated land</td>
<td>0.74</td>
<td>16</td>
<td>Wheat and oil seed rape from Jan to June and corn from June to Oct</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tea garden</td>
<td>0.59</td>
<td>45</td>
<td>Tea and grass</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>River outlet</td>
<td>510</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$H$ is the average height of the trees and crops in the monitoring sites (m); and $D_{1.3}$ is the average diameter of trees at the height of 1.3 m (cm)
manually, and the water level of each weir was measured to be used to correct the error of the water level in the recorder paper. Field observation data of water runoff at the five monitoring sites in the years 1999 and 2000 were used in this study.

2.3.2 Precipitation Observation

A manual rain gauge and an autographic rain gauge were set up to survey precipitation and rainfall intensities 100 m away from the weir at the monitored site of cultivated land and 400 m from the weir at the monitored site of the pine forest. Every morning at 8 o’clock the recorder paper of the autographic rain gauge was changed. For every rainfall event the precipitation was read from the recorder paper of automatic rain gauge and compared with the data from the manual rain gauge. Precipitation data for 1999 and 2008 obtained from the autographic rain gauge were used in this study. Precipitation data for 2001 and 2008 obtained from the manual rain gauge were also used.

2.3.3 Suspended Sediment Observation

A simple laboratory in the Shangshe catchment was equipped with an oven and electric scale. For each rainfall event in year 2000 that led to runoff, 500 ml of water was collected manually at intervals of 10 min from each weir and labeled with time, place, and water level. Then organic matter, such as grass and leaves, was picked out. From each of these samples, 30 ml was taken out. Sands were filtered through filter paper of 0.2 μm and put into the oven at 110°C for 24 h, to evaporate the water and measure the SC (sediment concentration, kg m⁻³). According to Mihara (1996), who studied the effects of agricultural land consolidation on erosion processes in semi-mountainous paddy fields of Japan, when the paddy field surface was flooded, leveling of adjacent surfaces and construction of levee slopes were essential, and erosion in paddy fields occurred mostly on levee slopes. Lu and Higgitt (2001) studied temporal changes in sediment yields in the Yiwanshui catchment in western China by using caesium-137 (¹³⁷Cs) dating techniques and found the reservoir to contain as much as 84% of the eroded soil, with the remaining 16% being attributed to valley-floor paddy fields. In our research, samples taken from the flushed paddy field showed similar SSC (suspended sediment concentration) between the flush in and flush out of the paddy field. Accordingly, soil erosion and sedimentation in the paddy field were not considered in this study.

Photo 2.2a is the sub-catchment of cultivated land, which encompasses an area of 7400 m². The crops in cultivated land are wheat from January to June and corn from the end of June until October. The main cultivation measures are carried out at the middle term of June for corn planting, after clearing of the wheat crop, and December for wheat planting. Fertility measures are carried out mainly in August with weeding and soil tilling. Photo 2.2b is the sub-catchment of pine forest with an average age of 20 years, which encompasses an area of 8600 m². Photo 2.2c is
the sub-catchment of tea garden, in which tea was planted after pine forest was cut in 1997. The catchment of the tea garden encompasses an area of 5900 m$^2$. Grass-clearing measures are carried out in March and August and the remains of the cut grass are put on the terraces as strip covers. Photo 2.2d is the sub-catchment of 16-year-old Chinese fir forest, which encompasses an area of 8900 m$^2$.

Field observation data of precipitation, sediment discharge, and runoff at different scales from micro-plot, USLE (universal soil loss equation) plot, sub-catchment to catchment scale were used in this study in 10 years from 1999 to 2008. Field observation data of sediment discharge and runoff at micro-plot and USLE-plot scales in each type of land use will be introduced in Chapters 4, 5, and 6, respectively.

### 2.4 Method to Calculate the Amount of Water and Sediment Discharge

#### 2.4.1 Method to Calculate the Amount of Water Discharge

For many years in China, the standard method to calculate the amount of runoff has been to use an averaging method in hydrological experiments (Fu, 1998) as

$$Q = \sum_{i=1}^{n} \frac{(Q_{s,i-1} + Q_{s,i})}{2} t_i$$  \hspace{1cm} (2.1)

where
- $Q$ is the amount of water discharge in a period (m$^3$);
- $Q_{s,i-1}$ and $Q_{s,i}$ are the fluxes of water discharge based on a $H - Q_s$ relationship at the beginning and the end of a period, respectively (m$^3$/s);
- $H$ is the water level (m);
- and $t_i$ is the time in the period (s).

However, an averaging method is most suitable for cases of small changes of water level at a weir, or of flow on an even keel. In the Dabie Mountains, it is suitable for the study of cases of runoff produced by small rainfall events. In the case of a storm, however, water levels of surface runoff of a triangular weir in forest and sloping land respond to rainfall rapidly, in only a few minutes. Because of the great change of water level in a short period, the averaging method will necessarily result in large errors in calculating the amount of water discharge. Figure 2.4 presents an example of the water level ($H$, m) – $Q_s$ (m$^3$/s) relationship of a 45° sharp-crested V-notch. It also shows a comparison between the methods of calculating the amount of water discharge in a certain period using Eq. (2.1), along with an integral calculus method. The triangular ABC in Fig. 2.4 represents the water discharge calculated with Eq. (2.1). The shade of ABC in Fig. 2.4 represents the water discharge calculated with an integral method.
The integral calculus method, which can obviate the shortcoming of Eq. (2.1), is suitable for all cases of the calculation of amount of water discharge with high accuracy (Fig. 2.4).

This method is as follows:

First, the water level line is divided into periods at 10-min intervals. Then the beginning water level $H_A$ and the ending water level $H_B$ in each period are recorded. The equation of the flux according to the water level of a triangular sharp-crested weir is as follows (Dong, 1995):

$$Q_s = m_0 b \sqrt{gH^{2.5}}$$  \hspace{1cm} (2.2)

where

- $Q_s$ is the flux of water discharge ($m^3/s$);
- $m_0$ and $b$ are parameters of the weir;
- the variable $g$ represents the acceleration of gravity; and
- $H$ is the water level (m).

When the sharp-crested weir has a 45° degree roof angle, the equation to calculate the flux is

$$Q_s = 0.58247H^{2.5}$$  \hspace{1cm} (2.3)

In a 90° case, the equation to calculate the flux is

$$Q_s = 1.4H^{2.5}$$  \hspace{1cm} (2.4)
2.4 Method to Calculate the Amount of Water and Sediment Discharge

where

\( Q_s \) is the flux of water discharge (m\(^3\)/s) ; and
\( H \) is the water level of a triangular weir with 45\(^\circ\) roof angle (m).

Using integral calculus from Eq. (2.3), Eq. (2.5) is obtained:

\[
\int Q_s \, dt = \int 0.58247H^{2.5} \, dt \tag{2.5}
\]

Because in any one interval the water level line at the weir is straight, or nearly straight, Eq. (2.6) pertains:

\[
H_t = H_A + ((H_B - H_A)/t_i)t \tag{2.6}
\]

where

\( H_t \) is the water level at any time of the AB period (m);
\( H_A \) is the water level at the beginning of the AB period (m);
\( H_B \) is the water level at the end of the AB period (m);
\( t \) is the time during the AB period (s); and \( t_i \) is the total time of AB period (s).

In Eq. (2.6), \((H_B - H_A)/t_i\) is the slope of a line. Let this be represented by \( K \); then Eqs. (2.7) and (2.8) are obtained:

\[
\int Q_s \, dt = \int 0.58247H^{2.5} \, dt = \int 0.58247(Kt)^{3.5} \, dt
\]

\[
\int_0^t Q_s \, dt = 0.58247 \times ((2/7)(1/K)(H_A^{3.5} - H_A^{3.5})) \tag{2.8}
\]

Similarly, Eq. (2.9) is used to calculate the flux for a 90\(^\circ\) sharp-crested weir:

\[
\int_0^t Q_s \, dt = 1.4 \times ((2/7)(1/K)(H_t^{3.5} - H_A^{3.5})) \tag{2.9}
\]

For a water level over the crested weir at the river outlet, which is made of a 90\(^\circ\) sharp-crested weir and a wide concrete weir, the total flux consists of two parts: the flux from the 90\(^\circ\) sharp-crested weir and that from wide concrete weir. Eq. (2.10) (Dong, 1995) is used to calculate the flux:

\[
\int_0^t (Q_s + 0.26) \, dt = \int_0^t (a(bH + H^{1.5}) \, dt + \int_0^t 0.26 \, dt \tag{2.10}
\]
$Q_s$ is the flux from the concrete weir (m$^3$/s); the value 0.26 represents the flux of water from the 90° sharp-crested weir, when the water level begins to spill over the weir (m$^3$/s); and $a$ and $b$ are parameters of the concrete weir, which are 25.734 and 0.933, respectively.

Figure 2.4 depicts the difference between the amount of runoff calculated using the averaging method and the integral calculus method. The amount of water discharge calculated using the averaging method is higher than that by the integral calculus method. In addition, the error increases with the variance of water level. For example, Table 2.2 shows the example of a storm on 15 Aug. 2000. When the water level varied from 9.3 to 39.2 cm in a 10-min interval, the amount of water discharge calculated by the averaging method was 9.96 m$^3$, which was 68.8% more than that calculated by the integral method. The total error on this day was 15.67 m$^3$. Table 2.3 shows that the integral calculus method can improve the accuracy of annual amount of water discharge by 8.7–17.1% at the five monitoring sites using the data in the year 2000.

Table 2.2 Comparison between values of water discharge calculated by two methods in the subcatchment of cultivated land on 15 Aug. 2000

<table>
<thead>
<tr>
<th>Date</th>
<th>Time (min)</th>
<th>Rain (mm)</th>
<th>Water level (cm)</th>
<th>$Q_s$ (m$^3$/s)</th>
<th>$Q_{av}$ (m$^3$)</th>
<th>$Q_{int}$ (m$^3$)</th>
<th>Error (m$^3$)</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aug. 15th</td>
<td>0</td>
<td>1.4</td>
<td>0.00000</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>9.3</td>
<td>39.2</td>
<td>0.05604</td>
<td>16.82</td>
<td>9.96</td>
<td>6.86</td>
<td>68.84</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>9.5</td>
<td>51.3</td>
<td>0.10979</td>
<td>49.75</td>
<td>48.63</td>
<td>1.12</td>
<td>2.29</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>18</td>
<td>35.34</td>
<td>0.00801</td>
<td>35.34</td>
<td>28.26</td>
<td>7.08</td>
<td>25.04</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>5</td>
<td>2.50</td>
<td>0.00033</td>
<td>2.50</td>
<td>1.88</td>
<td>0.62</td>
<td>32.93</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>104.4</td>
<td>88.74</td>
<td></td>
<td>15.67</td>
<td>12.6</td>
<td></td>
</tr>
</tbody>
</table>

$Q_{av}$ is the amount of water discharge of a period calculated using the averaging method (m$^3$) and $Q_{int}$ is the amount of water discharge of a period calculated using the integration formula (m$^3$)

Table 2.3 Comparison between annual fluxes calculated by the two methods of various land use in the year 2000

<table>
<thead>
<tr>
<th>Land use</th>
<th>Cultivated land</th>
<th>Chinese fir forest</th>
<th>Pine forest</th>
<th>Tea garden</th>
<th>River outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{av}$ (m$^3$)</td>
<td>2445.4</td>
<td>911.5</td>
<td>932.7</td>
<td>956.7</td>
<td>902, 123.5</td>
</tr>
<tr>
<td>$Q_{int}$ (m$^3$)</td>
<td>2211.7</td>
<td>797.1</td>
<td>796.8</td>
<td>872.2</td>
<td>830,067.3</td>
</tr>
<tr>
<td>Error (m$^3$)</td>
<td>233.8</td>
<td>114.4</td>
<td>136.0</td>
<td>84.5</td>
<td>72,056.2</td>
</tr>
<tr>
<td>%</td>
<td>10.6</td>
<td>14.4</td>
<td>17.1</td>
<td>9.7</td>
<td>8.7</td>
</tr>
</tbody>
</table>

$Q_{av}$ is the amount of water discharge of a period calculated using the average method (m$^3$) and $Q_{int}$ is the amount of water discharge of a period calculated using the integration formula (m$^3$)
2.4.2 Method to Calculate Sediment Load

Equation (2.11) is used to calculate the sediment discharge at the sub-catchment and catchment scale:

\[ SD = \sum_{i=1}^{n} SD_i = \sum_{i=1}^{n} C_i Q_{fi} \]  

(2.11)

where

- \( SD \) is the sediment discharge in the whole period (kg);
- \( SD_i \) is the sediment discharge in the \( i \)th period (kg);
- \( C_i \) is the average suspended sediment concentration in the \( i \)th period (kg m\(^{-3}\)); and
- \( Q_{fi} \) is the amount of water discharge calculated by integral equation in the \( i \)th period (m\(^3\)).

The sediment discharge was transformed into specific sediment discharge using the following equation:

\[ SSD = SD / S_A \]  

(2.12)

where

- \( SSD \) represents specific sediment discharge (t km\(^{-2}\));
- \( SD \) is the sediment discharge from each sub-catchment or catchment (ton, t); and
- \( S_A \) is the area of each sub-catchment or catchment (km\(^2\)).

Because the sediment discharge was calculated with the same SSC data at each monitoring sites in the Shangshe catchment, application of the integral calculus method can also improve the accuracy of the annual amount of sediment discharge by 8.7–17.1%.

2.5 Comparison of Soil Loss Among Various Types of Land Use

Using the USLE-plot field observation data in the Shangshe catchment, annual soil loss from different land types (pine forest, Chinese fir forest, cultivated land, and tea garden) was calculated (Fig. 2.5). Figure 2.5 shows farmland that was planted with wheat from November to June in the second year and corn from June to November, of which during the growing season, the coverage was more than 80%. During the harvest–planting–seedling period (after mid-June to early August and early November to late December), the plant cover is close to 0. In the year 2000 in the planting–seedling period of corn (June through August), because of highly
intense precipitation, soil erosion was very serious in the area, with runoff and erosion occurring when daily rainfall reached close to 10 mm. Soil loss in the year 2000 amounted to 35,000 t km\(^{-2}\). In the years of 2001 and 2002, because in the planting–seedling period, rainfall was reduced by 30–50% compared to that of the year 2000, soil erosion was correspondingly reduced by the amounts of 30–50%. In year 2003 a project of returning farmland on slopes to forests started in the sub-catchment of cultivated land. Poplar trees and ginseng were planted. Then a large decline in soil erosion was shown. The status of the sub-catchment of cultivated land before and after “the returning farmland to forest project” is shown in Photos 2.3, 2.4, and 2.5. Figure 2.5 shows that in the year 2005, despite heavy rainfall, the annual amount of soil erosion was still less than 4000 t km\(^{-2}\). In the years 2006 and 2007, the amount of soil erosion dropped to 504 and 309 t km\(^{-2}\), respectively. The effects of soil erosion control increased with the coverage of mulberry trees (\textit{Morus alba}) and poplar trees (\textit{Populus euramericana}) (Photo 2.5), which is very significant. In year 2005, in the runoff of a plot of mixed young Formosan sweet gum (\textit{Liquidambar formosana}) and pine afforestation, due to higher rainfall erosivity, soil erosion reached 4000 t km\(^{-2}\). In years of 2006 and 2007, due to the percentage of vegetation coverage increasing to 45 and 70%, on-site runoff and soil loss decreased significantly (Fig. 2.6) so that soil loss was reduced to 774 and 305 t km\(^{-2}\), respectively.

Soil erosion in the sloping land of a mixed young maple and pine plot and the control plot was much higher than that of tea, Chinese fir, pine forest, and bamboo forest. In order to better explain soil erosion from tea garden, Chinese fir forest, pine forest, and bamboo forest, in the years of 2000–2007, a detailed soil erosion comparison of four types of land use was made (Fig. 2.6). Figure 2.6 shows that soil erosion in the year 2001 was 55 t km\(^{-2}\) in the tea garden, much higher than other years (which was less than 10 t km\(^{-2}\)), mainly due to the grass cutting and
tending during the rainy season. For the Chinese fir forest, pine forest, and bamboo forest, the annual soil losses were less than 10 t km\(^{-2}\), except for the pine forest in 2005, the annual amount of which was 10.9 t km\(^{-2}\). Soil loss in the forest was very low because the multi-layer vegetation of forest intercepted rainfall, weakened the kinetic energy of the raindrops, and decreased the direct impact of the raindrops on
soil. At the same time, due to better soil structure for infiltration, and particularly due to the litter, a considerable part of the surface runoff was absorbed into the ground floor. The small amount of sediment loss of forests was also due to the well-developed root system for soil erosion control.
2.5 Comparison of Soil Loss Among Various Types of Land Use

Photo 2.6  A USLE plot of maple cross pine tree (2008.7)

Photo 2.7  Pine seeding entered the control plot and grew well (2008.7)
In years 1999 and 2000, the bush in the control plot became bare land. Serious soil erosion occurred when daily precipitation reached 7–8 mm. Soil loss in the year of 2000 amounted to 35,000 t km\(^{-2}\). After 2001 natural regeneration of vegetation gradually occupied the controlled plot and coverage increased to 70 and 80\% (Photos 2.5, 2.6, 2.7, and 2.8) by the years of 2006 and 2007, respectively, the amount of soil erosion decreased significantly to 305 and 303 t km\(^{-2}\).

References

Fu J (1998) Hand book for carrying out hydrological and environmental field observations in Anhui Province. Department of Forestry, Hefei, China

![Photo 2.8](image-url) Students of soil and water conservation of Nanjing Forestry University were making investigation of pine biomass in the control plot (2008.8)
Theory and Practice of Soil Loss Control in Eastern China
Zhang, Y.; DeAngelis, D.L.; Zhuang, J.Y.
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