Preface

Ranking issues are found everywhere! For example, chemicals can be as harmful to humans and the environment as they are useful. Therefore, it appears rather clear that only those chemicals should be used in the market that do not have an adverse impact on humans and the environment. How do we find out whether they are hazardous? There are many time-consuming and expensive investigations necessary to perform a risk assessment. Hence the question is: With which chemicals should we begin at first? Thus a ranking can be performed to give the more involved investigations a reasonable operating sequence. Once accepted that a ranking is needed, we discover that there is no intrinsic property of a chemical which tells us that it is hazardous. Still worse, one needs to know the hazard of chemicals in different scenarios. Hence, several aspects of a chemical need to be simultaneously considered. And thus the final and central question arises: How to rank chemicals characterized by several attributes.

Child well-being: In a report of UNICEF, a ranking of 21 rich nations was performed with respect to child well-being. For this purpose, six attributes were finally constructed by which the countries were ranked. It is clear that each of these six rankings need not be the same. Therefore, a composite indicator was defined, giving each of the six indicators the same weight. How far is this justified? What influence does this kind of aggregation have on the final result? Italy, for example, could get a better position if the indicator "family" would get more weight on the index. How can we analyze the role of weights?

Integrity of watersheds: Scientists of the Atlantic Slope Consortium (ASC) developed three levels of indicators to describe the health of watersheds. The indicators of the three levels increase in quality and accuracy of the data as well as the amount of cost and efforts needed to obtain the data. An important question is about how well level 1 or level 2 indicators perform compared with level 3 indicators. Partial order can help with this question.

Surface water management strategies: High concentration of nutrients in surface waters is of much concern for environmental protection agencies. What could be done to improve the situation? Clearly one has to study the release paths by which nutrients enter the surface waters. Then one has to develop strategies to control these and limit the emissions into surface waters. How well do such strategies work? In a chapter, we analyze 15 management strategies developed to reduce the concentration of nitrogen in the surface waters of the river. Each strategy is characterized by eight indicators according to the path through which nitrogen enters the surface water. Is there a best strategy? Can we compare, for example, rural and technical strategies? Unfortunately they turn out to be incomparable when partial order is applied. However, we use tools to facilitate the comparison without having to develop weights for all the eight indicators.

In this monograph, we discuss the following main topics of ranking, applying partial order theory:

1. Starting from a data matrix, partial order analysis renders methods to get insights into ranking without crunching the indicators by subjective weights into a composite indicator.
2. Partial order in terms of formal concept analysis renders knowledge about ordinal implication structures inherent in the data matrix.
3. Attributes may serve as proxies for a certain abstract concept. We analyze how well these attributes describe this unknown nevertheless desired concept.
4. Compared with composite indicators, partial order analysis enables comparability knowledge discovery.

These topics need a broad base and this book attempts to provide that. It is not a mathematical textbook about partial order in general but rather discusses how far simple partial order methods can be useful for the ordinal analysis of data matrices. Therefore, this monograph provides in the first 10 chapters axioms of partial order and some basic material, for example, consequences of "crisscrossing" of data profiles, the role of aggregations of the indicators, and the powerful method of formal concept analysis. The interested reader will learn how to apply fuzzy methods in partial order analysis and what "antagonistic indicator" means.

An exclusive chapter dwells on the concepts through some illustrative case studies. For example, we apply fuzzy partial order methods on biomanipulation in lakes. We consider chemicals and compare regions with respect to their pollution. We ask what to do to improve the ecological status of communes. Management strategies are compared to help improve the water quality in a river basin. One illustrative case study is concerned with the quality of information services for drinking water quality and another one with the human environment interface index. The role of a fish species for ranking creeks in a wetland is discussed, and we render how a Hasse diagram "sees" the ecological role of this fish species.

Ranking of complex, multifaceted items is often done by means of composite indicators. So, nowadays and we think in the future, composite indicators will be constructed and used more and more. Composite indicators tend to sit between advocacy (when they are used to draw attention to an issue) and analysis (when they are used to capture complex multidimensional phenomena), to quote Saltelli. Using composite indicators, objects of interest can be compared and an important application is to deduce through the scalar values of the composite indicators a ranking of the objects.

We would like our reader to know of partial order in this connection. Partial order theory is a discipline associated with graph theory and discrete mathematics. Partial order theory is the theory by which objects, characterized by multiple indicators, can be compared and ordered. Partial order as the theory of order is applied to the set of objects and it delivers insights which result in appropriate ranking of objects.

We can derive a measure by which the set of indicators can be checked for its appropriateness and completeness as a proxy for a non-measurable nevertheless important aim. The composite indicator depends on the functional form and especially if a linear combination is selected, it depends on the weights. Independent of whether there is uncertainty about the functional form or the weights, partial order can derive subsets of objects whose relative rankings are invariant with respect to the functional form selected or the weights. In this connection, a key concept in partial order theory is that of a chain. Because of the averaging process in the weighted sum, the individual role of a single indicator cannot be easily traced back. Partial order theory offers some tools to overcome this difficulty. These tools are developed within the context of stepwise aggregation: Start with an indicator, add the next, and see what happens until the composite indicator is finally attained. The averaging process, may affect objects in different ways if weights are uncertain. Uncertainty concerning the weight values results in a rank interval indicative of ambiguity in the ranks of objects. Partial order theory provides an upper limit for the ranges of ranks of objects. We also conduct a Monte Carlo simulation in changing the weights and in observing the corresponding rank frequency distributions for the objects in response to the varied weights. The crucial role and the consequence of weights are well known. Therefore, we offer a method to deduce weights from the data matrix alone, where the rows are defined by the objects and the columns by the indicators. Partial order theory also offers several methods to obtain linear orders of objects (with or without ties). Therefore, one of these partial order methods could be selected if a ranking is desired that does not need to weight the indicators. We may even compare the ranking due to a composite indicator with that obtained from partial order theory, if there are no uncertainties.

Partial order can work even if the data matrix consists of indicators with different scaling levels. This kind of a situation occurs often in scientific fields where quantitative measures are difficult to obtain. Partial order, in its own right, delivers insights to understand the impacts of indicators on the objects in a multi-indicator system. Such results are important, especially when crunching the indicators into a composite indicator is not an option, as it was the case in a study of pollution. There the measurements of single indicators were so expensive that an averaging into a composite indicator was seen to be too disadvantageous.

The Hasse diagram, which is a graph theoretical visualization of a partially ordered object set, is an ideal tool if the number of objects is not too large. Striking feature in Hasse diagrams is the concept of incomparability, which appears if the order of objects due to one indicator contradicts the order of another indicator. Chains can be easily identified from a Hasse diagram. If the Hasse diagram is too messy to get chains by inspection, software tools help find chains. Many concepts can be motivated just by discussing them within a simple Hasse diagram but are still valid, even when the Hasse diagram loses its visual appeal, because it is too complex and messy.

We are thinking of a reader who is concerned with ranking in the broadest sense. So we have in our mind stakeholders, statisticians, scientists, and instructors. We hope to render for them a monograph helpful in the application of tools, insightful with theoretical considerations and motivated by a series of case studies for further applications.

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http://www.springer.com/978-1-4419-8476-0

Ranking and Prioritization for Multi-indicator Systems
Introduction to Partial Order Applications
Brüggemann, R.; Patil, G.P.
2011, XXVI, 338 p., Hardcover
ISBN: 978-1-4419-8476-0