

Chapter 2

How to Measure the Temperature

The three main methods for measuring the temperature: by direct contact with a thermometer, by analyzing radiation spectra stemming from a thermal source, and by conjecturing from the observed chemical composition.

2.1 Thermoscopes and Thermometers

In order to develop or understand relevant mathematical models used to describe the laws of nature in theoretical physics it is of help to enlist first what the intuition says about the issue. Several of our concepts in science, in particular the basic thermodynamical quantities, like heat, energy, temperature, are also rooted in our perception – as a first, naturally given device to form our feelings and thoughts through our senses. Based on this, and following the push towards objectivity, man-constructed devices enlarge our sensory field and make different personal experiences more comparable at the same time. Finally, quantification and introduction of a standardized scale leads to the mature state of being a scientific (and technical) concept. Such concepts can be parts and actors of a scientific theory then, aiming at a logical system of explanations for phenomena which is free from the smallest contradiction.

This process takes its time. Historically, physics developed through several centuries to the science it is today. This is, in particular, true for the concepts related to thermal phenomena. In this section we deal with devices constructed and used for measurements of heat. Thermoscopes just react to the presence or transfer of heat, thermometers translate this process into a temperature scale. The very question of the arbitrariness of such scales has been answered in classical thermodynamics by pointing out the physical sense behind a universal, so called absolute temperature scale. This existence and constructibility statement is part of the problematics connected to the zeroth theorem, alias the zeroth law of thermodynamics.

2.1.1 Heat Perception

Human beings (and many others) do show sensitivity to heat, to this special form of energy change. Mostly – and most safely – radiated heat can be sensed; the best by hands or by the face. Since this happens without mechanical contact seen by our eyes or felt by our pressure sensitive cells – heat is considered to be something more mystical than the mechanical concepts, like force or energy. In fact, the interpretation of heat as a substance, the “caloricum,” was the predecessor theory to the “kinetic” one, interpreting heat as internal motion. Joseph Black worked out several important concepts of thermionic physics based on this – in retrospective abandoned – view. Among others he introduced quantities like specific heat, latent heat, melting and evaporation heat, and heat capacity [1].

Experiments have been designed to demonstrate and measure the heat substance. Benjamin Thompson (Lord Rumford) concluded from his experiments at the end of eighteenth century that the weight of heat, the change in weight by 1° temperature rise, has to be less than one to a million (10^{-6}). The final victory for the kinetic over the substantial interpretation of heat was brought by the formulation of the principle of equipartition, explaining not only the nature of heat, but interpreting both the principle of energy conservation and the irreversibility trend by dissipative phenomena in a unified framework. It is a remarkable irony of history that today we consider a mass equivalent to all type of energy, including the kinetic energy of microscopic (atomic) motion. This effect is, however, very small in the everyday life of SI units, $k_B T / mc^2$ being in the order of magnitude of 10^{-40} for 1° temperature difference in a body with a mass of one kilogram (about 2 pounds). The situation is quite different in particle physics, where this ratio approaches one in some situations.¹ For the latter it is unavoidable to work out a relativistic theory of heat and thermal phenomena.

Radiated heat is frequently accompanied by visible light; and from the modern physics we know that both light and heat radiation are in fact mediated by photons – just a little different in energy. This issue touches the question about the very nature of light: is it an electromagnetic wave or a corpuscle called photon? In fact both views led to predictions of radiation spectra of so called absolute black bodies – bodies that emit the same spectra of radiation they absorb. Max Planck has established his formula for the black body radiation spectrum by interpolating the entropy-energy relation, $S(E)$, between these complementary views [2, 3].

Also by touching we can sense whether a body is warm or cold; whether it is warmer or colder than oneself. Being in water cold and hot streams can be sensed quite sharply. In this case we meet with the flow nature of heat: it circumvents our sensory device (the skin), it flows from warmer to colder places. This flow is sometimes steady, sometimes turbulent. Already in the Middle Ages became clear that

¹ For example, pions have $mc^2 = 140$ MeV and experience in heavy ion collisions carried out in modern accelerators a temperature around $k_B T = 120 - 160$ MeV.

heat-related, or shortly thermal phenomena are associated to two main properties: one is more like an intensity (modern physics names the temperature as an intensive variable), the other is more like a quantity, increasing with the extension of the hot body (so called extensive quantity). The favorite example is flame: its thermal intensity is greater than that of a piece of hot iron, while the latter carries more substance of heat (and causes more damage to us than a flame at the same temperature).

And finally heat conduction, the third form of heat transfer, can be detected by our senses: the far better conductivity of metals is responsible for the fact that by touching them they appear appreciably colder than an insulator, like wool or wood or human skin, at the same temperature. This process leads much faster to thermal equilibrium than the previous ones, especially by radiative contact we never expect to be equilibrated to the temperature of the source (e.g. the Sun) in our lifetime.² And still, thermodynamics states that the thermal equilibrium is universal in the sense, that this state is independent of the way, of the material consistence and of the speed of changes by which we arrive at it. Furthermore, once equilibrium is achieved, it will also be maintained – at least in lack of serious disturbances.

Such a behavior really can only be understood on the basis of coupling our macroscopic, from the world of senses and human made human size devices stemming information to assumptions and models about the microscopical behavior of matter: the kinetic theory of heat and the concept of entropy as information about microscopical order and disorder are rooted very much in the above universality and maintenance properties of a thermal equilibrium state.

2.1.2 The First Thermometers

The first devices were just sensitive to the change of heat without actually measuring it, the so called thermoscopes. Thermometers connect this sensitivity to a scale; to a quantitative measurement between two fixed physical points [4].

Most devices make use of a physical sensitivity to heat, such as dilation (the common thermometers using mercury or alcohol), or change in the electric properties (digital thermometers). Phenomena related to dilation of air and vapor reacting to heat were already known in ancient civilizations. There are notes about Héron of Alexandria, who experimented with devices making use of the force of heat. Huge and heavy temple gates were secretly opened by the use of the work exerted by heat.

The use of dilation for the measurement of heat intensity – the temperature – had a long technical evolution. Reproducibility requires that such devices become robust, but still delicate enough to react fast to changes and delicate enough not to influence the measured object unduly. Instead of air water, later alcohol became the favorite signal material, just to give room at the end to mercury. First, thermoscopes

² Actually, one leaves the sunny spot before his/her own temperature becomes uncomfortable.

constructed in the renaissance Italy were open systems reacting to air pressure as well as to temperature. Such *baro-thermoscopes* were also constructed by Galilei and Toricelli, later by the Bernoullis – just to mention a few famous names.

The device known nowadays as “Galilei thermometer” was invented by Ferdinand II, Grand Duke of Tuscany. In a closed glass tube, filled with a mixture of water and alcohol, small, colored vesicles of different density swim. Depending on the temperature more of them float at the top and some of them sink to the bottom. The temperature is calibrated to the middle one. This device is already a thermometer, since numbers, measures of temperature, are associated to each floating sphericle. This device usually measures temperatures with a resolution of 2°C (cf. Fig. 2.1).



Fig. 2.1 So called Galilei Thermometer – actually invented by Ferdinand II Grand Duke of Tuscany

It is not clear who was the first constructor of a thermometer. Tradition names Galilei as well as Santorio, Avicenna, Cornelius Drebbel or Robert Fludd. Fludd's thermometer, introduced in 1638, uses a vertical tube with a bulb at the top, the other end immersed into water. The basic design with a bulb and a tube remained til the modern times; it separates the "reaction zone" in the bulb from the "read-out zone" in the tube. The first who put a scale besides the tube could have been Francesco Sagredo or Santorio.

The standardization of the temperature scale also has a long history. Christian Huygens suggested to use the melting and boiling of water as two characteristic points to fix the scale in 1665. In 1701, Isaac Newton proposed to use 12° between the melting of ice and the body temperature. The sexagesimal and the decimal system fought long. In the continent the decimal metric system has been established after the French revolution, while in England the dozen- and sixty-based counting remained more common. Regarding the temperature scales this evolution peaked in the Celsius (also called centigrade) and the Fahrenheit scales.

2.1.3 Réaumur, Fahrenheit and Celsius

Thermometers using the physical phenomenon dilation (of alcohol or mercury) attach a scale to the tube. The points on this scale has to be fixed. Assuming linearity – what is behind almost all scales in use – actually two points would suffice. Two dramatic and easily reproducible physical events can serve well to fix a temperature scale. More points may serve to control the linearity of the dilation subsequently. The number of subdivisions are absolutely arbitrary, different suggestions were made, thought to be "natural" for the contemporaries.

René Antoine Ferchault de Réaumur (1683–1757) suggested in 1731 a temperature scale using an octogesimal division between the freezing point of water (zero point, $0^\circ R$) and its boiling at normal atmospheric pressure ($80^\circ R$). The grads were designed to belong to one thousandth change of the volume contained in the bulb and in the tube up to the zero mark. The choice of 80 was quite natural for the French, especially before the Revolution, when they introduced the decimal metric system. Eighty has several divisors among whole numbers: $80 = 2 \times 40$, 4×20 , 5×16 and 8×10 . This helps for fast calculations and rapid perception.

Daniel Gabriel Fahrenheit proposed in 1724 his 96-based system. By using mercury filling, a finer grading was demanded. He fixed the scale to the melting point of salty ice at $-18^\circ C$ centigrades and to the human body temperature at $+36^\circ C$ centigrades. These are the zero point, $0^\circ F$ and the upper end, $96^\circ F$. Again these numbers are easily divided by a number of divisors: $96 = 2 \times 48$, 3×32 , 4×24 , 6×16 and 8×12 .

The scale suggested by Anders Celsius in 1742 takes one hundred subdivisions between the freezing and boiling point of water at normal atmospheric pressure. This became part of the metric system during the French revolution in 1790, and is in use all over the world today. Owing to the different fixing points and numbers of subdivisions, there are linear formulas for transforming temperatures between these scales. Obtaining Fahrenheit temperature from the Celsius one, the linear equation

$$F = aC + b \quad (2.1)$$

has to be fixed at two points. Zero Fahrenheit belongs to -18 centigrades, while 96°F to 36 centigrades:

$$\begin{aligned} 0 &= -18a + b \\ 96 &= 36a + b. \end{aligned} \quad (2.2)$$

Subtracting from the second line the first one we get $96 = 54a$, so the proportionality coefficient becomes $a = 96/54 = 16/9$. This value is often approximated by $a \approx 9/5$ intending to facilitate fast computation by heart ($16 \times 5 = 80$ while $9 \times 9 = 81$, it makes an error faintly larger than 1%). Substituting the result for a into the first line, the parameter b can be obtained as being $b = 18 \times 16/9 = 2 \times 16 = 32$. Finally the transformation formula is given by

$$F = \frac{16}{9}C + 32 \approx \frac{9}{5}C + 32. \quad (2.3)$$

Perhaps it is easier to remember a few special values at centigrades divisible by nine. They are collected in the following table (Table 2.1).

F	0	16	32	48	64	80	96
C	-18	-9	0	9	18	27	36

Table 2.1 Easy to remember values in the Fahrenheit and Celsius temperature scales

This table is very simple if the Fahrenheit values are written in hexadecimal (16-based) number system and the Celsius values in a nonal (9-based) system (cf. Table 2.2). The fast computation can be based on a special form (2.3) emphasizing that Fahrenheit degrees are at best grouped into sixteens while Celsius degrees into groups of nines:

$$\frac{F}{16} = \frac{C}{9} + 2 \quad (2.4)$$

As a consequence in both the nonal and the hexadecimal system the shift is two times the base, “20” (meaning $2 \times 16 = 32$ or $2 \times 9 = 18$ respectively).

${}_{16}\text{F}$	0	10	20	30	40	50	60
${}_{9}\text{C}$	-20	-10	0	10	20	30	40

Table 2.2 Hexadecimal and nonal number system values in the Fahrenheit and Celsius temperature scales

Since then the technology of thermometers undergone a process of refinement and diversion. The handy size clinical thermometer we know today was introduced in 1866. It delivers a result in five minutes with a precision of 0.1°C . There can be several sources for a thermometer being imprecise. The calibration has to be done with great care, since pressure influences the value of temperature. Distilled water at ice melting and boiling is regarded at standard atmospheric pressure according to the actually valid international standard. The linear interpolation between the calibrated points also may depend on the material used for dilation. The mercury in a glass tube may show the maximal deviance from the value measured by the electric resistance of platinum in the middle of the scale, at 50°C . Due to glass-technology a variation in the diameter of capillaries also cannot be excluded.

Modern electric thermometers, like the platinum resistance thermometer, has a resolution of $\Delta T = 0.1^\circ\text{C}$ and is calibrated at five points at $-18, 0, 40, 70$ and 100 centigrades. At the interpolation points it reaches an accuracy of $\pm 0.2^\circ\text{C}$. For scientific purposes and in the industry several other thermometers are in use. Infrared thermometers are very good at telemetry: they measure spot temperatures at a distance. They are particularly useful for measuring high temperatures (like in metal industry) or temperatures of moving objects. Their scale is based on the black body radiation formula; for shiny or gray surfaces corrections have to be made (usually included in the software). It is also of theoretical interest, the temperature of moving bodies is related to the relativity principle and will be discussed in some detail in Chap. 6.

Bi-metallic stemmed thermometers (so called thermocouples) are used in food-industry, thermistors (electronic devices with temperature sensitive resistance) by cooking and baking. Modern electronics and solid state physics also have developed a number of smart thermometers. Liquid crystal thermometers are in clinical and household use. Temperature measurement based on radiance is the principle behind phosphor thermometry. This plethora of methods and technologies is rather overwhelming than reassuring. Which is the correct temperature? Must physics depend on so many circumstances? Melting and boiling of one, dilation or electric conductivity of another material? One would very much welcome a universal temperature.

2.1.4 The Absolute Temperature

The above wish did not remain without fulfillment. Although the temperature scale is still arbitrary, there is an exclusive zero point on physical grounds. In order to

explain this fact a correspondence between the energy, as contained in internal microscopic motion, and the temperature, as an intensity property of this motion, had to be established. Furthermore also the heat, describing the substantial component in thermal phenomena had to be understood.

Studies about the nature of heat led to the formulation of energy conservation. The absolute temperature scale is zero when the internal motion is at its minimum: in the classical physics this energy is zero, in the quantum mechanics (established later than the introduction of the absolute temperature scale) a small zero point motion is present. In this stage the order is maximal, the number of ways of realizing this macrostate are minimal. The absolute zero point turned out to be at about -273°C . The absolute scale is in centigrades, just the starting point, the “absolute zero” differs from the Celsius scale. This temperature is named after Lord Kelvin (Thomson) and is denoted by “K”. Since the state of thermal equilibrium is universal, the “absolute” temperature is also universal in the sense of being independent of the material consistence [5].

According to present international agreement the Kelvin scale is fixed to two points: the absolute zero 0K is at -273.15°C and the value 273.16K at the triple point of standard water with a specific mixture of hydrogen and oxygen isotopes. This triple point, where ice is melting, is at 0.01°C . Practically this definition fixes the absolute temperature to be measured by the same scale as the Celsius one, just shifted by a constant amount of 273.15.

In physics, the absolute Kelvin-temperature is the only sensible temperature to talk about. It is often cited in the equivalent energy units using the Boltzmann constant:³ $k_B \approx 1.38 \times 10^{-23} \text{m}^2 \text{kg s}^{-2} \text{K}^{-1}$. In particle physics, often “temperature” is written but the energy $k_B T$ is meant.

2.2 Spectral Temperature

The classical concept of temperature rests on the concept of thermal equilibrium. The first thermometers and the first problems discussed in thermodynamics – stemming from demands to solve problems in everyday life and industry – were based on direct contact between large bodies; under such circumstances thermal equilibration happens fast. While a perfect reservoir has an infinite ability to give or absorb energy and heat (it has an infinite heat capacity) and therefore it keeps its own temperature during a thermal contact with other objects, a perfect thermometer on the other hand reacts immediately to changes in temperature and its own temperature equals to the temperature of the attached body (it has zero heat capacity). Building on these ideal properties, thermodynamical theories assume large objects to be investigated. So large that they themselves can serve as a perfect reservoir for their smaller parts. Whenever this approximation becomes physically meaningful, we consider the so called *thermodynamical limit*. Please note, that this definition of

³ This name was actually given by Max Planck.

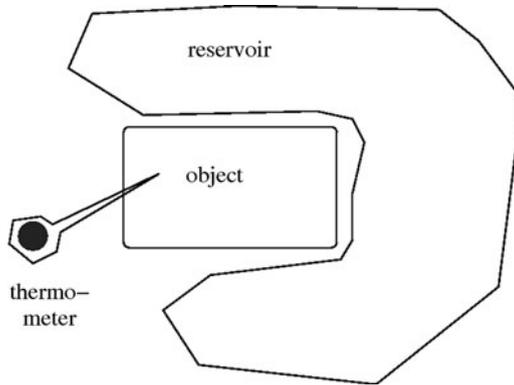


Fig. 2.2 Size relations between thermometer, object and heat reservoir. In the thermodynamical limit, all are realized in the same system

the thermodynamical limit is not restricted to large volume or large particle number only; if the behavior as its own heat reservoir is established by any means, this limit is considerable. As the reverse of this coin, a small part of a large enough object may serve as a thermometer. In ideal thermal equilibrium, it must have the same temperature as the rest of extended body, signals received from such a local spot in principle reflect information about the whole equilibrated object (cf. Fig. 2.2).

The direct and stationary contact between object and thermometer is a sufficient but not a necessary condition for a temperature measurement. Measuring the temperature from a distance is based on radiation. In this case, one assumes that the spectrum (energy distribution) of the radiation is characteristic to the emitting source and essentially was not distorted in its way to the detecting device. In the most common cases of astronomical spectroscopy this assumption is quite natural, since the outer space is very rarely polluted by objects. Yet, sometimes, it happens that some cloud, plasma or strong magnetic field shields the observed object from us and then the radiation can become distorted from its original shape. Fortunately, detecting and measuring a large enough part of the energy distribution in a radiation itself reveals whether it stems from an ideal object, called black body, at one fixed temperature, or not.

In order to rely on temperature measurements by spectral analysis, one utilizes a background knowledge established during the past two centuries. It includes a number of “laws” associated to different facets of thermal electromagnetic radiation, converging to the (quantum-)statistical theory of photons. In the next subsection, we follow this path and gradually introduce the most important concepts underlying the black body radiation. Then, analog to the statistical model of photons, we consider other particles. The main question to be discussed is how their spectral distribution reflects the temperature of the emitting source (provided no disturbance between emission and detection).

2.2.1 Black Body Radiation

Measuring temperature by radiation has the enormous advantage that it can be done from a distance. The theory behind radiation, however, also had to be developed first. An object, emitting radiation, first of all has to be in thermal equilibrium for the applicability of thermodynamics, otherwise we would already fail at the level of the zeroth law. The thermal equilibrium state of a radiating object is defined by Kirchhoff's law, formulated by Gustav Kirchhoff in 1859: the emitted and absorbed energy by the radiation must be equal. And not only as a total amount, but in detail: at each energy of the photons. Since the concept of photons was not yet established that time; the experience of equality was formulated in terms of wavelength. In each small interval around a selected wavelength, emission and absorption are equally efficient by a body in thermal equilibrium. The wavelength, λ and the (circular) frequency, ω are related by $\omega = 2\pi c/\lambda$ and $d\omega = -2\pi c d\lambda/\lambda^2$. A brief formulation of Kirchhoff's law is given as

$$\text{emissivity}(\omega) = \text{constant} \times \text{absorptivity}(\omega)$$

This is the *detailed balance* principle applied to radiation. The fraction of incident power (energy rate in time) may also depend on the angle; for the sake of simplicity it is often left out from the physical discussion. The laws of thermodynamics must be valid in general, therefore it is frequently enough to consider only the isotropic case.

An object fulfilling Kirchhoff's law is an absolute *black body*, it is an ideal, theoretical object. But reality is surprisingly close to this ideal. As a corollary to Kirchhoff's law, the emissivity of a real body cannot be higher than that of a black body: in thermal equilibrium the entropy is maximal. Kirchhoff's law can be interpreted as a detailed balance between a box with an emitting and absorbing wall filled with electromagnetic radiation. Denoting the frequency distribution of the black body radiation inside the box at (absolute) temperature, T , by $B(\omega, T)$, the energy incident to the wall defines the absorptivity coefficient, $A(\omega)$:

$$E_{\text{in}} = A(\omega)B(\omega, T). \quad (2.5)$$

Similarly, the emitted energy defines the emissivity coefficient, $\varepsilon(\omega)$:

$$E_{\text{out}} = \varepsilon(\omega)B(\omega, T). \quad (2.6)$$

The not absorbed power (energy per unit time) is called reflectivity, $R(\omega) = 1 - A(\omega)$. How big these frequency-dependent coefficients are, depends on the material quality of the wall; the ratio between emissivity and absorptivity is, however, a general statement about the thermal equilibrium of radiation. So is the *spectral distribution* of energy in the radiation, $B(\omega, T)$: It is determined by the temperature and

the detailed balance principle only. As we shall see later, it contains two constants of nature: Boltzmann's constant relating the absolute temperature to energy units and Planck's constant describing the relation between frequency and energy units.

Now we turn to the determination of the black body radiation spectrum, $B(\omega, T)$. At first global characteristics of the radiation power became known. The current density of emitted energy in thermal black body radiation of temperature T was discovered to be proportional to the fourth power, $J_e = \sigma T^4$, around the mid of nineteenth century. Based on experiments John Tyndall had found out that a body radiates at 1,473 K about 11.7 times more energy than at 798 K.⁴ Jozef Stefan realized in 1879 that $11.7 \approx (1,473/798)^4$. Stefan also determined the coefficient to be $\sigma = 5.67 \times 10^{-8} \text{ J sm}^{-2} \text{ K}^4$. Finally, Ludwig Boltzmann gave a thermodynamical explanation assuming ideal heat equipartition between matter and light in 1884. The coefficient σ is called "Stefan–Boltzmann constant." Another global description of the black body radiation is given by Wien's law, presented in 1893. It establishes a scaling relation between the frequency (wavelength) and temperature dependence and hence makes a prediction about the frequency of maximal intensity at a given temperature. According to Wien's law $\lambda_{\max} T$ is constant. Equivalently, ω_{\max}/T is also constant.

Approximate descriptions of the spectrum $B(\omega, T)$ of black body radiation were given by Wien, Raleigh, and Jeans. A radiation spectrum fulfilling Wien's law is emerging if the dependence is given as

$$B(\omega, T) = \omega^3 f(\omega/T), \quad (2.7)$$

i.e. the temperature dependence occurs only via the ratio ω/T . The prefactor ω^3 is set by the requirement that total (frequency integrated) energy current density (which is simply related to the total energy density by the velocity of light) stays in accordance with the Stefan–Boltzmann law:

$$\int \omega^3 f(\omega/T) d\omega = \sigma T^4, \quad (2.8)$$

which is easy to prove by using $x = \omega/T$ as the integration variable. The position of maximum is determined by setting the derivative $\frac{\partial B}{\partial \omega}$ to zero:

$$\frac{\partial}{\partial \omega} B(\omega, T) = 3\omega^2 f(\omega/T) + \frac{1}{T} \omega^3 f'(\omega/T) = 0. \quad (2.9)$$

Dividing this equation by T^2 we gain a condition purely in terms of ω/T :

$$3 \frac{\omega^2}{T^2} f\left(\frac{\omega}{T}\right) + \frac{\omega^3}{T^3} f'\left(\frac{\omega}{T}\right) = 0. \quad (2.10)$$

Its solution reveals the frequency of maximal intensity at a given temperature and reproduces Wien's law: ω_{\max}/T is constant.

⁴ Allegedly, Tyndall had not measured really a black body and the precise value should have been a factor of 18.6. But the $\sim T^4$ law was already established by the time of more precise measurements.

Based on this expectation Wien undertook a determination of the function $f(\omega/T)$ in 1896. He assumed that (1) the radiation law $B(\omega, T)$ can be connected to the Maxwell–Boltzmann distribution of the kinetic energy of particles in an ideal gas, and (2) this energy has to be replaced by an expression depending solely on the frequency of radiation. Choosing the simplest, i.e. a linear correspondence, he had guessed the following spectral law

$$B(\omega, T) = b\omega^3 e^{-a\omega/T}. \quad (2.11)$$

Max Planck had tried to establish this formula. He knew that the black body radiation is independent of material quality of the wall enclosing the radiation, it has to be determined by general principles valid in thermal equilibrium. Therefore he choose a simple model for the matter part: harmonic oscillators. These oscillators have eigenfrequency ω and the negative and positive charges, bound by these oscillators, are the source for the electromagnetic radiation in the box. The energy density of the radiation in the box and the average energy of oscillators become then in equilibrium:

$$B(\omega, T) = \frac{2\omega^2}{\pi c^3} E(\omega, T). \quad (2.12)$$

According to the classical equipartition theorem the average energy per degree of freedom is $k_B T/2$. Counting two independent polarization states of electromagnetic waves one considers $E(\omega, T) = k_B T$. This way

$$B(\omega, T) = \frac{2\omega^2}{\pi c^3} k_B T, \quad (2.13)$$

not having a maximum. This result is the Raleigh–Jeans radiation law, established in 1900. Raleigh obtained this result by counting standing waves around the frequency ω in a cavity.

Planck [2, 3] had found a way from the relation (2.12) to the Wien spectrum (2.11) from the following assumption about the entropy carried by the oscillators:

$$S(E) = \frac{E}{a\omega} \ln \frac{E}{be\omega}, \quad (2.14)$$

with e being Euler’s number. Applying now the definition $1/T = \partial S/\partial E$ one obtains Wien’s spectral law. Naturally, Planck had chosen the entropy formula for the oscillators just to arrive at the given spectral law. He was not satisfied with this result. Later, in 1900, he obtained his famous interpolating formula between the Raleigh–Jeans and the Wien spectra. Seeking for an entropy maximum, he inspected the second derivatives:

$$\begin{aligned} \frac{\partial^2 S}{\partial E^2} &= \frac{\text{const}}{E}, \\ \frac{\partial^2 S}{\partial E^2} &= \frac{\text{const}}{E^2}, \end{aligned} \quad (2.15)$$

for the Wien–Planck and Raleigh–Jeans distributions, respectively. Planck took the interpolating formula

$$\frac{\partial^2 S}{\partial E^2} = -\frac{a}{E(E+b)}. \quad (2.16)$$

This formula reconstructs Wien’s result for low average oscillator energy, $E \ll b$, at a fixed frequency. Likewise for high energy the Raleigh–Jeans result is obtained. For the general case Planck’s interpolating formula leads to

$$\frac{1}{T} = \frac{\partial S}{\partial E} = -\int \frac{a}{E(E+b)} dE = -\frac{a}{b} \ln \frac{E}{E+b}. \quad (2.17)$$

Inverting this result we obtain the average oscillator energy as a function of temperature

$$E = \frac{b}{e^{b/aT} - 1}. \quad (2.18)$$

Using Wien’s scaling form one obtains $E = \omega f(\omega/T)$, and due to this $b \propto \omega$ and $b/a \propto \omega$. Finally, relying on the equipartition formula, (and counting for the $d^3k \sim \omega^2 d\omega$ elementary phase space cells) Planck’s law for radiative energy density distribution is given as

$$B(\omega, T) = b' \frac{\omega^3}{e^{a'\omega/T} - 1}. \quad (2.19)$$

Here, a' and b' are frequency independent constants. Comparing the Planck law with the Stefan–Boltzmann law and Wien’s “maximum shift” formula a new constant of nature can be derived, today called Planck’s constant: $h = 6.55 \times 10^{-27}$ erg sec. Denoting by $\hbar = h/2\pi$, the constant a' is identified as being $a' = \hbar/k_B$. The scaling variable in electromagnetic radiation spectra is given by $x = \hbar\omega/k_B T$. Finally, Planck re-derived his formula based on permuting partitions of energy quanta of $E = \hbar\omega$ among oscillators. We introduce a similar derivation in the next chapter discussing the statistical foundations of thermodynamics.

Below we give a short summary of formulas related to these classical thermodynamical theories of black body radiation:

Stefan – Boltzmann	$cE/V = \sigma T^4$	global property	
Raleigh – Jeans	$\frac{\partial^2 S}{\partial E^2} = -\frac{1}{E^2}$,	$\frac{1}{T} = \frac{\partial S}{\partial E} = \frac{1}{E}$,	$E = k_B T$
Wien	$\frac{\partial^2 S}{\partial E^2} = \frac{a}{E}$,	$\frac{1}{T} = \frac{\partial S}{\partial E} = a \ln E$,	$E = b\omega e^{-\omega/aT}$
Planck	$\frac{\partial^2 S}{\partial E^2} = \frac{a}{E} - \frac{a}{E+b}$,	$\frac{1}{T} = \frac{\partial S}{\partial E} = a \ln \frac{E}{E+b}$,	$E = \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1}$

2.2.2 Particle Spectra

Following the advent of quantum theory by Planck's formula in 1900, it soon became interpreted as an ideal gas of photons by Einstein and Bose. The Planck constant, \hbar is the proportionality constant between the energy of a quantum, thought to be a particle, and the frequency of the radiation wave: $E = \hbar\omega$. Soon the wave – particle duality was extended to known elementary particles, first to the electron by de Broglie. Niels Bohr, in order to explain the stability of atoms inspite the fact that elementary charges, the electrons, seemed to move on curved (circular) trajectories around the nucleus and therefore should have lost their energy by electromagnetic radiation, as “normal” accelerating charges do, invented the quantization principle: the electrons may change their energy only by packets representing a finite value, which is related to the frequency of emitted or absorbed photon by the Planckian conjecture, $E = \hbar\omega$. De Broglie discovered that interpreting the electrons bound in atoms as standing particle waves with wave number, k (and a wavelength, $\lambda = 2\pi/k$), corresponding to their momenta as $p = \hbar k$, Bohr's quantization principle follows. This way there can be a general correspondence between particles and waves, relating energy and momenta to frequency and wave number. In particular a statistical counting of states of particles freely moving in a large volume, V , is equivalent to a counting of possible standing waves in a large cavity. This is the origin of the habit, that by analyzing modern accelerator experiments [6], the energy distributions of detected particles is called *particle spectra*.

Free particles in a large box constitute an ideal gas. The possible states are characterized by possible values of momenta, in a finite box it is a discrete spectrum related to standing sine waves. In the large volume limit, however, the sum over discrete momentum states is well approximated by integrals over continuous (often quoted as “classical”) momenta. As a reminiscent to the quantum theory the integral measure, the elementary cell in phase space, is normalized by the original Planck constant, $h = 2\pi\hbar$. This way particles freely moving in an “infinite” volume in all three spatial directions represent continuously distributed states with the following elementary measure (cell) in phase space:

$$d\Gamma = \frac{d^3p \, d^3x}{(2\pi\hbar)^3}. \quad (2.20)$$

Ignoring the Planck constant, this coincides with the classical phase space for the mechanical motion of mass points. In fact, particles in an ideal gas are treated as non-interacting ones; and they do so – most of the time. This peaceful existence is, however, interrupted by very short dramatic events, the collisions. Collisions play one important role: they ensure equipartition by exchanging individual particle momenta. For the macroscopic view this occurs as randomizing the momenta. This way they come close to the Gibbsean ideal: long term repeated observations of a macroscopic system pick up images of a randomized statistical ensemble.

The simplest, so called *thermal models* of particle spectra stemming from energetic collisions, consider an ideal gas of particles or particle mixtures at a given temperature [6–11]. This simple idea leads to predictions for the energy and momentum distributions of detected and identified particles in modern accelerator experiments. Slow particles, moving with velocities of negligible magnitude compared to the speed of light, can be considered as non-relativistic; their kinetic energy practically satisfies the $E = \mathbf{p}^2/2m$ relation. The distribution of energies, momenta and velocities resemble that of ordinary atomic gases at room temperature. Regarding the velocity distribution it is called Maxwell-distribution; as a distribution of energy the Gibbs distribution. Obtaining them from equal probabilities for phase space cells is due to Boltzmann; therefore, it is also called Maxwell–Boltzmann and Boltzmann–Gibbs distribution. In particle physics it is shortly called Boltzmann-distribution.

A particle with mass m , three-momentum \mathbf{p} and kinetic energy $E = \mathbf{p}^2/2m$ is distributed according to the Gibbs factor $e^{-E/k_B T}$ in a large volume at absolute temperature T . Using $k_B = 1$ units the differential probability to find a particle around the momentum \mathbf{p} (and anywhere in the coordinate space) is given by a Gaussian formula

$$f(\mathbf{p}, \mathbf{x}) = C e^{-\mathbf{p}^2/2mT} d\Gamma. \quad (2.21)$$

Here, C is a normalization constant related to the total particle number, $N = \int f d\Gamma$ in the large volume $V = \int 1 d^3x$. The above distribution, f , is the one-particle distribution giving the differential probability to find a representative particle near to a point in the phase space. This probability is solely a function of the energy of particle – such systems have a chance to be thermodynamic systems.

This distribution leads to a temperature dependent average particle density

$$n = \frac{N}{V} = C d^3 p e^{-\mathbf{p}^2/2mT}. \quad (2.22)$$

From here on we use the $\hbar = 1$ convention, counting frequencies in energy units. The above integral can be factorized into Gauss-integrals over the momentum components $\mathbf{p} = (p_x, p_y, p_z)$ and results in

$$n = C \left(\frac{mT}{2\pi} \right)^{3/2}. \quad (2.23)$$

The energy density can be obtained similarly by evaluating the integral

$$e = \frac{E}{V} = C d^3 p \frac{\mathbf{p}^2}{2m} e^{-\mathbf{p}^2/2mT}. \quad (2.24)$$

Instead of carrying out more complicated Gauss integrals it is enough to observe that

$$\frac{\partial}{\partial T} e^{-\mathbf{p}^2/2mT} = \frac{1}{T^2} \frac{\mathbf{p}^2}{2m} e^{-\mathbf{p}^2/2mT}. \quad (2.25)$$

This way we obtain

$$e = T^2 \frac{\partial n}{\partial T} = T^2 C \frac{3}{2} \left(\frac{mT}{2\pi} \right)^{1/2} \frac{m}{2\pi}. \quad (2.26)$$

This result simplifies to $e = 3nT/2$ revealing the formula for the average kinetic energy per particle

$$\frac{E}{N} = \frac{e}{n} = \frac{3}{2} T. \quad (2.27)$$

Since the particles move in three independent directions, this result conforms with the more general statement that due to the equipartition of energy in an ideal system to each degree of freedom of motion there is an average energy of $k_B T/2$.

The average energy per kinetic degree of freedom, $k_B T/2$ is a *measure of temperature* in ideal systems.

Finally, the pressure of an ideal gas can be obtained by using a relation derived from gas heating and expansion experiments: the Boyle–Mariotte law,

$$pV = NT \quad (2.28)$$

in $k_B = 1$ units. From here, using the general thermodynamical definition for the chemical potential μ as a variable associated to particle number, we get

$$n := \frac{\partial p}{\partial \mu} = \frac{p}{T}. \quad (2.29)$$

This means that the pressure must have the following form:

$$p(\mu, T) = e^{\mu/T} g(T). \quad (2.30)$$

Comparison with the result (2.23) leads to the conclusion that the normalization constant C also contains the factor $e^{\mu/T}$, and this is its only dependence on the temperature. All this, of course, is true only as long as the classical filling pattern of the phase space is assumed; the formula (2.30) is not valid when quantum statistics is considered. In particular it is not valid for radiation, for a quantum gas of photons.

In practical accelerator experiments the particle velocities are relativistic, they achieve a large portion of the speed of light. The Maxwell–Boltzmann statistics therefore cannot be applied for analyzing momentum spectra. Nor the Planck law is valid, since the particle mass and the characteristic temperature are also in the same order, $k_B T \sim mc^2$, especially for pions in high energy experiments. The hadronization temperature is around $k_B T = 170$ MeV, and the hadronic fireball does not cool down much before disintegrating into a non-interacting system of newly produced hadrons. Realistic estimates consider $k_B T = 120 - 140$ MeV for this “break up” temperature. The pion mass on the other hand is around $m_\pi c^2 \approx 140$ MeV, too.

As in the case of radiation, a temperature may be conjectured upon studying the whole spectrum of different momenta stemming from such events. This, however, requires a huge amount of exclusive data: Energy and all momentum components, for each particle from each elementary collision in the accelerated beam, have to be separately stored and analyzed. A fast estimate of the temperature can be obtained instead by inspecting global features of particle spectra. In analogy to Wien's law, the position of maxima might be studied, but this also requires some detailed data, albeit only near to the spectral maxima. In order to gain fast information, one rather utilizes the concept of average energy per particle as a measure of temperature.

More precisely, the average of momentum components squared are used for first estimates. In the non-relativistic Maxwell–Boltzmann distribution, one determines

$$\langle p_x^2 \rangle = \langle p_y^2 \rangle = \langle p_z^2 \rangle = mT. \quad (2.31)$$

Since the source of particles, the hadronic fireball may not be spherically symmetric but rather distributed alongside the beam axis, and – to begin with – it is hard to measure particles close to the beam direction, quite often the momentum components transverse to this direction are collected and averaged. This leads to the following expectation value, assuming a non-relativistic ideal gas:

$$\langle p_T^2 \rangle = 2mT. \quad (2.32)$$

Plotting the average transverse momentum squared of identified hadrons against their respective masses, the slope of the linear approximation delivers the temperature (cf. Fig. 2.3).

This estimate is improved in the framework of a relativistic thermal model. The single particle kinetic energy depends on the momentum according to the relativistic kinematic formula

$$E = \sqrt{\mathbf{p}^2 + m^2} - m \quad (2.33)$$

in $c = 1$ units.⁵ This energy formula does not contain the rest mass energy $E_0 = m$, its low-momentum approximation is given by $E = \mathbf{p}^2/2m + \dots$. The velocity vector of a relativistic particle is given by $\mathbf{v} = \mathbf{p}/E$ and its magnitude never exceeds one, the speed of light. The high-energy (as well as high-momentum and high-velocity) approximation of this formula coincides with the low-mass expansion, $E = |\mathbf{p}| + \dots$. These two extreme cases are realized by classical gases and by radiation at such high temperatures where the averages are well approximated by Wien's law instead of the Planck distribution.

The Boltzmann–Gibbs energy distribution factor, $e^{-E/T}$, for relativistic ideal gases contains the square root formula (2.33). Splitting the momentum vector to components parallel and transverse to the beam, $\mathbf{p} = (p_L, p_T \cos \varphi, p_T \sin \varphi)$.

⁵ We have arrived at the practical unit system of high energy particle physics assuming $k_B = \hbar = c = 1$ and therefore measuring temperature, time and distance uniformly in energy units of MeV, or in its respective powers.

Introducing further the transverse mass, satisfying $m_T^2 = m^2 + p_T^2$, one arrives at the following differential density of particles in momentum space:

$$d^3n = \frac{C}{(2\pi)^3} e^{-\sqrt{m_T^2 + p_L^2}/T + m/T} m_T dm_T dp_L d\phi. \quad (2.34)$$

Here, we utilized the fact that $p_T dp_T = m_T dm_T$. Integrating over the azimuthal angle ϕ and selecting out those particles which have approximately zero momentum component in the beam direction, $p_L \approx 0$, one arrives at a Boltzmann–Gibbs-like exponential distribution in the transverse mass (being the total energy in this case):

$$\left. \frac{d^2n}{p_T dp_T dp_L} \right|_{p_L=0} = \frac{C}{(2\pi)^2} e^{(m-m_T)/T}. \quad (2.35)$$

The average of p_T^2 – or equivalently m_T^2 – can now be easily obtained weighted by these transverse spectra (almost) perpendicular to the beam. One has

$$\langle m_T^2 \rangle = \frac{\int_m^\infty m_T^3 e^{-m_T/T} dm_T}{\int_m^\infty m_T e^{-m_T/T} dm_T} \quad (2.36)$$

and $\langle p_T^2 \rangle = \langle m_T^2 \rangle - m^2$. The factor $e^{m/T}$ occurs both in the numerator and denominator and therefore cancels out. The exponential integrals are in general incomplete Euler Gamma functions, in the above special case they contain polynomials of m/T . The final result is

$$\langle p_T^2 \rangle = 6T^2 + 2mT \frac{m}{m+T}. \quad (2.37)$$

For massive particles this result approaches the one obtained using the two-dimensional, non-relativistic Boltzmann formula, $2mT$. For very high temperatures, $T \gg m$, however, it relates the transverse momentum square to the temperature universally, independent of the particle mass: the leading term $6T^2$ reflects an ideal gas obeying Wien's law.

Inspecting Fig. 2.3 one realizes that in hadronic fireballs emerging from accelerator experiments most particles can be treated as non-relativistic with respect to their thermal distribution, but the lightest ones, like the pion. The spectral thermometer is roughly given by a linear dependence of the average momentum squared per particle on the particle rest mass.

Summarizing, elementary particle spectra give information about the temperature of their emitting source with which they had their last thermal contact. A temperature can be reflected in the spectral shape of momentum or energy distribution, but also in average values. Most prominently the momentum squared per particle is a linear function of mass at not too relativistic temperatures $k_B T < mc^2$. The average energy per particle per kinetic degree of freedom is about $k_B T/2$ in the non-relativistic, about $k_B T$ in the extreme relativistic limit.

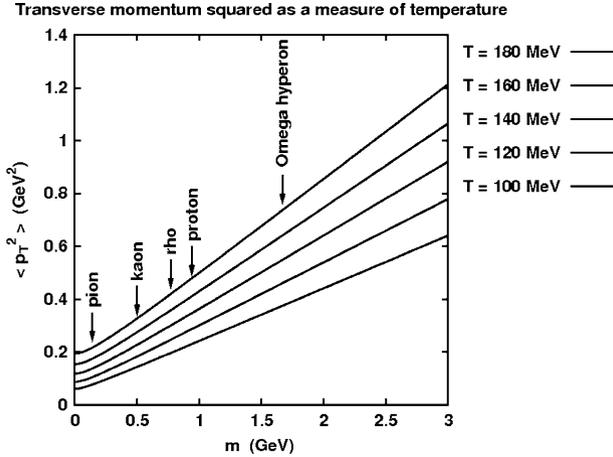


Fig. 2.3 Average transverse momentum squared, $\langle p_T^2 \rangle$, of elementary particles stemming from a hadronization in high-energy accelerator experiments according to the simplest thermal model. Characteristic temperatures, $T = 120, 140, 160,$ and 180 MeV are shown in the mass range from the pion to lowest mass baryons

2.3 Chemical Temperature

Since heat and temperature do have an effect on chemical reactions, also chemical signals may be used for indicating, scaling and – with some further calculational work – to measure temperature. Most spectacular are films or liquid solutions changing their color by heating and cooling. This, in general, can be achieved by changing the concentration of some components relative to the others.

Such a method, based on comparing multiplicities (relative numbers) of different components in a (supposedly) thermalized mixture, also occurs in high energy accelerator experiments. In particular, a “chemical decoupling temperature” is conjectured, the temperature when the hadron species – detected afterward – were equilibrating with each other the last time before they have lost contact. This moment in the evolution of a hadronic fireball is called chemical decoupling [7–11]. In what follows we demonstrate this effect in simplified models of non-interacting thermal mixtures of different kinds of particles.

Ideal mixtures realize a peaceful coexistence of several different components. It means that the respective probabilities of finding one or the other component, specific particle, in a given phase space cells factorize: These are independent statistical events. Considering an ideal gas of different particles, the i -th sort having a number density n_i , the total ideal pressure is additive

$$P = \sum_i n_i T \tag{2.38}$$

in the non-relativistic Boltzmann limit (still using $k_B = 1$ units). Each number density is determined by a common temperature, T , and the individual particle properties, most prominently by a particle rest mass, m_i and a chemical potential, μ_i . The respective ideal densities can be obtained as being

$$n_i = e^{\mu_i/T} d_i \int d^3 p e^{-E_i(p)/T}. \quad (2.39)$$

Here, $E_i(p) = \sqrt{\mathbf{p}^2 + m_i^2}$ is the energy of a single, freely and relativistically moving particle with rest mass m_i and momentum \mathbf{p} . The factor d_i counts for a possible degeneracy due to internal degrees of freedom, not taking part in the particles motion but distinguishing quantum states. The best example is a polarization, with the degeneracy factor $d_s = (2s + 1)$ for a particle with spin s . The chemical potential, μ_i , depends on the charges the particle carries and on the chemical potentials associated to the total density of those conserved charges:

$$\mu_i = \sum_a q_i^a \mu^a, \quad (2.40)$$

summed over all sorts of charges. In high energy accelerator experiments, the majority of hadrons is produced in a very short and energetic event governed by the elementary strong interaction; during this there are three conserved charges: the electric charge (which is strictly conserved in all interactions known to date), the baryonic charge, associated to the quark content of the hadrons, and the hypercharge, correlated with an exotic property, the so called ‘‘strangeness’’ of the particle. This way the chemical potential of a given particle is a linear combination of three basic chemical potentials, μ_3 , μ_B , and μ_S associated to the electric charge, to the baryonic number and to the strangeness number of the particle:

$$\mu_i = q_i^{\text{electric}} \mu_3 + q_i^{\text{baryonic}} \mu_B + q_i^{\text{strangeness}} \mu_S. \quad (2.41)$$

For charge balanced hadronic fireballs these chemical potentials are negligible, but due to the incomplete detection in fluctuating events they are never exactly zero. A sizable value is expected in the first place for μ_B by using heavy nuclei for initiating the reactions. In particular, future experiments are devoted to the study of the strongly interacting hadronic and quark matter at finite total baryon density [12, 13].

By inspecting particle number ratios of identified particles, the masses m_i are known and the chemical potential and temperature dependence can be conjectured to conform with the ideal gas mixture formula (2.38). It is a natural assumption to consider that all hadrons coming from the same fireball were thermalized in about the same volume. In this case, the particle number ratios are given by the density ratios

$$\frac{N_i}{N_j} = \frac{n_i}{n_j} = e^{(\mu_i - \mu_j)/T} \frac{f(m_i/T) d_i}{f(m_j/T) d_j}, \quad (2.42)$$

if comparing two particles having the same charges with respect to electric, baryonic and strangeness degrees of freedom. The function, $f(m/T)$ can be obtained by evaluating the integral (2.39). Assuming an isotropic fireball, i.e. an equally thermalized motion in all of the three spatial directions, one has to calculate the following integral:

$$n = e^{\mu/T} \frac{1}{2\pi^2} \int_m^\infty E(E^2 - m^2)^{1/2} e^{-E/T} dE. \quad (2.43)$$

The result of this integral is proportional to a special function, the modified Bessel function of second kind with index 2, denoted by $K_2(x)$. We obtain

$$n = e^{\mu/T} \frac{1}{2\pi^2} m^2 T K_2(m/T). \quad (2.44)$$

At low temperatures, $T \ll m$ (in $k_B = c = \hbar = 1$ units), this formula leads to the Maxwell–Boltzmann result (2.23). In the other extreme, i.e. at extreme relativistic temperatures, $T \gg m$, one obtains the following average particle density

$$n = e^{\mu/T} \frac{1}{\pi^2} T^3. \quad (2.45)$$

The total energy per particle, defining the kinetic temperature for each particle mass, also can be obtained by evaluating relativistic integrals. For the sake of completeness we give here the result

$$\frac{E}{N} = \frac{e}{n} = 3T + m \frac{K_1(m/T)}{K_2(m/T)}. \quad (2.46)$$

It is particularly simple if one studies ratios of particle sorts having the same charges but different mass. This can be a comparison with a so called resonance of an elementary particle – but here the resonance decay after the chemical decoupling has to be taken into account (this happens without any interaction, too). Stable particles with all the same charge but different masses, unfortunately do not exist; in fact, the higher mass particle decays into the lower mass partner with otherwise the same properties. As an example, let us compare the ρ and π mesons in an ideal, thermal ensemble. On a very short time scale both can be treated as stable particles in thermal equilibrium. The equilibrium ratio of their numbers is given by

$$\frac{N_\rho}{N_\pi} = 3 \frac{f(m_\rho/T)}{f(m_\pi/T)}, \quad (2.47)$$

which in the non-relativistic limit becomes

$$\frac{N_\rho}{N_\pi} = 3e^{(m_\pi - m_\rho)/T} \left(\frac{m_\rho}{m_\pi} \right)^{3/2}. \quad (2.48)$$

Due to the rest mass energy factor, $e^{-m/T}$, such a ratio may be used to define a chemical temperature

$$T_{\text{chem}|\rho/\pi} = \frac{m_\rho - m_\pi}{\ln \frac{3N_\pi}{N_\rho} + \frac{3}{2} \ln \frac{m_\rho}{m_\pi}}. \quad (2.49)$$

Substituting the particle masses $m_\pi = 140 \text{ MeV}$ and $m_\rho = 770 \text{ MeV}$, the chemical temperature is related to the particle ratio as

$$T_{\text{chem}|\rho/\pi} = \frac{630 \text{ MeV}}{2.557 + \ln \frac{3N_\pi}{N_\rho}}. \quad (2.50)$$

For half as much rho meson than pion this gives an estimate of $T \approx 145 \text{ MeV}$.

Of course, in case of chemical and thermal equilibrium all possible ratios have to comply with the common temperature and the few chemical potentials associated to conserved charge-like quantities. In different thermal models of heavy ion reactions, further parameters, like an excluded volume of each particle sort, or fluctuating volumina can also be taken into account: the details change, but the main principle of obtaining the chemical temperature remains.

2.3.1 Challenges

In principle, the different methods for measuring the temperature, namely by direct contact, by analyzing radiation spectra and by determining from component decomposition, should deliver identical results. However, only the direct contact based measurement fulfills the requirement of the zeroth law of thermodynamics, only in this case is established the reading of the temperature value in a thermal equilibrium state between the measured object and the thermometer, also treated as a physical system. The solely idealization is the vanishing heat capacity of the thermometer; it is a task for the technology to come close to this ideal.

By conjecturing the temperature from radiation, the direct contact and hence the thermal equilibration between the thermometer, represented by the detector, and the measured object, e.g. a distant star, is not achieved during the measurement. In theory, the knowledge of the entire spectrum would reveal whether we observe a black body, – a radiation being in thermal equilibrium with its source, – but in practice only parts of a spectrum can be detected. Furthermore, both the shining surface and the motion of the observed body may distort the result of the spectral measurement. Although the latter may be disentangled by observing spectral lines, transition frequencies stemming from quantum mechanical processes between atomic states, the radiation based temperature measurement is clearly less certain than the classical one based on direct contact.

Finally, a temperature determination based on the chemical concept of analyzing component particle ratios is even less reliable. Assumptions have to be made about the statistical independence and equation of state contributions from the individual

components to the mixture. Furthermore, the chemical decoupling has to happen nearly instantly (and before the kinetic decoupling), in order to trust that the detected particle sort ratios still carry information about a thermal state of the mixture. In particular by confronting with a distributed decoupling during or an instant one at the end of an evolution, which includes strong changes of temperature in space and time, the interpretation of the calculated chemical temperature has to be handled with care. The undeniable fact of a *global evolution* is unified with the theoretical concept of *local equilibrium* in such cases. For too small mesoscopic systems this concept has severe limitations. For example, there is a longstanding debate among researchers in particle physics about the question that how far can a temperature be associated to the fireball state from which observed hadronic spectra stem. While for reactions initiated by the collision of heavy atomic nuclei (lead, gold or uranium) the researchers' common sense tends to accept the thermodynamic interpretation of absolute temperature in describing experimental hadron spectra, for the reactions emerging from collisions of smaller systems (like proton on antiproton or electron–positron scattering) many indicate well argued doubts about the applicability of such a concept. But where exactly lies the borderline between these cases?

Problems

- 2.1. Convert by heart (and fast) the following temperature values between the Celsius and Fahrenheit scales: 36°C , 27°C , 22°C , 100°C , 32°F , 64°F , 80°F , 71°F , 451°F .
- 2.2. Derive Kirchoff's law from the equality of intensities of emitted and absorbed radiation between two bodies in equilibrium.
- 2.3. Using Wien's law determine the wavelengths of maximal intensity for the Sun's surface, for a light bulb, for the human body, and for the cosmic microwave background.
- 2.4. What is the average energy carried by a photon in thermal radiation according to Planck's law, according to Wien's law and according to a Raleigh–Jeans law cut at the maximal frequency of Wien's formula?



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