Chapter 2
Magnetic Rotation

2.1 Introduction

Sequences of rotational-like bands with strong M1 transitions (\(\Delta I = 1\) bands) were observed in nearly spherical nuclei in the neutron-deficient Pb isotopes in the early 1990s [1–7]. These sequences were later interpreted [8] as occurring due to a new nuclear excitation mode—Magnetic Rotation. A \(\Delta I = 1\) rotational band was already reported in the literature in the year 1986 in \(^{83}\text{Kr}\) [9] which was later found to possess all the experimental features to qualify as a magnetic rotational band [10].

These \(\Delta I = 1\) bands have a number of interesting properties. One of the most interesting aspects of these bands is the rotational-like behaviour which was till then a concept associated only with normally deformed and super-deformed nuclei. One of the first examples of such \(\Delta I = 1\) bands in the Pb isotopes was found in \(^{199}\text{Pb}\) nucleus [1, 3]. Figure 2.1 shows the gamma-ray spectrum of a \(\Delta I = 1\) magnetic rotational band in \(^{199}\text{Pb}\) [11]. The striking regularity in energy spacing between the consecutive gamma-rays is surprising. These gamma-ray transitions were later found to be of M1 character [12, 13]. In Fig. 2.2 is shown the gamma-ray spectrum of a super-deformed band in \(^{196}\text{Pb}\) [11]. The spectrum in Fig. 2.1 is amazingly similar in the regularity of energy spacing to that in Fig. 2.2. But in the latter figure the connecting gamma-ray transitions are of E2 character. In both the \(\Delta I = 1\) band and the super-deformed band, the observed energy levels follow the relation \((E-E_0) \propto I(I+1)\), where \(E\) is the excitation energy of a state, \(E_0\) is the energy of the lowest energy state (\(E_0 = 0\) for the ground state of a normally deformed band in an even–even nucleus) and \(I\) is the spin of the excited state. The above relationship holds when the nucleus does not change its structure. In normally deformed nuclei, for example in the mass \(A \sim 160\) region, M1 gamma-ray transitions are observed between the signature partners of a deformed band. These M1 transitions are in competition with strong E2 intra-band transitions, indicating substantial quadrupole deformation of the nucleus. In the \(\Delta I = 1\) bands, as in \(^{199}\text{Pb}\), the strong M1 transitions are associated with weak or very weak cross-over E2 transitions. This indicates small deformation of the nucleus depicting the \(\Delta I = 1\) bands.

A large number of \(\Delta I = 1\) magnetic rotational bands have been observed in different mass regions [14] in nuclei near magic numbers where the nuclei are weakly deformed. In the sections to follow, a review of the available experimental informa-
**Fig. 2.1** Gamma-ray spectrum of a magnetic rotational band in $^{199}$Pb. The gamma-ray peak energies are marked in keV. (Figure reproduced with permission from [11])

**Fig. 2.2** Gamma-ray spectrum of a super-deformed band in $^{196}$Pb. The gamma-ray energies of the peaks are marked in keV. (Figure reproduced with permission from [11])
tion on the properties of magnetic rotational bands in Pb region will be given. A brief mention about such bands observed in the lighter mass regions will be made. This will be followed by theoretical interpretation of these $\Delta I = 1$ bands.

Excellent review articles on magnetic rotational bands are available in the literature [11, 15].

2.2 Magnetic Rotational Bands in the Pb Region

2.2.1 General Nuclear Structure in Light Pb Nuclei

For a discussion of the properties of magnetic rotational bands in the neutron-deficient Pb region $A \sim 190–200$, it is helpful to first know the general nuclear structure in these single closed shell $Z = 82$ nuclei. For $N \leq 125$ odd-$A$ Pb nuclei, the low-lying spherical states are the $p_{1/2}$, $f_{5/2}$ and $p_{3/2}$ single neutron hole states. In addition to these, the $3/2^-$, $5/2^-$, $7/2^-$ and $9/2^-$ states may be described as weak-coupling multiplets—$f_{5/2}$, $p_{3/2}$, $p_{1/2}$ neutron hole states coupled to the $2^+$ state of an even–even core.

The $J^p = 13/2^+$ states in these odd-mass nuclei are the $(\nu i_{13/2})^{-1}$ one-quasi-neutron hole states. The $33/2^+$ states are the $(\nu i_{13/2})^{-3}$ three-quasi-neutron hole states, whereas in the even–even Pb nuclei, the $12^+$ and the $20^+$ states are $(\nu i_{13/2})^{-2}$ two-quasi-neutron and the $(\nu i_{13/2})^{-4}$ four-quasi-neutron hole states, respectively.

In many closed shell and near closed shell nuclei in the $Z = 82$ Pb region, intruder states have been observed. Low-lying $0^+$ states have been found in $^{190–198}\text{Pb}$ isotopes [16, 17]. In addition, $8^+$ and $11^-$ isomeric states have been found in many of the even isotopes of Pb (e.g. see [18]). These have been interpreted as proton $2p–2h$ (two particle–two hole) excitations across the $Z = 82$ shell gap; that is two $3s_{1/2}$ protons have been excited to the $h_{9/2}$ orbital ($8^+$ state) or one proton to $h_{9/2}$ and the other to $i_{13/2}$ orbital ($11^-$ state) leaving two proton holes in the $s_{1/2}$ orbital. An interesting feature of the Pb nuclei is that these $2p–2h$ excited states which coexist with the other spherical $\nu i_{13/2}$ neutron hole states show different shapes. Enhanced E3 transitions from the $11^-$ states were reported in $^{190–196}\text{Pb}$ [19, 20] and were interpreted as evidence that these states are of oblate deformed shape. Spectroscopic quadrupole moments were measured for the $11^-$ states in $^{192–196}\text{Pb}$ [21, 22]. Assuming oblate deformation, the average quadrupole deformation was obtained to be $\beta_2 = -0.146(14)$ for these states. The basic nucleon configurations assigned [23–25] to these proton intruder states are as follows:

<table>
<thead>
<tr>
<th>$J^p$</th>
<th>Configuration</th>
<th>Nilsson configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8^+$</td>
<td>$\pi [h_{9/2}^2]_{8^+}$</td>
<td>$[{\pi h_{9/2} [505] 9/2^- } \otimes {\pi h_{9/2} [514] 7/2^- }]_{K^\pi = 8^+}$</td>
</tr>
<tr>
<td>$11^-$</td>
<td>$\pi [h_{9/2} i_{13/2}]_{11^-}$</td>
<td>$[{\pi h_{9/2} [505] 9/2^- } \otimes {\pi i_{13/2} [606] 13/2^+ }]_{K^\pi = 11^-}$</td>
</tr>
</tbody>
</table>

It may be mentioned that no rotational bands are built on these intruder states.
So far, the independent coexistence of spherical states and the weakly collective oblate 2p–2h states in the light Pb isotopes has been described here. In the next section, properties of the $\Delta I = 1$ magnetic rotational bands built on the 2p–2h proton states coupled with the $(v_{13/2})^n (n = 1, 2, 3, \ldots)$ neutron hole states will be discussed.

2.2.2 The Magnetic Rotational Bands—Experimental Results

Soon after the discovery of a $\Delta I = 1$ band in $^{199}$Pb [1, 3], intense experimental investigations of these bands were undertaken by several groups [11 and references therein] and, as a result, a large number of these bands were found in the odd and even light Pb isotopes and in several other medium and light mass near spherical nuclei [14]. In this section, spectroscopic observations of such $\Delta I = 1$ bands and their properties in the odd Pb isotopes $^{193, 195, 197, 199}$Pb and the even Pb isotopes $^{194, 196, 198}$Pb will be described and discussed. In Figs. 2.3, 2.4, 2.5, 2.6, 2.7 and 2.8 the level schemes of $^{197}$Pb [26], $^{199}$Pb [11 and references therein], $^{194}$Pb [27], $^{196}$Pb [28] and $^{198}$Pb [26] are shown. These level schemes were investigated using high detection efficiency gamma detector arrays GAMMASPHERE and different versions of the EUROGAM and EUROBALL. The identified $\Delta I = 1$ bands are marked by band numbers in the figures. In addition to the measurement of gamma-ray energies of the $\Delta I = 1$ transitions in the bands and of the cross-over $\Delta I = 2$ transitions wherever these could be detected along with their relative gamma-ray intensities, a number of different measurements, such as the $g$-factor and quadrupole moment of the bandhead states and of the component neutron and proton states, internal conversion coefficients and linear polarisation of gamma-ray transitions and angular correlation ratios $R_{DCO}$, were done in separate experiments. Another important class of measurements was that of accurate lifetimes of the excited states of the $\Delta I = 1$ bands.

Let us now talk about the properties of these $\Delta I = 1$ bands as deduced from the above-mentioned experiments.

2.2.2.1 Multipolarity and Character of the Inter-connecting Transitions

For the determination of multipole order of the gamma-ray transitions, angular distribution coefficients and directional correlations from oriented states (DCO ratios—$R_{DCO}$) have been measured in a number of $\Delta I = 1$ bands in the Pb isotopes [23, 26–30]. Such measurements take advantage of the fact that in heavy-ion fusion reactions, anisotropy of the gamma-ray angular distributions is induced as a result of preferential population of some of the magnetic sub-states of excited nuclear state, causing nuclear alignment. Let us take the case of band 1 in $^{197}$Pb (see Fig. 2.3) which gives a good representation of the properties of $\Delta I = 1$ bands in Pb region. This band in this nucleus will often be referred as a ‘case study’. In Fig. 2.9 a plot of $R_{DCO}$ ratios versus gamma-ray transition energy for the band [26] is shown. The
Fig. 2.3 Level scheme of $^{197}$Pb [26]. (Figure reproduced with permission from [26])
Fig. 2.4 Partial level scheme of $^{199}$Pb [11 and references therein]. (Figure reproduced with permission from [11]).
2.2 Magnetic Rotational Bands in the Pb Region

low-energy in-band gamma-ray transitions are dipoles as the $R_{DCO}$ values for these lie around a value $R_{DCO} = 0.6$ for pure stretched dipole transitions for the EUROGAM 2 array geometry. The high-energy cross-over transitions, for example 729.0, 750.2, 754.9 and 975.1 keV, are quadrupoles with the $R_{DCO}$ values which scatter around $R_{DCO} = 1$ for pure stretched quadrupole transitions.

Relative parities of excited nuclear levels can be determined by the measurement of linear polarisation of gamma-ray transitions between states as these measurements depend upon the electric or magnetic character of the transition and supplement the multipole order information obtained from the $R_{DCO}$ ratio measurement.

Fig. 2.5 Partial level scheme of $^{194}$Pb [27]. (Figure reproduced with permission from [27])
Fig. 2.6 Level scheme of $^{196}$Pb [28]. (Figure reproduced with permission from [11])
Fig. 2.7 Partial level scheme (Part 1) of $^{198}$Pb [26]. (Figure reproduced with permission from [26])
Linear polarisation measurements have been done for the in-band low-energy gamma-ray transitions in the $\Delta I = 1$ bands in $^{196}$Pb [28] and $^{197}$Pb [26] by using the Clover detectors of the EUROGAM 2 and EUROBALL–IV spectrometers. General description of the details of linear polarisation measurement of gamma-ray transitions using Compton polarimeters, like a Clover detector or three Ge-detectors, can be found in [26, 31 and references therein]. The nuclear alignment in heavy-ion reactions, which induces the anisotropy of angular distribution, also manifests linear polarisation of gamma-rays. In linear polarisation measurements, advantage is taken of the fact that Compton scattering depends upon the polarisation of gamma-rays. The linear polarisation effect is largest near 90° to the beam direction where the Clover detectors are placed. In a Clover detector, the Ge crystal which is hit by an emitted gamma-ray acts as a scatterer and the two adjacent Ge crystals act as absorbers. A reference plane for each gamma-ray is defined by the beam direction.
and the direction of emission of the gamma-ray. Intensity ($I_h$) of the full energy gamma-ray peak formed by Compton-scattered gamma-rays parallel (or horizontally) to the reference plane in one adjacent absorber Ge crystal in coincidence with the partial energy events in the scatterer and ($I_v$) of the peak in similar Compton scattering coincidences vertically to the plane are analysed separately. Such a polarimeter can be used to determine the electric or magnetic character of a gamma-ray as magnetic transitions favour parallel (horizontal) Compton scattering whereas the electric transitions favour perpendicular (vertical) Compton scattering [32]. The asymmetry ratio

$$A(E_{\gamma}) = \frac{[I_v(E_{\gamma}) - I_h(E_{\gamma})]}{[I_v(E_{\gamma}) + I_h(E_{\gamma})]}$$

defined in [33] is measured [26]. The polarisation asymmetry ratio $A(E\gamma)$ is negative for unmixed stretched magnetic transitions and positive for electric transitions. In order to deduce the linear polarisation, $P = A(E\gamma)/Q(E\gamma)$, where $Q(E\gamma)$ is the polarisation sensitivity of the Compton polarimeter which in the experiment on $^{197}$Pb for the Clover detectors is $Q \approx 0.2–0.25$ in the gamma-ray energy range 300 to 700 keV [26 and references therein]. In Fig. 2.10 is shown the difference gamma-ray spectrum of the linear polarisation spectra, vertical and horizontal, of the same band 1 in $^{197}$Pb (see Fig. 2.3). Gamma-ray transitions with magnetic character result in negative gamma-ray peaks whereas electric transitions give positive peaks [26]. The low-energy transitions are the in-band magnetic transitions and the high-energy cross-over transitions 750.2 and 754.9 keV (marked as ‘x’ in the figure) are of electric character. The other positive peaks of energies 556.6, 626.8 and 738.4 keV in the figure are some E2 transitions in the low-energy part of the level scheme. The 432.5 keV positive peak is an E1 transition which depopulates band 1.
Multipole order and the electric or magnetic character of gamma-ray transitions can also be determined by the measurement of internal conversion coefficients of the transitions and their comparison with the theoretical values for pure transitions. Such measurements have been done for the $\Delta I = 1$ bands in $^{197}$Pb and $^{199}$Pb [12, 13]. Let us again consider band 1 in $^{197}$Pb (Fig. 2.3). Figure 2.11 shows the experimental values of the K-shell internal conversion coefficients, $\alpha_K$, for the low-energy in-band gamma-ray transitions in this band versus transition energy. The solid lines in the figure are the theoretical values [34] of $\alpha_K$ for the pure M1, E2 and E1 transitions in the energy range ~100 to 400 keV. From this figure, it is clear that these in-band transitions are M1 transitions.

**Fig. 2.10** Difference gamma-ray linear polarisation spectra for parallel and perpendicular Compton scattering of transitions in band 1 in $^{197}$Pb. Transitions with magnetic character result in negative peaks whereas the electric transitions result in positive peaks [26]. (Figure reproduced with permission from [26])

**Fig. 2.11** The experimental values of K-shell internal conversion coefficients, $\alpha_K$ [13] versus transition energy, for the in-band, low-energy gamma-ray transitions for band 1 in $^{197}$Pb. The *solid lines* are the corresponding theoretical values [34] for the pure M1, E2 and E1 transitions.
The spins and parities of the $\Delta I = 1$ bands in most of the Pb isotopes have been assigned from the information on multipole order and character of the in-band transitions and those of the decay-out transitions connecting the $\Delta I = 1$ band to the low spin states in the nucleus and from the knowledge of $J^e$ of the low spin states.

Having now known the level schemes, the magnetic dipole nature of the in-band transitions and the assignment of spins and parities of the levels in the $\Delta I = 1$ bands in the Pb isotopes, attention will now be paid to the nucleon configuration and the coupling of angular momenta of the participating nucleons for the bandhead states of these bands. We shall then talk about deformation of the bandhead states.

### 2.2.2.2 Nucleon Configurations and Coupling of Angular Momenta

Amongst the several $\Delta I = 1$ bands observed in Pb nuclei, the most intense are the lowest lying negative parity and the positive parity bands. These bands are built on $[\pi (s_{1/2}^{-2} h_{9/2} i_{13/2}) 11^- \otimes \nu i_{13/2}^{-1} n]$ configuration (where $n = 1, 2, 3,...$) with the possible addition for some bands of one or more low-$j$ neutrons of $f_{5/2}$ and $p_{3/2}$ origin. For the negative parity $\Delta I = 1$ bands in the odd Pb isotopes, the suggested [26 and references therein] configuration of the band head states is $[\pi (s_{1/2}^{-2} h_{9/2} i_{13/2}) 11^- \otimes \nu i_{13/2}^{-1} i_{13/2}^{-1}]$. The $g$-factor of the $29/2^- T_{1/2} = 9$ ns band head state of band 1 in $^{193}\text{Pb}$ was determined [35] by the time-dependent perturbed angular distribution (TDPAD) method. The result obtained is

$$g_{\text{exp}}[2584 \text{ keV level}] = 0.68 \ (3).$$

The $g$-factor of the state can be calculated using the experimental values of the $g$-factors of the individual components of the nucleon configuration for which the average values [36] are

$$g[\pi (h_{9/2} i_{13/2}) 11^-] = 1.11 \ (2)$$

and

$$g(\nu i_{13/2}^{-1}) = 0.150 \ (6).$$

Using the additivity relation for magnetic moments

$$g_{\text{calc}} [\pi (h_{9/2} i_{13/2}) 11^- \otimes \nu i_{13/2}^{-1}] = 0.71 \ (4),$$

which agrees well with the experimentally determined value of $g$-factor. Other plausible nucleon configurations give $g$-factor values in disagreement with the measured result. This confirms the above-mentioned-suggested configuration for the band head state on which $\Delta I = 1$ band 1 in $^{193}\text{Pb}$ is based.

The lowest energy for this repulsive particle–hole coupling is expected for a perpendicular orientation of the proton and neutron spins. Since the $g$-factor is determined by component of magnetic dipole moment along the spin vector, the
agreement between the experimental and the calculated g-factors confirms approximately perpendicular coupling of the proton particle and the neutron-hole spins.

The nucleon configuration $\left[ \pi \left( \frac{h}{2} i_{13/2} \right) 11^- \otimes \nu \left( i_{13/2} \right) \right]$ and the approximately perpendicular coupling of the proton-particle and neutron-hole spins have, therefore, been adopted for all the negative parity $\Delta I = 1$ bandhead states in odd-mass Pb isotopes.

For the lowest lying positive parity $\Delta I = 1$ bands in odd Pb isotopes, the suggested configuration [26 and the references therein] for the bandhead states is

$$\left[ \pi \left( \frac{h}{2} i_{13/2} \right) 11^- \otimes \nu \left( i_{13/2} \right) f_{5/2} \right].$$

Let us now consider the lowest energy $\Delta I = 1$ bands in the even-mass Pb isotopes. A glance at the 16$^-$ bandhead states in $^{194}$Pb (band 1a in Fig. 2.5), $^{196}$Pb (band 2 in Fig. 2.6) and $^{198}$Pb (band 3 in Fig. 2.8) shows that most of the band intensity passes through the $\pi \left( \frac{h}{2} i_{13/2} \right) 11^-$ states. This suggests the involvement of the $\pi \left( \frac{h}{2} i_{13/2} \right) 11^-$ configuration in the 16$^-$ bandhead states of the negative parity $\Delta I = 1$ bands. The suggested [26 and references therein] configuration of the bandhead states for these bands is

$$\left[ \pi \left( \frac{h}{2} i_{13/2} \right) 11^- \otimes \nu \left( i_{13/2} \right) f_{5/2} \right].$$

2.2.2.3 Deformation of Bandhead States

Deformation of the bandhead states of the $\Delta I = 1$ bands can be estimated through the measurement of spectroscopic quadrupole moments, $Q_s$, of the states. The level lifetimes for most of the states are too short for such measurements. As mentioned earlier, the 29/2$^-$ bandhead state of band 1 in $^{193}$Pb has a halflife of 9 ns which is just long enough for the measurement of quadrupole moment. The $Q_s$ for this state has been measured by the TDPAD method [37] in an attempt to determine the deformation of the state. The value obtained is $|Q_s| = 2.84 (26)$ eb. This value is much larger than that for the $\nu \left( i_{13/2} \right)$ 12$^+$ isomers in the neighbouring even–even Pb isotopes. The spectroscopic quadrupole moments, $Q_s$, for the 12$^+$ states in $^{192}$Pb, $^{194}$Pb and $^{196}$Pb have been measured [21, 38, 39], the values for which are given below:

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$Q_s$ [12$^+$ state] (eb)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{192}$Pb</td>
<td>0.32 (4)</td>
<td>[21]</td>
</tr>
<tr>
<td>$^{194}$Pb</td>
<td>0.48 (3)</td>
<td>[38]</td>
</tr>
<tr>
<td>$^{196}$Pb</td>
<td>0.65 (5)</td>
<td>[39]</td>
</tr>
</tbody>
</table>
These $Q_s$ values indicate small deformation for the $12^+$ states. It, therefore, appears that the deformation of the bandhead state of band 1 in $^{193}$Pb, for which the nucleon configuration is

$$\left[\pi (h_{9/2} i_{13/2}) 11^- \otimes \nu i_{13/2}^{-1}\right],$$

is dominated by the deformation of the $\left[\pi (h_{9/2} i_{13/2}) 11^-\right]$ proton particle state. The spectroscopic quadrupole moments were measured for the $11^-$ isomeric states [21, 22] in the neighbouring even–even Pb isotopes. The values obtained are as follows:

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>$Q_s$ [$11^-$ state] (eb)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{192}$Pb</td>
<td>2.9 (3)</td>
<td>[21]</td>
</tr>
<tr>
<td>$^{194}$Pb</td>
<td>3.6 (4)</td>
<td>[21]</td>
</tr>
<tr>
<td>$^{196}$Pb</td>
<td>3.41 (66)</td>
<td>[22]</td>
</tr>
</tbody>
</table>

The intrinsic quadrupole moment, $Q_0$, can be estimated from $Q_s$ experimental using the strong coupling formalism, assuming here $K = I$ and using the relation

$$Q_s = Q_0 \frac{3K^2 - I(I + 1)}{(2I + 3)(I + 1)};$$

The quadrupole deformation parameter $\beta_2$ can be derived from $Q_0$ using the relation

$$Q_0 = \frac{3ZR^2\beta_2}{\sqrt{5\pi}},$$

where $R = r_o A^{1/3}$ and $r_o = 1.2$ fm. Assuming oblate shape, the average quadrupole deformation obtained for the $11^-$ states is $\beta_2 = -0.146 (14)$ [21]. The $Q_s$ value obtained for the $29/2$ state in $^{193}$Pb is similar to the values obtained for the $11^-$ isomers in the above even–even Pb isotopes. Therefore, the quadrupole deformation of the $29/2$ bandhead state in $^{193}$Pb can be taken as $\beta_2 \sim -0.14$. It may be mentioned here that the deformation of the $29/2$ state cannot be directly determined from the measured $Q_s$ as this state is not a high $K$ isomer—$K$ is not a good quantum number and so the strong coupling model cannot be applied to determine $\beta_2$.

### 2.2.2.4 Bandcrossings in $\Delta I = 1$ Bands

Shorthand notations for band configurations, which are the standard cranked shell model band classifications, as introduced in [30], will be used below for a discussion on bandcrossings in the $\Delta I = 1$ bands in the Pb isotopes. Since in the Pb region, the Fermi level is located well below the shell gap $N = 126$, neutron pairing plays an important role. Therefore, the lowest energy positive parity neutron configurations of $i_{13/2}$ origin are labelled by the letters A, B, C, D, ..., for quasi-neutrons and by E,
F for natural parity (negative parity) quasi-neutrons of predominantly $f_{5/2}$ and $p_{3/2}$ origin. The proton pairing is neglected as the two proton particles involved in the $\Delta I = 1$ band configurations are the $2p-2h$ excitations across the $Z = 82$ closed shell gap. The proton configuration is, therefore, labelled by the spin quantum number.

For example, the negative parity proton configuration $\pi(h_{9/2} i_{13/2})^{11-}$ is abbreviated as 11. Bandhead configurations for the low-lying strongest $\Delta I = 1$ bands in this notation are as follows:

<table>
<thead>
<tr>
<th>Nucleus and band</th>
<th>Configuration</th>
<th>Shorthand notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Odd Pb isotopes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative parity bands 1</td>
<td>$[\nu(i_{13/2}) \otimes \pi(h_{9/2} i_{13/2})]^{11-}$</td>
<td>A11</td>
</tr>
<tr>
<td>Positive parity bands 2</td>
<td>$[\nu(i_{13/2}^{-2} f_{5/2}^{-1}) \otimes \pi(h_{9/2} i_{13/2})]^{11-}$</td>
<td>ABE11</td>
</tr>
<tr>
<td><strong>Even Pb isotopes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative parity bands</td>
<td>$[\nu(i_{13/2}) \otimes \pi(h_{9/2} i_{13/2})]^{11-}$</td>
<td>AB11</td>
</tr>
<tr>
<td>Positive parity bands</td>
<td>$[\nu(i_{13/2}^{-1} f_{5/2}^{-1}) \otimes \pi(h_{9/2} i_{13/2})]^{11-}$</td>
<td>AE11</td>
</tr>
</tbody>
</table>

In the left panel of Fig. 2.12 [26] is given a plot of angular momentum as a function of rotational frequency which is taken as $h\omega = E_\gamma$ for the dipole transitions, for the strongest negative parity $\Delta I = 1$ bands 1 in odd-mass Pb isotopes. These bands have configuration A11 at low spins and show a bandcrossing (except in the case of $^{201}$Pb) at a rotational frequency, $h\omega \sim 0.3$ MeV with a gain in alignment of $\sim 8\hbar$. This has been interpreted [26 and references therein] as the decoupling and rotational alignment of the BC neutron pair. Therefore, the configuration of band 1

![Fig. 2.12](image-url) Plots of spin versus rotational frequency for the strongest negative parity (left panel) and strongest positive parity (right panel) $\Delta I = 1$ bands in odd-mass Pb isotopes. (Figure taken with permission from [26])
is $A_{11}$ before bandcrossing and $A_{11}C_{11}$ after the bandcrossing. An irregularity is seen only in the case of band 1 of $^{197}\text{Pb}$, at $\hbar\omega \sim 0.35 \text{ MeV}$ with an alignment gain of $\sim 2\hbar$. This has been attributed [26 and references therein] to the alignment of the natural parity neutrons EF. In the same band, another irregularity is observed at $\hbar\omega \sim 0.6 \text{ MeV}$ indicating a configuration change in the neighbourhood of this rotational frequency. Data to such high rotational frequencies do not exist for band 1 in the other odd-mass Pb isotopes.

The right panel of Fig. 2.12 shows a similar plot as in the left panel for the strongest positive parity bands (bands 2) in the odd-mass Pb isotopes. At low spins, the configuration of the bands is $AB_{E11}$. At a rotational frequency of $\hbar\omega \sim 0.45 \text{ MeV}$, this band in $^{199}\text{Pb}$ is crossed by a new structure which involves the decoupling and rotational alignment of the CD ($i_{13/2}$) neutron pair with a large alignment gain. Similar bandcrossing with less alignment gain also occurs for band 2 in $^{197}\text{Pb}$. The configuration of bands 2 above this bandcrossing is $ABC_{DE11}$ [26 and references therein].

Let us now consider the bandcrossing phenomenon for the $\Delta I = 1$ bands in the even-mass Pb isotopes. Plots of angular momentum as a function of rotational frequency for the strongest negative parity bands (left panel) and the strongest positive parity bands (right panel) in the even-mass Pb isotopes are shown in Fig. 2.13. The configuration of the negative parity bands is $AB_{11}$ below and $AB_{11}C_{11}D_{11}$ above the bandcrossing respectively [26 and references therein] due to the rotational alignment of the CD neutron pair at $\hbar\omega \sim 0.42 \text{ MeV}$. No bandcrossing is observed in $^{200}\text{Pb}$. For the positive parity bands, the configuration is $AE_{11}$ below and $ABCE_{11}$ above the bandcrossing [26 and references therein]. This bandcrossing is due to

![Fig. 2.13 Plot of angular momentum, $I$, as a function of rotational frequency, $\hbar\omega$, for the strongest negative parity bands (left panel) and the strongest positive parity bands (right panel) in the even-mass Pb isotopes. (Figure taken with permission from [26])](image-url)
the occurrence of the alignment of the BC neutron pair at a rotational frequency of \( \hbar \omega \sim 0.3 \) MeV with an alignment gain of \( \sim 8\hbar \).

### 2.2.2.5 Electromagnetic Properties of \( \Delta I = 1 \) Bands

Electromagnetic properties of the \( \Delta I = 1 \) bands, which play an important role in the understanding of nuclear structure of these bands, can be obtained from the absolute values of the reduced M1 and E2 transition probabilities, \( B(M1) \) and \( B(E2) \), respectively. These reduced transition probabilities can be deduced from lifetime measurements of excited states of the band from where the branching into M1 and E2 transitions occur. In most of the cases in \( \Delta I = 1 \) bands, the M1 transitions are strong whereas the cross-over E2 transitions are either weak or cannot be detected because of very weak gamma-ray intensities. The \( B(M1) \) values are sensitive to the details of nucleon configuration of the coupled multi-quasi particle high-\( j \) state, coupling scheme of nucleon angular momentum vectors and their contribution to the component of magnetic dipole moments perpendicular to the total angular momentum vector. The larger is this component, the larger is the M1 strength (actually, \( B(M1) \propto \) the square of the magnitude of this component). The \( B(E2) \) values provide information about the deformation of the system. It is always not possible to measure lifetimes of nuclear states and deduce the absolute \( B(M1) \) and \( B(E2) \) values. In such a situation, the \( B(M1)/B(E2) \) ratios are very useful to get an insight into the details of nuclear structure. It may be mentioned here that bulk of such information has been obtained through these ratios in a large number of deformed rotational bands in nuclei.

Although accurate lifetime measurements are now available for excited states in many of the \( \Delta I = 1 \) bands in the Pb region, we will still talk about the \( B(M1)/B(E2) \) ratios which can be deduced from experiments using the expression [40]

\[
B(M1)/B(E2) = \frac{0.6968 E_{\gamma_2}^5}{\lambda E_{\gamma_1}^3 (1 + \delta^2)} (\mu_N/e_b)^2,
\]

where \( E_{\gamma_1}, E_{\gamma_2} \) are the gamma-ray energies in MeV for the M1(\( I \rightarrow I - 1 \)) and E2(\( I \rightarrow I - 2 \)) transitions, respectively, from a state \( I \), and \( \lambda \) is the measured gamma-ray intensity ratio \( [I(\gamma_2)/I(\gamma_1)] \). The mixing ratio \( \delta^2 \) can be neglected since the in-band transitions are predominantly M1 (e.g. see Fig. 2.11) and strong compared to the E2(\( I \rightarrow I - 2 \)) transitions.

In Fig. 2.14 are shown the \( B(M1)/B(E2) \) ratios as a function of angular momentum for the ‘case study’ \( \Delta I = 1 \) band, band 1 in \(^{197}\)Pb [26]. Considering also the other \( \Delta I = 1 \) bands in \(^{197}\)Pb and \(^{198}\)Pb [26], the average value of the \( B(M1)/B(E2) \) ratio lies around 30 (\( \mu_N/e_b \))^2. This is much larger than the ones in rotational bands in deformed nuclei, e.g. it is \( \sim 0.5–5 \) (\( \mu_N/e_b \))^2 in \(^{157}\)Ho [41]. The uncertainties in the ratios shown in the figure are large because of weak gamma-ray intensities of the E2 cross-over transitions. Note an abrupt increase in the \( B(M1)/B(E2) \) ratio at the bandcrossing at \( I \sim 20\hbar \).
Accurate mean lifetimes in the pico- to sub-picosecond region have been measured for excited states in several $\Delta I = 1$ bands in the Pb isotopes using the Doppler shift methods and high photopeak detection efficiency arrays GAMMASPHERE and the EUROBALL. The Recoil Distance Method (RDM) has been used for low spin members in some of these bands while for the high spin states, the Doppler Shift Attenuation Method (DSAM) was employed. From the measured lifetimes, absolute experimental values of reduced transition probability $B(M1)$, assuming the $\Delta I = 1$ in-band transition of pure M1 character, is obtained from the expression

$$B(M1) = \frac{0.05697 B_{\gamma_1}}{E_{\gamma_1} \tau [1 + \alpha_t(M1)]} (\mu_N)^2,$$

where $E_{\gamma_1}$ is the gamma-ray energy in MeV of the M1 transition, $\tau$ is the level lifetime in ps, $\alpha_t(M1)$ is the total theoretical M1 internal conversion coefficient and the branching ratio $B_{\gamma_1} = I\gamma(M1)/[I\gamma(M1) + I\gamma(E2)]$ is obtained from the measured gamma-ray intensities of the M1 and the cross-over E2 transitions. In

![Fig. 2.14 Plot of $B(M1)/B(E2)$ ratios as a function of angular momentum for the $\Delta I = 1$ band 1 in $^{197}$Pb [26]]
some cases the cross-over E2 transitions are very weak and not observed. In such cases the M1 branching ratio is unity.

The experimental $B(E2)$ reduced transition probability is also deduced from the measured lifetime $\tau$, using the expression [40]

$$B(E2) = \frac{0.08156 B \gamma^2}{E \gamma^2 \tau [1 + \alpha_t(E2)]} (eb)^2.$$  

The total internal conversion coefficient $\alpha_t(E2)$ is small and can be neglected as the energy of the cross-over E2 transition is large (~0.5–1.2 MeV).

Figure 2.15 shows plots of experimental values of reduced M1 transition probability $B(M1)$, for intraband transitions in the $\Delta I = 1$ bands in $^{193–199}$Pb, deduced from lifetime measurements [42–47], as a function of spin of the initial state. The values are from DSAM measurements except shown by diamonds which are the RDM determinations. This figure is taken from [48]. The curves are the results of tilted axis cranking (TAC) calculations [48]. This will be discussed in the section on interpretation of $\Delta I = 1$ bands (Sect. 2.2.3). In all the above-mentioned Pb isotopes, for the mentioned band configurations, the experimental $B(M1)$ values show a characteristic decrease with spin—a result of immense significance for the

---

**Fig. 2.15** Plots of experimental reduced transition probabilities, $B(M1)$, as a function of spin of the initial state for in-band M1 transitions in $\Delta I = 1$ bands in $^{193–199}$Pb. See text for details. (Figure taken with permission from [48])
phenomenon of generation of angular momentum in these $\Delta I = 1$ bands. It can also
be seen from this figure that the $B$(M1) values differ for different configurations
(given in shorthand notation in the figure) of the bands and are drastically different
for band configurations with a different number of aligned neutrons. For band 1 in
$^{196}$Pb, the band configuration is AE11 before and ABCE11 after the band crossing
(see Fig. 2.13). In this band, below the bandcrossing, the $B$(M1) values decrease
from 2.4 to 0.7 $\mu N^2$. Above the bandcrossing, the $B$(M1) value jumps to $\approx 9 \mu N^2$
and drops to 1.8 $\mu N^2$ with increasing spin. As will be explained later (see Sect. 2.2.4),
this big jump in the $B$(M1) values is due to the shears opening as a result of the
rotational alignment of an $i_{13/2}$ BC neutron pair. In $^{197}$Pb, band 1 and band 2, before
bandcrossing, the configurations are A11 and ABE11, respectively (see Fig. 2.12).
Note the different $B$(M1) values for these configurations in the figure (Fig.
2.15).

Figure 2.16 depicts the reduced $B$(E2) transition probabilities, deduced from life-
time measurements [42–47], for intraband transitions in $\Delta I = 1$ bands in $^{193–199}$Pb,
as a function of spin of the initial state. The symbols have the same meaning as in
Fig. 2.15. The curves are the results of TAC calculations [48]. As the $B$(E2) values
have large errors because of the cross-over transitions being weak, it is not possible
to know whether the $B$(E2) values are constant as a function of spin. In band 1 in
$^{197}$Pb, at low spins, where the $B$(E2) values have relatively smaller errors, these are
seemingly constant. In general, the $B$(E2) values are small which indicates a small

![Figure 2.16](image-url)

Fig. 2.16 Same as in Fig. 2.15 but for the reduced transition probabilities, $B$(E2). See text for
details. (Figure taken with permission from [48])
deformation of the nucleus. However, this point will be discussed further in the section on the interpretation of these \( \Delta I = 1 \) bands.

### 2.2.2.6 \( \Delta I = 1 \) Bands in Lighter Mass Regions

Several \( \Delta I = 1 \) bands have been observed in the nearly spherical nuclei in mass regions \( A \sim 80, 100–110 \) and 140 [14 and references therein]. All these \( \Delta I = 1 \) bands in the lighter mass regions have properties similar to those observed for the \( \Delta I = 1 \) bands in Pb isotopes which have been discussed at length in the previous sections.

The nucleon configurations for the \( \Delta I = 1 \) bands in the mass regions are as follows:

<table>
<thead>
<tr>
<th>Mass region</th>
<th>Bandhead configuration</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>( \pi(g_{9/2}^n) \otimes \nu(g_{9/2}^-) )</td>
</tr>
<tr>
<td>100–110</td>
<td>( \pi(g_{9/2}^n) \otimes \nu(h_{11/2}^-) )</td>
</tr>
<tr>
<td>140</td>
<td>( \pi(h_{11/2}^n) \otimes \nu(h_{11/2}^-) )</td>
</tr>
</tbody>
</table>

Very recently, four \( \Delta I = 1 \) bands have been found in \( ^{60}\text{Ni} \) [49] which possibly have nucleon configurations involving one \( 1f_{7/2} \) proton hole and \( 1g_{9/2} \) neutron particle(s).

### 2.2.3 Theoretical Interpretation of the \( \Delta I = 1 \) Bands

One of the important experimental observations in \( \Delta I = 1 \) bands in the Pb region and in the lighter mass regions, as mentioned in the earlier sections, is that these bands are always built upon bandhead states which are excitations of high spin particles coupled to high spin hole states and never upon only the high spin particle or the high spin hole states alone. A \( g \)-factor measurement [35] of the band head state of band 1 in \( ^{193}\text{Pb} \) established the nucleon configuration of this state to be \( \left[ \pi \left( h_{9/2}^{13/2} \right) 11^- \otimes \nu \left( i_{13/2}^{-1} \right) \right] \). This experiment also confirmed an approximately perpendicular coupling of the proton particle and neutron-hole spins.

The observed \( \Delta I = 1 \) bands are rotational-like bands.

The bands have strong in-band M1 transitions and the cross-over E2 transitions are either weak and, in some cases, not even observed. The bandhead states are weakly oblate deformed \( (\beta_2 \approx -0.14) \) as indicated by the spectroscopic quadrupole moment measurement [37] of the bandhead state in band 1 in \( ^{193}\text{Pb} \) and those for the proton particle \( 11^- \) states in the even Pb isotopes [21, 22].

Another significant observation has come from the accurate lifetime measurements of excited states in the \( \Delta I = 1 \) bands in the Pb isotopes. The experimentally deduced reduced transition probabilities \( B(\text{M1}) \) are large (up to several \( \mu_N^2 \)) and decrease characteristically with increasing spin (see Fig. 2.15). The obtained \( B(\text{E2}) \) values are small \((\sim 0.1 \text{ e}^2\text{b}^2)\) in many cases (see Fig. 2.16) indicating small deformation.
The above-mentioned experimental findings greatly helped in the theoretical understanding of these $\Delta I = 1$ bands.

Amongst the various theoretical approaches which provide insight into the phenomenon of $\Delta I = 1$ bands and their properties in near spherical light Pb isotopes and in the lighter mass regions is the pioneering work of Frauendorf [8] within the framework of the TAC calculations. Frauendorf et al. [50] also did spherical shell model calculations in a reasonable configuration space to interpret the observed $\Delta I = 1$ bands. Later, Macchiavelli et al. [51–53] presented a semi-classical analysis of the process of generation of angular momentum in these bands, predictions of reduced transition probabilities $B$(M1) and $B$(E2), and an effective interaction between the neutron and proton forming each ‘blade’ of the system to provide an answer to the rotation-like behaviour of these $\Delta I = 1$ bands. An integrated view of these ideas is contained in a review article by Clark and Macchiavelli [15].

Still another approach has been the calculations using the particle rotor model. In [54], competition between the process generating angular momentum in the $\Delta I = 1$ bands and the core rotation was investigated through the particles-plus-rotor model. The phenomena of signature splitting and signature inversion in some of the $\Delta I = 1$ bands in the Pb isotopes were studied in [55]. In the many particle plus rotor model description [56] of the $\Delta I = 1$ bands in the Pb isotopes, the reduced transition probabilities $B$(M1) and $B$(E2) have been calculated as a function of initial spin and the results are compared with experimental data on band 1 in $^{199}$Pb. This theory also considers the question of band termination in these bands.

In this section, we will consider the TAC approach.

2.2.3.1 Angular Momentum Coupling Scheme

Let us consider the coupling of the high-$\Omega$ proton particles and low-$\Omega$ neutron hole orbitals in the tilted axis representation. The density distribution of the $h_{9/2}$ and $i_{13/2}$ protons is torus-like and that of the $i_{13/2}$ neutron holes dumb-bell like. The proton particle spin, $j_\pi$, prefers deformation alignment, i.e. it is parallel to the symmetry axis (3-axis) of the nucleus which is weakly oblate deformed. The neutron-hole spin, $j_\nu$, prefers alignment perpendicular to $j_\pi$ (rotation aligned). This type of coupling results in maximum overlap of the density distributions of the proton particles and the neutron holes and minimum system energy (bandhead energy). This situation is shown in the schematic drawing of the coupling scheme in Fig. 2.17a. The total angular momentum, $I$, then lies along a tilted axis at an angle $\theta$ (tilt angle) with respect to the symmetry axis. The angular momentum is restricted to lie in the principal plane formed by the 3- and the 1-axis. The angular momentum and energy in the $\Delta I = 1$ band are increased by a step-by-step alignment of $j_\pi$ and $j_\nu$ towards the total angular momentum vector, $I$. The angle between the $j_\pi$ and $j_\nu$ vectors decreases but as will be shown later, the tilt angle, $\theta$, remains nearly constant. This is shown in Fig. 2.17b. Finally (see Fig. 2.17c), the particle and hole spins fully align parallel to the total angular momentum, $I$. This corresponds to the maximum spin state of the band of the assigned nucleon configuration. The band then termi-
nates at this maximum spin. This whole process of closing of proton and neutron vectors (blades), $j_\pi$ and $j_\nu$, is similar to the closing of sheep shears and, therefore, this process of angular momentum generation in $\Delta I = 1$ bands has been called the ‘shears mechanism’ and the bands the ‘shears bands’ [30].

The magnetic dipole moment of the high-$j$ protons is large and positive, i.e. in the direction of $j_\pi$, whereas that of high-$j$ neutrons is small, as a result of no orbital contribution, and negative. The magnetic dipole moment components of the protons and neutrons are also shown schematically in the coupling scheme diagrams. The addition of components, $\mu_\pi^\perp$ and $\mu_\nu^\perp$, perpendicular to the total angular momentum, $I$, gives a large value of $\mu_{tot}^\perp$ which breaks the rotational symmetry of the quantal system (similar to what the charge distribution does in deformed nuclei).

![Diagram](image_url)

Fig. 2.17 Angular momentum coupling scheme of shears mechanism in a $\Delta I = 1$ band in a weakly oblate deformed nucleus in the Pb region a at low rotational frequency (at bandhead), b at intermediate rotational frequency and c at maximum rotational frequency. The perpendicular magnetic dipole moments are also shown schematically. See the text for details. (Figure taken with permission mainly from [14]; figure c courtesy Professor A. K. Jain)
Since in the TAC approach, uniform rotation of the nucleus is considered around an axis parallel to $I$, the total perpendicular component of the magnetic dipole moment $\mu_{\text{tot}}^\perp$, rotates around $I$. In analogy with the rotation of deformed nucleus where the electric quadrupole moment rotates around the total angular momentum, the new excitation mode is called ‘magnetic rotation’ and the $\Delta I = 1$ bands the ‘magnetic rotational bands’.

As shown in Fig. 2.17a–c, the perpendicular component of the total magnetic dipole moment $\mu_{\text{tot}}^\perp$ decreases in a characteristic manner with increase in spin and excitation energy in the $\Delta I = 1$ band, due to the closing of the shears (alignment of $j_\pi$ and $j_\nu$ towards $I$). The perpendicular component $\mu_{\text{tot}}^\perp$ finally vanishes for the situation when $j_\pi$ and $j_\nu$ align along $I$ (see Fig. 2.17c). Since $B(M1)$ is proportional to $|\mu_{\text{tot}}^\perp|^2$, the $B(M1)$ values should also decrease characteristically with increasing spin in the band. This is what is observed experimentally through accurate measurement of lifetimes of excited states in the $\Delta I = 1$ bands in neutron-deficient Pb isotopes (see Fig. 2.15). These measurements, therefore, provide a strong experimental evidence for the existence of the shears mechanism of the generation of angular momentum in such bands. The $B(M1)$ values obtained in the TAC model will be discussed later.

### 2.2.3.2 Recent Results from TAC Calculations

Recently, Chmel et al. [48] made detailed calculations of tilt angles, deformation parameters, angular momenta and reduced magnetic dipole and electric quadrupole transition probabilities, $B(M1)$ and $B(E2)$, respectively, within the framework of the TAC model for the shears/magnetic rotational bands ($\Delta I = 1$ bands) in the neutron-deficient Pb isotopes $^{193}$Pb to $^{202}$Pb. In this work [48], the pairing-plus-quadrupole-quadrupole version of the tilted axis cranking (PQTAC) model [8, 57, 58] was applied for the above-mentioned calculations. Details of the various parameters used in the calculations are also discussed in [48].

In the following, the results of the PQTAC model calculations will be discussed. Fig. 2.18 gives a plot of the tilt angles $\theta$ as a function of rotational frequency, $\hbar \omega$, for the A11 (see Fig. 2.12) and the AB11 (see Fig. 2.13) configurations of the $\Delta I = 1$ bands in the Pb isotopes. It is observed that the tilt angle varies little with rotational frequency for a given isotope and for a particular configuration. This suggests that the shears mechanism plays a dominant role in the generation of angular momentum in these bands. The tilt angle varies with neutron number of the different isotopes. This is due to the changes of neutron Fermi level. The tilt angle is different for the A11 $[\nu(i_{13/2}^{-1}) \otimes \pi (h_{9/2} \otimes i_{13/2})11^-]$ and the AB11 $[\nu(i_{13/2}^{-2}) \otimes \pi (h_{9/2} \otimes i_{13/2})11^-]$ configurations. It becomes larger when an $i_{13/2}$ neutron hole is added which gives an extra spin component perpendicular to the 3-axis. In band 1 in $^{197}$Pb (see Fig. 2.3 for level scheme), before bandcrossing, the configuration is A11 and after bandcross-
ing, it is ABC11 with the rotation alignment of a BC $i_{13/2}$ neutron pair. The tilt angle nearly doubles in magnitude after bandcrossing (see Table 1 in [48]).

The deformation parameters $\varepsilon_2$ (=$0.95 \beta_2$ for small deformations) and $\gamma$ have also been calculated in the PQTAC model [48]. These values for A11 and AB11 configurations are shown as a function of rotational frequency, $\hbar \omega$, in Fig. 2.19. The quadrupole deformations are small, typically $\varepsilon_2 \sim -0.10$ and it remains constant within the bands. This value is somewhat smaller than a value of $\beta_2 \sim -0.14$ adopted in Sect. 2.2.2.3 for the bandhead state of band 1 in $^{193}$Pb. The triaxiality is found to be small in all the cases considered. There is less than about 10% variation from the value for axial symmetry ($\gamma = -120^\circ$ with $\varepsilon_2$ negative).

In Fig. 2.15, plots of experimentally deduced $B$(M1) values as a function of initial state spin, for the $\Delta I = 1$ in $^{193-199}$Pb, are shown. It is observed that the $B$(M1) values characteristically decrease with increasing spin (see also Sect. 2.2.2.5). The
calculated $B$(M1) values from the PQTAC model [48] are shown as curves in the figure. The calculations reproduce the decrease in $B$(M1) values with increasing spin reasonably well. This is a convincing evidence of the existence of the shears mechanism of generation of angular momentum in the $\Delta I = 1$ bands.

The calculated $B$(E2) values for the Pb isotopes using the PQTAC model [48], as a function of initial state spin, are shown as curves in Fig. 2.16. These are compared with the experimentally deduced values from lifetime measurements (see also Sect. 2.2.2.5). For band 1 in $^{197}$Pb, at low spins (below bandcrossing), where the experimental $B$(E2) values have relatively small errors, the calculated values are in good agreement with the experimental data. Both the values show a constant trend with increasing spin. This is an indication that the calculated almost constant quadrupole deformation of $\varepsilon_2 \approx -0.1$ could be accepted at least for the $\Delta I = 1$ magnetic rotational band—band 1 in $^{197}$Pb. The question whether the quadrupole deformation is constant for all magnetic rotational bands, away from the bandcrossing regions, can be decided when accurate values of $B$(E2) are available from experiments.

### 2.2.4 Band Termination

Maximum spin state for a given configuration in a shears band is reached when the proton particle and the neutron-hole spins align parallel to the total angular momentum vector, $I$ (i.e. when the shears close). This is schematically shown in Fig. 2.17c. The shears band then terminates. Let us take the example of band 2 in $^{199}$Pb (see level scheme in Fig. 2.4). For this positive parity shears or magnetic rotational band, angular momentum, $I$, is plotted as a function of transition energy, $E_\gamma$ of the in-band M1 gamma-ray transitions, in Fig. 2.20. As mentioned in Sect. 2.2.2.4, the nucleon configuration of the band below bandcrossing is $[\nu(i_{13/2}^{-2} f_{5/2}^{-1}) \otimes \pi (h_{9/2} i_{13/2})]^{11^-}$ and after the rotational alignment of a CD $v_{i_{13/2}^{-2}}$ neutron hole pair, it is $[\nu(i_{13/2}^{-4} f_{5/2}^{-1}) \otimes \pi (h_{9/2} i_{13/2})]^{11^-}$. To start with, the bandhead spin is 35/2, which is obtained by a near 90° coupling of proton spin of 11 and neutron spin of 29/2. The maximum spin as a result of closing of the shears is 51/2. However, as observed experimentally, the band terminates at spin 57/2. The difference of spin is contributed by weakly deformed core rotation through many nucleons with small spins, as $I = I_{\text{shears}} + R_{\text{core}}$. As seen in the level scheme (Fig. 2.4), an irregularity occurs near band termination. Two closely spaced 57/2 states and three 59/2 levels are populated. With the rotational alignment of two $v_{i_{13/2}^{-2}}$ neutrons, the shears open up to ~90° coupling and a new shears band with $[\nu(i_{13/2}^{-4} f_{5/2}^{-1}) \otimes \pi (h_{9/2} i_{13/2})]^{11^-}$ starts to build-up. This band again exhausts its spin content at maximum spin of 67/2 by shears closing. The actual maximum spin state observed is at 71/2 due to the core contribution. The core contributions are shown as horizontal arrows at maximum spins in the figure.

The band termination phenomenon in deformed rotational nuclei is in some sense different from that discussed above. For a detailed discussion of the former type of band termination, see ref. 2 under Band Termination on p. 114.
Another type of shears mechanism than the one discussed above was proposed by Frauendorf [59] which he called ‘Antimagnetic rotation’ (AMR). In this mechanism, two shears—like systems—are formed. Best examples of this are predicted in the light Cd isotopes, although AMR is expected to occur in the same mass regions and under similar conditions as magnetic rotation [57]. Here, two $h_{11/2}$ neutron particles align their spin along the symmetry axis and the two $g_{9/2}$ proton holes are aligned perpendicular to the symmetry axis, but their spins are anti-parallel. Each of the proton holes combines with a neutron particle, thus forming a pair of back to back shears. In contrast to the shears mechanism in magnetic rotation where there is a large component of magnetic dipole moment ($\mu_\perp$) perpendicular to the total angular momentum vector, in these two pair of shears, the perpendicular component of magnetic dipole moment from each pair is equal and opposite, and thus they cancel each other, i.e. $\mu_\perp = 0$. Therefore, the $B(M1)$ values vanish and so no M1 transitions are observed. The magnetic dipole moments of one of the shears specify the orientation. The angular momentum in an antimagnetic rotational band is generated through the simultaneous step-by-step closing of the two shears. Since the

\[ \nu(i_{13/2} - f_{5/2}) \times \pi(h_{3/2} i_{13/2}) \]

\[ \nu(i_{13/2} - f_{5/2}) \times \pi(h_{3/2} i_{13/2}) \]

Fig. 2.20 Angular momentum, $I$, as a function of $E_\gamma$ for band 2 in $^{199}$Pb (see Fig. 2.4 for level scheme). (Figure taken with permission from [11])

2.3 Antimagnetic Rotation
two shears system is symmetric with respect to rotation by $\pi$ about the total angular momentum axis, the rotational band should consist of sequences of energy levels with $\Delta J = 2\hbar$. The levels in the band should decay by weak E2 gamma-ray transitions as the core has small deformation. The phenomenon of antimagnetic rotation is characterised by a rapid decrease of the $B$(E2) values with increasing spin.

Experimental investigations on antimagnetic rotation have been done in $^{100}$Pd [60] and in $^{106, 108, 109, 110}$Cd [61–65]. In $^{106}$Cd [66], lifetimes were measured [61, 62] by the DSAM for excited states between spins 18–26 in the collective band based on $10^+$ bandhead state which is a two-quasi-neutron rotation aligned $\nu(h_{11/2})^2$ structure. The partial level scheme of $^{106}$Cd [66] is shown in Fig. 2.21 [67]. A further alignment of a pair of $g_{7/2}$ neutrons was observed in this band at a rotational frequency $\hbar \omega \sim 0.45$ MeV. The neutron configuration $\nu[(h_{11/2})^2_{10} (g_{7/2})^2_{6}]$ was, therefore, assigned to this band above spin of 16. The Cd isotopes with

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Fig. 2.21 Partial level scheme of $^{106}$Cd [66]. (Figure taken with permission from R. Wadsworth NS06 [67])
Z = 48 have two proton holes in the $g_{9/2}$ orbital. The aligned four quasi-particle neutron configuration is coupled to the two $g_{9/2}$ proton holes. The coupling scheme is represented in Fig. 2.22 [65]. The four aligned quasi-neutrons are denoted by the vector $j_\nu$ and each of the two $g_{9/2}$ proton holes $\pi(g_{9/2})^{-1}$ by $j_\pi^{-1}$. The proton hole angular momentum vectors are antiparallel and nearly perpendicular to the neutron angular momentum vector at spin 16. This system has total $\mu_\perp = 0$ and no M1 transitions are observed. The angular momentum generation above spin 16 in the band is by the shears mechanism. The solid line (dashed line) arrows show the coupling at low (high) rotational frequency. The spin in the band increases from 16 to 26 on complete alignment of the proton–hole spin vectors. Actually, the aligned spin contribution of the pair of proton–holes is $(9/2 + 7/2)\hbar = 8\hbar$, which will make the spin $16\hbar + 8\hbar = 24\hbar$ at band termination. The deformed core contributes an extra $\sim 2\hbar$. The TAC calculations [62] yield $\beta_2 \approx 0.16$ at spin $I \sim 17$ and there is a rapid decrease in $\beta_2$ with increase in spin. The $\beta_2$ value becomes $\sim 0$ around spin 26. The experimental values of $B(E2)$ deduced from the lifetime measurements [62] are plotted in Fig. 2.23 [67] as a function of spin. The $B(E2)$ values decrease rapidly with spin as predicted by theory. This provides evidence for the shears mechanism and thus for the antimagnetic rotation phenomenon. The experimental results are in good agreement with the cranked Nilsson–Strutinsky (CNS), TAC and the semiclassical (SCM) calculations. The arrows in the figure show the rela-
tive orientation of angular momentum of the $g_{9/2}$ proton holes as predicted by the TAC calculations.

The number of investigations in antimagnetic rotation is not very large, and so there is a need to investigate and find more evidence for this new type of shears coupling through the measurement of lifetimes in particular, in this and the other mass regions.

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