Preface

The beginning of stochastic programming, and in particular stochastic linear programming (SLP), dates back to the 50's and early 60's of the last century. Pioneers who—at that time—contributed to the field, either by identifying SLP problems in particular applications, or by formulating various model types and solution approaches for dealing adequately with linear programs containing random variables in their right–hand–side, their technology matrix, and/or their objective's gradient, have been among others (in alphabetical order):

E.M.L. Beale [12], proposing a quadratic programming approach to solve special simple recourse stochastic programs;
A. Charnes and W.W. Cooper [41], introducing a particular stochastic program with chance constraints;
G.B. Dantzig [49], formulating the general problem of linear programming with uncertain data and
G.B. Dantzig and A. Madansky [53], discussing at an early stage the possibility to solve particular two-stage stochastic linear programs;
G. Tintner [326], considering stochastic linear programming as an appropriate approach to model particular agricultural applications; and
C. van de Panne and W. Popp [333], considering a cattle feed problem modeled with probabilistic constraints.

In addition we should mention just a few results and methods achieved before 1963, which were not developed in connection with stochastic programming, but nevertheless turned out to play an essential role in various areas of our field. One instance is the Brunn-Minkowski inequality based on the investigations of H. Brunn [36] in 1887 and H. Minkowski [235] in 1897, which comes up in connection with convexity statements for probabilistic constraints, as mentioned e.g. in A. Prékopa [266]. Furthermore, this applies in particular to the discussion about bounds on distribution functions, based on inequalities published by G. Boole in 1854 and by C.E. Bonferroni in 1937 (for the references see A. Prékopa [266]), and on the other hand, about bounds on the expectation of a convex function of a random variable, leading to a lower bound by the inequality of J.L. Jensen [148], and to the Edmundson–Madansky upper bound due to H.P. Edmundson [83] and A. Madansky [210].

Among the concepts of solution approaches, developed until 1963 for linear or nonlinear programming problems, the following ones, in part after appropriate modifications, still serve as basic tools for dealing with SLP problems:

Besides Dantzig's simplex method and the Dantzig–Wolfe decomposition, described in detail in G.B. Dantzig [50], the dual decomposition proposed by J.F. Benders [14], cutting plane methods as introduced by J.E. Kelley [180], and feasible direction methods proposed and discussed in detail by G. Zoutendijk [355], may be recognized even within today's solution methods for various SLP problems. Of course, these methods and in particular their implementations have been revised and improved meanwhile, and in addition we know of many new solution approaches, some of which will be dealt with in this book.

The aim of this volume is to draw a bow from solution methods of (deterministic) mathematical programming, being of use in SLP as well, through theoretical properties of various SLP problems which suggest in many cases the design of particular solution approaches, to solvers, understood as implemented algorithms for the solution of the corresponding SLP problems.

Obviously we are far from giving a complete picture on the present knowledge and computational possibilities in SLP. First we had to omit the area of stochastic integer programming (SILP), since following the above concept would have implied to give first a survey on those integer programming methods used in SILP; this would go beyond the limits of this volume. However the reader may get a first flavour of SILP by having a look for instance into the articles of W.K. Klein Haneveld, L. Stougie, and M.H. van der Vlerk [189], W. Römisch and R. Schultz [288], M.H. van der Vlerk [335], and the recent survey of S. Sen [301].

And, as the second restriction, in presenting detailed descriptions we have essentially confined ourselves to those computational methods for solving SLP problems belonging to one of the following categories:

Either information on the numerical efficiency of a corresponding solver is reported in the literature based on reasonable test sets (not just three examples or less!) and the solver is publicly available;

or else, corresponding solvers have been attached to our model management system SLP-IOR, either implemented by ourselves or else provided by their authors, such that we were able to gain computational experience on the methods presented, based on running the corresponding solvers on randomly generated test batteries of SLP's with various characteristics like problem size, matrix entries density, probability distribution, range and sign of problem data, and some others.

Finally, we owe thanks to many colleagues for either providing us with their solvers to link them to SLP-IOR, or for their support in implementing their methods by ourselves. Further, we gratefully acknowledge the critical comments of Simon Siegrist at our Institute. Obviously, the remaining errors are the sole responsibility of the authors. Last but not least we are indebted to the publisher for an excellent co-

operation. This applies in particular to the publisher's representative, Gary Folven, to whom we are also greatly obliged for his patience.

Zürich,
September 2004

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Comments on the 2nd edition

Since fall 2004, when we finished the 1st edition of this volume, the scope of features for the field of stochastic optimization has broadened substantially, extending the variety of model types considered and the corresponding solvers designed, as well as spreading the areas of application and the related models.

Just to mention a few of these recent activities, we list the following topics:

– *Risk measures and dominance concepts.*
There is an ongoing discussion on using various kinds of risk measures as well as stochastic dominance concepts within stochastic optimization models. Particular concepts of dealing with risk are for instance the ICC, the *Integrated Chance Constraints* (joint as well as individual ICC) due to W.K. Klein Haneveld and M.H. van der Vlerk [191]. Furthermore, CVaR, the *Conditional Value at Risk*, as analyzed e.g. in T.R. Rockafellar and S.P. Uryasev [283], is increasingly included into stochastic optimization problems, either within constraints or else (additively) in the objective. On the other side, stochastic dominance of first or higher order as discussed for instance in D. Dentcheva and A. Ruszczyński [66] receives more attention in modeling risky situations. More generally, the class of polyhedral risk measures, favourable for stochastic programs with risk measures in the objective, is analyzed in A. Eichhorn and W. Römisch [84].

— *Increasing consideration of risk in applications.*
The above mentioned risk measures and dominance concepts got a wider impact in modeling stochastic programs for real situations in various areas, thus aiming towards more realistic results for the respective problems.
As examples for using risk measures in stochastic programming models within finance we just mention the investigations of Klein Haneveld – Streutker – Van der Vlerk [190] dealing with ICC in ALM (asset liability management), A. Künzi-Bay [197] aiming for CVaR minimization in multistage ALM models for Swiss pension funds, Mansini – Ogryczak – Speranza [215] involving the CVaR in portfolio optimization, or Dentcheva and

Ruszczyński [67] considering portfolio optimization with dominance constraints.
To give just one example of risk considerations in energy problems, we mention the thesis of M. Densing [64] discussing a coherent multi-period risk measure, as generalization of the one-period CVaR, to be used for a hydroelectric power plant dispatch problem incorporated into a multi-stage stochastic program.
Another broad area of applications is concerned with comparing efficiency among finitely many similar ventures called DMU (*decision making unit*) and modelled in the frame of DEA, i.e. *Data Envelopment Analysis*. Originally stated as models combining inputs and outputs by deterministic linear constraints for any particular DMU to check whether it is efficient (*non dominated*), this setup became questionable for various applications, and the first DEA models incorporating—at least partly—chance conctraints were discussed (where obviously efficiency with respect to chance constraints had to be defined appropriately). For this setup there are many references; a recent one is e.g. Talluri – Narasimhan – Nair [323]. A more general view was taken in the thesis of S. von Bergen [339], considering to model the case of stochastic outputs via constraints on special *risk functions*, which encompass chance constraints, ICC constraints and CVaR constraints, at least. Efficiency was redefined, taking into account the risk function formulation of the model constraints, yielding conditions being verifiable by solving two-phase nonlinear programs.

— *Growing need for stochastic optimization in engineering*
Formerly the attention of engineers regarding randomness was focused mainly on analyzing reliability of systems or structures. Meanwhile, an increasing interest in stochastic optimization models and methods can be observed in various fields of engineering, like for instance in structural optimization or in robot control. The reader may get an impression of the particular approaches in this area of applications in the recent volume of K. Marti [225].

— *Tools for modelling SLP problems and links to solvers*
In principle there are two kinds of modelling tools:

- Systems, controlled by programming languages, containing declarations of data structures and model types, typically used in SLP, including the syntax to specify the particular problem features (like various single-stage versions, multi-stage recourse including the recourse type, etc.) and to manipulate probability distributions used in the current model on the one hand, and on the other hand
- menu-driven systems, for instance with pull down menus, allowing to declare model types and data structures, to edit data (arrays), to specify the random model data and edit the corresponding distributions, etc.

As to the first variant, following earlier work of H.J. Gassmann, K. Fourer, et al. on languages usable in addition to AMPL (A Mathematical Programming

Language of Fourer – Gay – Kernighan [97]), there have been recently some further attempts as e.g. the paper of Colombo – Grothey – Hogg – Wooksend – Gondzio [45] and the language StAMPL (A filtration-oriented modeling tool for multistage stochastic recourse problems) of Fourer – Lopes [99]. Concerning the second variant, we have continued to work on our system SLP-IOR, extending its features. In this context, in his thesis M.T. Bielser [19] designed a programming language SEAL (Stochastic Extensions for Algebraic Languages) which is aimed to create (as its output) the information necessary to supply to SLP-IOR as input for starting up the system.

Fortunately it is not necessary to deal in detail with all these results (and others) —although containing very interesting considerations—in this edition, since the material presented should be sufficient to follow most of the recent developments.

Following a suggestion of F.S. Hillier, we have added exercises at the end of several sections, where we thought it could be helpful. At the end of the volume the reader finds hints for dealing with them in the Chapter *Exercises: Hints for answers*.

In this context we considered it as meaningful to provide access for the reader to the features of our model management system; we therefore prepared an executable version "SLP-IOR" (student version) for open access (download), for which on the COSP web page `http://stoprog.org` (see section Software & TestSets) the corresponding link can be found. Questions or comments concerning this software are welcome to `mayer@ior.uzh.ch`.

Finally, we are greatly indebted

- to many colleagues for pointing out various mistakes and/or inaccuracies in the system SLP-IOR as well as in the 1st edition of this book,
- to Fred S. Hillier, the editor of this series of books, for his encouragement to prepare the 2nd edition of this volume,
- to Neil Levine and his colleagues for their support on behalf of the publisher,
- and not least to Silvia von Bergen, formerly assistant at our Institute, for the careful reading of parts of the manuscript and for several helpful comments.

Nevertheless, the sole responsibility for any inconsistencies lies with the authors.

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