Chapter 2
Structured Laser Radiation (SLR)

2.1 Main Types of SLR

*Structured Laser Radiation* is spatially amplitude-modulated radiation obtained with the aid of classical optical elements, DOE, or structured screens [1, 2].

The main elements of structured laser radiation are listed in Table 2.1. These are classified by the shape of the spatial geometrical figures formed by the beams from the source as line-, plane-, or cone-structured laser radiation. The two-dimensional figures presented in the table are cross-sections of the beams formed by families of geometrical optics beams from the source.

By combining the main SLR elements, one can obtain other types of SLR source adapted to the structure of the inhomogeneity at hand and the shape of the body in whose vicinity boundary layers are being studied [3]. To diagnose bulk inhomogeneities, it is advisable to produce “measuring grids” from elementary sources.

Obviously, the idealized representation of SLR cross-sections in the form of geometrical figures is only valid in the geometrical optics approximation, and so when handling actual measuring setups, one should evaluate the errors due to diffraction effects in order that the applicability limits of the method can be determined. For example, plane-structured laser radiation, also referred to as the laser plane, is in fact an astigmatic laser beam of elliptical cross-section whose diffraction divergence is determined by the well-known quasioptical methods [4]. Considered in the following sections are the physical principles of forming simple SLR on the basis of Gaussian laser beams.
2.2 Gaussian Beams

2.2.1 Properties of Laser Radiation

Laser radiation differs from radiation produced by the ordinary thermal and luminescent light sources by high coherence, i.e., monochromaticity and directivity. Both these properties are of great importance in laser refractography. The monochromaticity of radiation allows one to disregard the dispersion of the medium, and its narrow directivity makes it possible to produce a new type of radiation, namely, structured radiation, by means of simple optical systems.

The laser is a system consisting of an active (amplifying) medium and a resonator (cavity) comprising one or more high-reflectivity mirrors. If the amplification (gain) of the medium exceeds losses, and the cavity provides for a positive feedback, a narrowly directed monochromatic radiation—the laser beam—is then formed at the exit from the laser [5]. In most cases, this radiation is polarized.

The spatial characteristics of laser radiation are determined by its mode composition. In the laser cavity transverse electromagnetic waves are generated, designated as TEM$_{mn}$ modes, where $m$ and $n$ are integers: $m, n = 1, 2, 3, ...$. Two lasing (laser generation) regimes are distinguished, namely, multi- and single-mode ones. Single-mode lasing is characterized by sets of indices $m$ and $n$ of one and the same value, whereas multimode generation by those of different values. For a TEM$_{mn}$ mode and rectangular cavity mirrors, the distribution of the electric field amplitude in the beam cross-section is described by the following expression [5]:

\[
A(x, y) = A_0 H_m \left( \frac{\sqrt{2}x}{w} \right) H_n \left( \frac{\sqrt{2}y}{w} \right) \exp \left\{ -\frac{x^2 + y^2}{w^2} \right\}, \tag{2.1}
\]

where $A_0$ is the coefficient determining the field amplitude, $H_m(x)$ and $H_n(x)$ are Hermitian polynomials of degrees $m$ and $n$, and $w$ is the laser beam radius. The Hermitian polynomials of the first three degrees have the form

\[
H_0(x) = 1, \quad H_1(x) = 2x, \quad H_2(x) = 4x^2 - 2, \quad H_3(x) = 8x^3 - 12x.
\]

The field amplitude distribution over the beam cross-section for the first four low-order modes is illustrated in Fig. 2.1.

<table>
<thead>
<tr>
<th>Source type</th>
<th>Linear</th>
<th>Plane</th>
<th>Conical</th>
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<tr>
<td>SLR cross-section</td>
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Table 2.1 Main elements of structured laser radiation
The laser beam radius in expression (2.1) is a scale parameter that depends on the laser cavity configuration and varies along the beam axis. The beam radii $w$ at the mirrors and $w_0$ at the beam waist, where the beam size is a minimum, are governed by the radii of curvature of the mirrors and the distance between them. If one of the mirrors is flat, the beam waist is located at it; if the mirrors are of the same radius of curvature, the waist is at the center of the cavity.

The laser radiation characteristics considered above are idealized. Actually, deviations are observed from the ideal field amplitude distribution over the beam cross-section and radiation spectrum, due to various kinds of fluctuations in the active medium and cavity parameters. All fluctuations in lasers are customarily classified under two types, natural and technical. Natural fluctuations are due to the atomic structure of the active medium and the quantum character of laser radiation; and technical ones are caused by the slow variations of the cavity and active medium parameters (amplification coefficient and refractive index fluctuations over the cross-section of the laser beam). Technical fluctuations can be reduced by improving the stability of the cavity and the parameters of the active medium.

The spatial coherence of laser radiation means correlation between perturbations at two space-apart points at one and the same instant of time. It affects the visibility of interference patterns and is measured by the Junge method [4]. The complex-degree spatial coherence module for single-mode gas lasers is close to unity. However, account should be taken of the fact that the spatial coherence of laser beams is substantially affected by the medium they pass through.

The divergence of laser beams depends on the shape and arrangement of the cavity mirrors. If these are flat, the divergence is determined by the diffraction loss. The beam divergence in industrial gas lasers ranges between 5 and 20 min, and it can be as great as a few tens of degrees in semiconductor lasers.
The polarization of radiation in gas lasers built around gas-discharge tubes equipped with Brewster-angle windows is linear. The plane of polarization of laser radiation here lies in the plane of incidence of the beam upon the window.

The polarization of the laser beam in gas lasers using gas-discharge tubes with intracavity mirrors depends on the cavity adjustment. As a rule, such lasers use axial magnetic field, which makes possible their two-frequency operation. Radiation of frequencies $\nu_1$ and $\nu_2$ is circularly polarized.

### 2.2.2 Characteristics of the Gaussian Beam

The processes of laser diagnostics of flows can most conveniently be analyzed using as an example laser beams of the principal, TEM$_{00}$, mode that are referred to as Gaussian beams, for the variation of the field amplitude in any cross-section of such a beam is described by a Gaussian curve (Fig. 2.2).

The Gaussian beam is characterized by the following parameters: the beam radius $w$, the radius of the wave front curvature, $R(z)$, the position of the beam waist where the beam radius is a minimum, the beam waist radius $w_0$, the beam confocal parameter given by

$$R_0 = \frac{k w_0^2}{2}, \tag{2.2}$$

where $k$ is the wave number, and the far-field divergence angle $\theta$.

![Figure 2.2](image_url) Variation of the parameters of the Gaussian beam along its propagation axis
The properties of Gaussian beams were examined by Evtikhiev and coworkers [4], Karlov [5], and Goncharenko [6]. The radius of such a beam is defined as the distance from the beam axis at which the field amplitude is reduced by a factor of \( e \) as compared with the amplitude on the axis. (The beam radius is sometimes defined as the distance at which the field strength is reduced by a factor of \( e \) in comparison with the field strength on the beam axis.)

The Gaussian beam has its minimal diameter of \( 2w_0 \) at its waist, where the wave front is flat, or plane. The variation of the beam radius with distance from the waist is described by the following relation:

\[
w(z) = w_0 \left[ 1 + \left( \frac{z}{R_0} \right)^2 \right]^{1/2}, \tag{2.3}
\]

where \( z \) is the distance from the waist. At great distances from the waist, where \( (z/R_0)^2 \gg 1 \), the beam radius varies with the distance \( z \) as

\[
w(z) = w_0 z / R_0. \tag{2.4}
\]

The radius \( R(z) \) of the wave front curvature depends on the distance \( z \) reckoned from the waist:

\[
R(z) = z \left[ 1 + \left( \frac{R_0}{z} \right)^2 \right]. \tag{2.5}
\]

As the distance from the waist increases, the radius of curvature of the wave front diminishes to assume its minimal value of \( R_{\text{min}} = 2R_0 \) at a distance of \( z = R_0 \). Thereafter the radius of curvature increases and asymptotically tends to \( z \). Thus, at a great distance from the waist, the wave front of the Gaussian beam takes the form of a sphere with its center at the waist and its radius of curvature equal to \( z \). The region where the radius of curvature decreases with the increasing distance \( z \) is called the near field of the Gaussian beam; and the region where this radius increases with the distance \( z \) is referred to as the far field of the beam. The near field covers the region \( z \leq R_0 \) and the far field ranges from \( z = R_0 \) to infinity. Figure 2.3 shows the relationship between \( R_0 \) and \( w_0 \) for various lasers. At a great distance from the waist (far field), the Gaussian beam can be characterized by the divergence angle \( \theta = w_0 / R_0 \).

The distribution of the field strength in the Gaussian beam has the form

\[
E(x, y, z, t) = A_0 \left[ \frac{w_0}{w(z)} \right] \exp\{-j\omega t\} \times \exp\left\{ j \left[ kz + k (x^2 + y^2) / (2R(z)) + \varphi \right] - \left( x^2 + y^2 \right) / w^2(z) \right\}. \tag{2.6}
\]

Here \( A_0 \) is the field amplitude on the beam axis at the waist, \( \omega \) is the circular frequency, and \( \varphi = \arctan(z/R_0) \) is the phase shift on the beam axis.

The dependences of \( w(z) \), \( \varphi \), and \( R(z) \) on the distance \( z/R_0 \) are presented in Fig. 2.4.
The field amplitude \( A_0 \) on the beam axis at the waist may be expressed in terms of the laser beam power \( P \) as follows. The distribution of the field strength at the Gaussian beam waist is given by

\[
E(x, y, 0, t) = A_0 \exp(-j\omega t) \exp\left\{-\left(\frac{x^2 + y^2}{w_0^2}\right)\right\},
\]

then, according to [2], \( P = QA_0^2 \pi w_0^2 / 2 \), whence we have

\[
A_0 = \sqrt{2P / (\pi w_0^2 Q)}, \quad (2.7)
\]

where \( Q = 1.35 \times 10^{-3} \). For instance, if a laser beam 10 mW in power is focused into a spot 50 \( \mu \)m in radius, then \( A_0 = 4.4 \times 10^4 \) V/m.
2.2.3 Propagation of the Gaussian Beam through Optical Elements

Consider the change the Gaussian beam undergoes during the course of its passage through a lens with a focal length of \( f \) placed at a distance of \( l_1 \) from its waist (Fig. 2.5). Such a lens transforms a Gaussian beam with a waist radius of \( w_{01} \) into a beam having a waist radius of \( w_{02} \), this being given by [6]

\[
w_{02} = w_{01} \left[ (1 - l_1/f)^2 + R_{01}^2/f^2 \right]^{-1/2}.
\]  

The location of the beam waist is defined by the relation

\[
1 - l_2/f = (1 - l_1/f) \left[ (1 - l_1/f)^2 + R_{01}^2/f^2 \right]^{-1}.
\]  

In these formulas, the ratios \( l_1/f \) and \( l_2/f \) are positive if the lens is a collective type and negative if it is dispersive. If the distance \( l_2 \) proves negative in calculation, the beam continues to diverge upon leaving the lens. If we put \( R_{01} = 0 \) in expression (2.9), we get the well-known lens formula for homocentric beams.

Analyzing the results obtained, note that the distance \( l_2 \) for the Gaussian beam can be shorter or longer than the distance \( l_2' \) for the homocentric one, depending on the ratio \( l_1/f \): at \( l_1/f > 1 \) we have \( l_2 > l_2' \), and if \( l_1/f < 1 \), \( l_2 < l_2' \).

Let us determine the conditions wherein a lens with a focal length of \( f \) transforms the Gaussian beam so that \( w_{02} = w_{01} \). It is well known from geometrical optics that the linear magnification of an optical system equals unity if \( l_1 = 2f \). For the Gaussian beam, we have from expression (2.9) that in this case the lens should be moved away from the beam waist for a distance of \( l_1 \) such that

\[
l_1 = f \pm \left( f^2 - R_{01}^2 \right)^{1/2},
\]

that is, the distance \( l_1 \) depends on the confocal parameter \( R_{01} \) of the beam. Specifically, if \( R_{01} = f \), the lens should be placed at a distance of \( f \) from the beam waist. It is well known that for a homocentric beam in the case where \( l_1 = f \), a parallel beam is

![Fig. 2.5](image-url) Transforms of a Gaussian and homocentric beams by thin lens. a Gaussian beam. b homocentric beam
formed at the exit from the lens. At \( l_1 = f \), in the case of the Gaussian beam, we have \( w_{02} = w_{01} f / R_{01} \); i.e., the magnification of the optical system is governed by the ratio between the focal length of the lens and the confocal parameter of the beam. The beam waist radius after the lens can be either smaller or greater than that before the lens. It follows from expression (2.8) that to minimize the spot size, one should take a short-focus lens and move it away for a great distance. If \( l_1 \gg R_{01} - f \), then \( w_{02} = w_{01} f / (f + l_1) \). But one should bear in mind here that a beam waist less than 5 \( \mu \)m in size is difficult to obtain because of the aberrations of the lens and the diffraction phenomenon.

It frequently proves necessary to transform a Gaussian beam with a waist radius of \( w_{01} \) into a beam with a waist radius of \( w_{02} \) by means of a lens with a focal length of \( f \). Such transformation takes place if the lens is moved away from the waist for a distance of \( l_1 \) such that

\[
l_1 = f \pm R_{01} \left( f^2 / f_0^2 - 1 \right)^{1/2}, \tag{2.10}
\]

the waist with a radius of \( w_{02} \) occurring at a distance of \( l_2 \) from the lens, given by

\[
l_2 = f \pm R_{02} \left( f^2 / f_0^2 - 1 \right)^{1/2}, \tag{2.11}
\]

where \( f_0^2 = R_{01} R_{02} \), and the signs in the formulas are the same. It should be noted here that such a transformation is only possible if \( f > f_0 \).

More often than not, it is the inverse problem that is being solved in practice, namely, which lens one should take and how to place it in order to obtain a beam waist of desired size at a specified distance. From formulas (2.8) and (2.9) we get the following expressions to calculate the focal length of the lens and its location, given the original waist radius \( w_{01} \), the waist radius \( w_{02} \) that is necessary to obtain, and the distance \( l_2 \) between the lens and the waist:

\[
f = \left[ l_2 R_{01} - \sqrt{R_{01} R_{02} (R_{02}^2 - R_{01} R_{02} + l_2^2)} \right] / (R_{01} - R_{02}),
\]

\[
l_1 = f + \left[ R_{01} (f^2 - R_{01} R_{02}) / R_{02} \right]^{1/2}.
\]

A single lens can help achieve the parameters specified if

\[
l_2^2 \geq R_{02}^2 \left[ R_{01} / R_{02} - 1 \right].
\]

This means that it is not always possible to obtain a Gaussian beam with a waist of desired size by means of a single lens.

The double-lens system shown schematically in Fig. 2.6 has great functional capabilities.

The transformation of the Gaussian beam in this system is described by the following expressions [7]:
where $f_1$ and $f_2$ are the focal lengths of the first and the second lens, respectively, $l_1$ is the distance from the waist of the original Gaussian beam to the first lens, $l_2$ is the distance from the first to the second lens, and $l_3$ is the distance from the second lens to the new beam waist.

These formulas can also help obtain the relations necessary for the solution of the inverse problem, namely, what are the lens parameters $f_1$ and $f_2$ and the distances $l_1$, $l_2$, and $l_3$ necessary to obtain the desired waist radius $w_{03}$, given the waist radius $w_{01}$. Calculation examples for a simple double-lens optical system capable of ensuring Gaussian beam parameters, unattainable with single-lens systems can be found in [7].

When calculating the characteristics of optical schemes for laser refractographic systems, one has to determine the parameters of the Gaussian beam passing through inclined plane-parallel plates, for example, windows, which limit the flow of interest, and individual regions of the medium under study that differ in refractive index, whose interface is inclined relative to the incident beam axis. In this case, the Gaussian beam becomes astigmatic; i.e., the waist positions of the beam and its radii of curvature in the meridional and sagittal planes fail to coincide. The formulas to calculate the beam parameters in such cases can be found in [7]. The classical Gaussian beams serve as a basis for formation, with the help of optical elements, the simplest types of SLR that are considered in the subsequent sections of the book.

2.3 SLR Formation on the Basis of Optical Elements

2.3.1 Formation of the Laser Plane

Laser refractographs based on astigmatic laser beams—laser planes—are at present the most widespread type. Depending on the problem at hand, the requirements for
the parameters of the laser plane differ widely, and so various systems are being used to form it.

**Single-Lens System.** Consider the characteristics of the single-lens system used to form the laser plane (Fig. 2.7). Placed in the path of the low-divergence beam of laser 1 is a negative or positive cylindrical short-focus lens 2 that first focuses the beam and then widens it in a single plane only.

For an aberration-free lens, the width $h(z)$ of the laser plane along the $x$-axis and its thickness $t(z)$ along the $y$-axis are calculated by the following formulas:

$$h(L) = 2w_1 \left[ 1 + \left( \frac{l_2 - l_1}{R_{01}} \right)^2 \right]^{1/2},$$  \hspace{1cm} (2.12)

$$t(L) = 2w_0 \left[ 1 + \left( \frac{L}{R_0} \right)^2 \right]^{1/2},$$  \hspace{1cm} (2.13)

where $w_0$ is the waist radius of the original laser beam, $w_1$ is the waist radius of the beam after the lens, $l_0$ is the distance from the waist of the original laser beam to the lens, $l_1$ is the distance from the lens to the waist of the beam after the lens, $l_2$ is the distance from the lens to the image recording plane, $R_0$ is the confocal parameter of the original laser beam, and $R_{01}$ is the confocal parameter of the focused laser beam. The confocal parameters $R_0$ and $R_{01}$ are given by

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**Fig. 2.7** Single-lens optical system for forming the laser plane. **a** View in the $xoz$ plane. **b** View in the $yoz$ plane. **c** Cross-section of the SLR formed. 1—laser; 2—cylindrical lens; 3—image recording plane
2.3 SLR Formation Based on Optical Elements

\[ R_0 = \frac{\pi w_0^2}{\lambda}, \quad R_{01} = \frac{\pi w_1^2}{\lambda}. \]

The radius \( w_1 \) of the new waist is calculated by the Gaussian beam transformation formula 2

\[ w_1 = w_0 \left[ \left( 1 - \frac{l_0}{f} \right)^2 + \frac{R_0^2}{f^2} \right]^{-1/2}, \]

where \( f \) is the focal length of the cylindrical lens.

The position of the waist of the beam after the lens is defined by the relation:

\[ 1 - \frac{l_1}{f} = \left( 1 - \frac{l_0}{f} \right) \left[ \left( 1 - \frac{l_0}{f} \right)^2 + \frac{R_0^2}{f^2} \right]^{-1}. \] (2.14)

If \( R_{01} > R_0 \), the divergence of the astigmatic beam after the lens along the \( x \)-axis will be greater than that along the \( y \)-axis. The ratio between the dimensions of this beam is

\[
\frac{h(L)}{t(L)} = \frac{\left[ 1 + \left( \frac{L - l_0 - l_1}{R_{01}} \right)^2 \right]^{1/2}}{\left[ 1 + \left( \frac{L}{R_0} \right)^2 \right]^{1/2} \left[ \left( 1 - \frac{l_0}{f} \right)^2 + \left( \frac{R_0}{f} \right)^2 \right]^{1/2}}. \] (2.15)

Figure 2.8 shows the astigmatism of a Gaussian beam as a function of its propagation distance.

The single-lens laser plane forming system has the advantage of simplicity, but suffers from a substantial shortcoming: because of the strong divergence of the beam, its power density lowers materially as its propagation distance grows longer.

**Double-Lens System.** A laser plane with a small divergence angle can be obtained by means of two cylindrical lenses forming a telescopic system (Fig. 2.9).

The dimensions of an astigmatic beam are defined by the relations

\[ h(L) = 2w_1 \left[ 1 + \left( \frac{l_2 - l_1}{R_{01}} \right)^2 \right]^{1/2}, \quad t(L) = 2w_0 \left[ 1 + \left( \frac{L}{R_0} \right)^2 \right]^{1/2}, \]

where \( L = l_0 + (n-1)h_1 + l_1 + l_2 + (n-1)h_2 + l_3 \), \( h_1 \) is the thickness of the first lens, \( h_2 \) is the thickness of the second lens, and \( n \) is the refractive index of the lenses. The distances \( l_1 \) and \( l_2 \) are calculated by the Gaussian beam transformation formulas.

**Triple-Lens Optical System.** In the single- and double-lens optical systems analyzed above, the beam thickness in the \( xoz \) plane at a distance of \( L \) is governed by
the divergence of the beam issuing from the laser. To reduce the thickness of the laser plane at the location of the optical inhomogeneity of interest, it is necessary to additionally use a spherical long-focus lens. The optical scheme of the triple-lens system for forming the laser plane is presented in Fig. 2.10. It comprises three

Fig 2.8 Astigmatism of a Gaussian beam as a function of its propagation distance for \( \lambda = 0.6328 \, \mu m, w_0 = 0.3 \, mm, \) \( f = 5 \, mm, \) and \( l_0 = 10 \, mm \)

![Graph showing astigmatism of a Gaussian beam](image)

Fig. 2.9 Optical system to form a low-divergence laser plane. a View in the xoz plane. b View in the yoz plane. l—laser; 2 and 3—cylindrical lenses; 4—plane of observation
The Besselian beam is a laser radiation model having a number of specific features [8], namely, the distribution of intensity over its cross-section is described with the aid of Bessel functions of varying order, the beam is centrally symmetric and retains its cross-sectional structure during the course of propagation (in practice, this property maintains only for a certain distance from the radiation source). Such beams were first studied and obtained experimentally in 1987.

Distinguished are several kinds of Besselian beams, from the simplest classical Besselian beam described by a Bessel function of order zero to complex tubular models that are classed with Besselian beams only as regards the complete set of the attributes described above [9].

The use of Besselian beams in the diagnostics of optically inhomogeneous media holds much promise, thanks to their high spatial properties (high spatial concentration of radiation within a narrow laser pencil or tubular beam). This makes it possible to obtain narrow laser probes, and study with them inhomogeneities in media at long distances.

There are several optical schemes for forming Besselian beams, most widespread among them being those built around conical lenses—axicons—and round apertures [9].

The field of a Besselian beam propagating along the \( z \)-axis in the cylindrical coordinate system \((r, \psi, z)\) is described as follows:
where $A$ is the maximum field amplitude, $J_0(x)$ is the Bessel function of the first kind of order zero, $\theta_0$ is the angular parameter of the beam, $n$ is the refractive index of the medium wherein the beam propagates, and $k_0$ is the wave number in vacuum. The beam amplitude is independent of the angle $\psi$; i.e., it is centrally symmetric. Figure 2.11 shows the transverse intensity distribution in the zero-order Besselian beam.

The Besselian beam is peculiar in that its phase varies in the $z$-direction at a rate of $v=c/(n \cos \theta_0)$ higher than $v=c/n$, the shape of the beam remaining unchanged during propagation (the amplitude being independent of $z$). This situation is akin to the one wherein the interference of two plane waves is observed, the only difference being that the interference field fails to occupy the entire space, but is practically wholly concentrated in a certain zone ($k_0nr \sin \theta_0 < 2.4$, where 2.4 is the first zero of the function $J_0$). Thus, the Besselian beam can be represented as the result of interference of a multitude of plane waves propagating at an angle of $\theta_0$ to the $z$-axis. Let us demonstrate how expression (2.16) can be derived from this assumption.

Let us assume that in the spherical coordinate system $\rho$, $\theta$, $\varphi$, where the angle $\theta=0$ corresponds to the positive direction of the $z$-axis, there are a set of plane waves of the same amplitude $A d\varphi$, whose propagation directions, defined as $s=\alpha i + \beta j + \gamma k$, make the same angle $\theta_0$ with the $z$-axis. Each of such fields may be written down in the rectangular coordinate system as

$$E(r, \psi, z) = AJ_0(k_0nr \sin \theta_0) \exp(jk_0nz \cos \theta_0), \quad (2.16)$$

Fig. 2.11 Transverse intensity distribution in the Besselian beam
\[ E(x, y, z) = A \exp(jk_0n(\alpha x + \beta y + \gamma z)) d\varphi, \] (2.17)

where \( \alpha = \sin \theta_0 \cos \varphi, \beta = \sin \theta_0 \sin \varphi, \gamma = \cos \theta_0. \) If we integrate expression (2.17) with respect to \( d\varphi \) from 0 to \( 2\pi \), then, with the properties of the Bessel function (the central symmetry of the beam and the presence of the principal maximum at the center) known, we arrive at expression (2.16):

\[
A \int_0^{2\pi} \exp(jk_0n(\alpha x + \beta y + \gamma z)) d\varphi
\]

\[
= A \exp(jk_0nz \cos \theta_0) \left( \int_0^{2\pi} A \exp(jk_0n \rho (\cos \varphi \cos \psi + \sin \varphi \sin \psi) \sin \theta_0) d\varphi \right)
\]

\[
= 2\pi AJ_0(kn\rho \sin \theta_0) \exp(jk_0nz \cos \theta_0). \] (2.18)

One of the most important problems associated with Besselian beams is the problem of their formation. A most simple way to obtain a Besselian beam is to use an axicon (Fig. 2.12). If a conical lens—an axicon—is placed in the path of a wide Gaussian beam, the beams deflected by it will interfere, producing a complex intensity distribution pattern. It will have the form of a bright central maximum surrounded by a system of rings.

In numerical modeling, one can use Besselian beam models specified in the MathCAD program by Bessel functions of order \( n, J_n(k \sin(\theta) \sqrt{x^2 + y^2}). \) In the MathCAD environment, the function \( J_n(x) \) is sought as the solution of the differential equation \( x^2(d^2/dx^2)y + x(d/dx)y + (x^2 - n^2)y = 0, \) where the function \( y(x) \) is the solution. In addition, the beam is specified by the wave number \( k \) and the axicon angle \( \theta. \) The axicon angle is the parameter determining the cross-sectional size of the beam. The physical meaning of this angle is illustrated in Fig. 2.13. In the general case, the Besselian beam results from the interference of plane waves propagating, thanks to the axicon, at the same angle to the optical axis, their propagation directions forming a cone. Its apex angle is the axicon angle.
Figure 2.14 shows the intensity distribution for Besselian beams of the first three orders. One can see from the above plots that the principal Besselian mode—the zero-order beam—has its principal maximum at the center, whereas the modes of higher orders are tubular beams, the beam “smearing out” with increasing mode order. Note that as the axicon angle increases, the cross-section of the beams narrows.

2.4 Formation of SLR on the Basis of Diffraction Gratings

Amplitude Gratings. Multiple-beam SLR can be obtained by means of diffraction gratings and an objective lens. Use can be made of both one- and two-dimensional amplitude and phase gratings. Figure 2.15a shows the optical scheme of formation.
of a multitude of individual beams with a two-dimensional amplitude grating. Here radiation from laser 1 successively passes through long-focus collective lens 2, diffraction grating 3, and fast lens 4 at whose exit a set of parallel laser beams 5 are formed that can be observed on diffusing screen 6. The cross-sectional appearance of the SLR formed is shown in Fig. 2.15b.

As demonstrated in Sect. 2.12, the double-lens system can make laser beams have the desired size at the location of the optical inhomogeneity under study in an optimal way.

The distances $\Lambda_x$ and $\Lambda_y$ between the beams along the $x$- and $y$-axes, respectively, are governed by the periods of the diffraction grating and the focal length $f$ of the objective lens: $\Lambda_x = \lambda f/d_x$, $\Lambda_y = \lambda f/d_y$, where $d_x$ and $d_y$ are the periods of the grating along the $x$- and $y$-axes, respectively.

**Phase Gratings.** SLR can be formed by means of dynamic phase gratings. Such gratings are formed owing to the acousto-optic effect occurring during propagation of ultrasonic waves in transparent media. Such media may be represented by liquids, glasses, and crystals. Ultrasonic waves are generated by piezoelectric emitters. The Raman-Nath diffraction observed in the frequency range 3–20 MHz yields a multitude of diffracted beams. Figure 2.16 illustrates the formation of SLR by means of a phase diffraction grating. Here radiation from laser 1 passes through long-focus lens 2 and ultrasonic modulator 3 supplied from high-frequency generator 4. Following the modulator there a set of diffracted divergent beams appear that are transformed with objective lens 5 into SLR 6 observed on screen 7.

When ultrasonic waves 10 MHz in frequency propagate in water with a velocity of 1,500 m/s, a periodic variation of the refractive index of the medium takes place, the variation period being equal to the length of the acoustic wave, 150 $\mu$m in our case. The angle the first diffracted wave makes with the original beam (the diffraction angle) for a wavelength of $\lambda = 0.6328$ $\mu$m amounts to 0.0042 rad. If use is made of an objective lens 500 mm in focal length, the distance between the parallel beams will be 2.1 mm. The number of diffracted waves depends on the power capacity of the generator and can come to a few tens.
This method of formation of linear SLR has the advantages of simplicity and capability of readily changing the distance between the parallel beams by varying the frequency of the generator. The shortcomings include the nonuniform distribution of laser power among the diffracted beams and narrow variation range of the grating period.

2.5 Formation of SLR on the Basis of Diffraction Elements

The diffraction optical elements (DOEs) that have only recently become commercially available have the form of a thin phase plate with a special phase relief laser engraved in it [10]. The diffraction of laser radiation by such an optical element produces various kinds of spatially modulated radiation known as structured laser radiation. DOEs are used with both gas and semiconductor lasers generating highly astigmatic beams. DOEs can help obtain SLR of practically any structure, adapted for solving problems in the diagnosis of fields of gradient refractive indices and other physical quantities.

At present, DOEs are used to split laser radiation, in spectroscopy, metrology, and multi-perspective measuring systems. DOEs were first used to obtain SLR for studies into optically inhomogeneous media, and also for their visualization from scattered radiation in [1, 2]. The original elliptical or high-divergence beam from a semiconductor laser is transformed with a corrector lens into an almost axially symmetric beam that is next converted with a diffraction optical element equivalent to a linear diffraction grating into a set of divergent beams arranged in a single plane. The angle between the adjacent beams is \( \Theta = \frac{\lambda}{\Lambda} \), the total divergence angle being given by

![Diagram](image-url)
where \(2m + 1\) is the number of diffracted beams, \(m\) is the diffraction order, \(\lambda\) is the laser radiation wavelength, and \(\Lambda\) is the period of the diffraction grating. To illustrate, at \(m = 5\), \(\lambda = 0.68\ \mu\)m, and \(\Lambda = 6.8\) mm the total divergence angle for eleven beams is \(\Theta_{\text{tot}} = 27^\circ\).

A diffraction optical element equivalent to a cylindrical lens can help obtain from a laser beam a divergent laser plane whose cross-section is a straight line. A cylindrical-lens-cum-diffraction-grating combination makes it possible to obtain a number of such lines, usually 5, 9, 11, 15, 33, 99. The number of lines can exceed 100. These beams are divergent. To illustrate, for a diffraction optical element with a period of 200 \(\mu\)m that produces 99 lines, the angle between the adjacent lines amounts to 0.15\(^\circ\), the total divergence angle being equal to 14.47\(^\circ\).

The use of DOEs with more intricate diffraction reliefs allows obtaining SLR of more complex forms: a cross-shaped beam comprising two laser planes arranged at right angles, a conical beam, or a set of conical beams, etc. [10].

Table 2.2 presents the main types of SLR obtained with commercially available DOEs [3].
References


5. N. V. Karlov, Lectures on Quantum Electronics (Nauka, Moscow, 1983) [in Russian].

6. A. M. Gonchaenko, Gaussian Light Beams (Nauka i Tekhnika, Minsk, 1977) [in Russian].


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Rinkevichus, B.S.; Evtikhieva, O.A.; Raskovskaya, I.L.
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