Chapter 2
Lead Screws

2.1 Screw Threads

The screw is the last machine to joint the ranks of the six fundamental simple machines. It has a history that stretches back to the ancient times. A very interesting historical account of the development of screws from Archimedes’ water snail to the works of Leonardo da Vinci and up to the twentieth century is given by Mac Kenzie [33].

The mechanics of a screw is similar to two other simple machines, namely; the inclined plane and the wedge. As shown in Fig. 2.1, a screw can be considered as an inclined plane wrapped around a cylinder. Similar to the inclined plane, the horizontal force \( F \) needed to raise a weight \( W \) is

\[
F \geq \left( \frac{\mu + \tan \lambda}{1 - \mu \tan \lambda} \right) W,
\]

where \( \mu \) is the coefficient of friction of the two rubbing surfaces and \( \lambda \) is the lead angle (equivalent to the angle that the inclined plane makes with the horizon).

Figure 2.2 compares a screw with a wedge. Here, instead of moving the load, the wedge is pushed under the load to raise it. The screw equivalent of this mechanism operates by applying a torque \( T \) to the screw to push the load upward turn by turn. Here the torque \( T \) needed to raise a weight \( W \) is

\[
T \geq r \left( \frac{\mu + \tan \lambda}{1 - \mu \tan \lambda} \right) W.
\]

The above force mechanisms are shared by both fastening screws and translating screws. The screws in the latter group – studied in this monograph – are commonly known as lead screws and are used for transmitting force and/or positioning by converting rotary to translational motion. In power transmission applications, lead screws are also known as “power screws” [34, 35]. When used in vertical applications, these systems are sometimes called “screw jacks” [1].

There are a number of thread geometries available for lead screws that are designed to address various requirements such as ease of manufacturing, load-carrying capacity, and the quality of fit [33]. The most popular of these geometries are the Acme and stub-Acme threads.\(^1\) Figure 2.3 shows the basic dimensions of symmetric trapezoidal threads (e.g., Acme threads). The thread angle (\( \psi_a \)) for Acme and stub Acme thread is 14½°. The basic relationships defining the screw geometry are given next for future reference.\(^2\) The lead angle (or helix angle), \( \lambda \), is defined as

\[
\tan \lambda = \frac{l}{\pi d_m},
\]

\(^1\)This design is further discussed in Sect. 2.3.

\(^2\)See [33] for specifications of other types of screw threads.
where \( d_m \) is the pitch diameter and \( l \) is the lead and it is defined as

\[
l = n_s \cdot p,
\]

(2.2)

where \( p \) is the screw pitch (distance between identical points of two consecutive threads) and \( n_s \) is the number of starts (or starts). Figure 2.4 shows three 1-in. lead screws with one, two, and ten starts.

Increasing the number of starts increases the lead thus increasing the translational velocity of the nut for a given fixed angular velocity of the screw. Based on (2.1) and (2.2) the lead angles for these screws are found as follows: \( \lambda_{(A)} \approx 5.20^\circ \), \( \lambda_{(B)} \approx 10.31^\circ \), and \( \lambda_{(C)} \approx 18.52^\circ \). In these examples, the pitch diameter was found according to the following equation: \( d_m = D - (p/2) = d + (p/2) \).

### 2.2 Lead Screw Engineering

For design and selection purposes, the mechanical analysis of lead screws usually is limited to the factors affecting their static or quasi-static performance, such as efficiency, driving torque requirements, and load capacity [33–35]. There are
numerous important aspects involved in the successful design of a lead screw drive system. Some of these issues are summarized in Fig. 2.5. It is important to mention that, to some degree, almost all of these issues influence the other aspects of the lead screw design.

Manufacturers offer a wide range of products in response to the diverse applications where lead screws are utilized. For positioning stages, high precision ground lead screws with or without anti-backlash nuts are offered as an alternative to the more costly but much more efficient ball screw-driven stages [36, 37].

In addition to their lower cost compared to ball screws, there are a number of distinct features that make a lead screw drive the favorable choice – if not the only choice – in many applications. These features include [38–40] the following:

- Quieter operation due to the absence of re-circulating balls used in ball screws.
- Smaller moving mass and smaller packaging.
- Availability of high helix angles resulting in very fast leads.
- Availability of very fine threads for high resolution applications.
- Possibility of self-locking to prevent the drive from being backdrivable thus eliminating the need for a separate brake system.
- Lower average particulate generation over the life of the system.
- Elimination of the need for periodic lubrication with the use of self-lubricating polymer nuts.
- Possibility to work in washed-down environments.

Design factors given in Fig. 2.5 are discussed by the manufacturers as part of their public technical information or product selection guidelines (see, e.g., [41–46]). There is, however, a major exception: friction-induced vibration. Only a few published works are found in the literature that discuss the dynamics of lead screw drive systems and the effect of friction on their vibratory behavior.3

Wherever sliding motion exists in machines and mechanisms, friction-induced vibration may occur, and when it does, it severely affects the function of the system. Excessive noise, diminished accuracy, and reduced life are some of the adverse consequences of friction-induced vibration. To this end, lead screw systems are no exception; the lead screw threads slide against meshing nut threads as the system operates.

One of the common issues in using lead screws – especially for the positioning applications – is backlash. As shown in Fig. 2.6, backlash is the axial distance the nut can be moved without turning the lead screw. Among the problems caused by backlash are the deterioration of the positioning accuracy and diminished repeatability of the performed task by the lead screw drive. Both design and/or manufacturing factors may contribute to the presence of backlash in a lead screw drive. Various anti-backlash nuts are designed and offered by the manufacturers to address these problems. These nuts generally are made of two halves connected

3See Sect. 1.2.
with preloaded springs that can move with respect to one another to compensate backlash and wear \cite{36, 41–44}. The drawback of using these nuts is in the increased friction force, which lowers the efficiency and increases the required driving torque.\footnote{See Sect. 5.4 for a mathematical model of a lead screw with an anti-backlash nut.}

\textbf{Fig. 2.5} Lead screw design and selection factors
2.3 Lead Screw and Nut: A Kinematic Pair

The rotary motion is converted to linear translation at the interface of lead screw and nut threads. The kinematic relationship defining a lead screw is simply

\[ x = r_m \tan \lambda \theta, \]

(2.3)

where \( \theta \) is the lead screw rotation, \( x \) is the nut translation, \( \lambda \) is the lead angle, and \( r_m \) is the pitch circle radius.

The interaction between the contacting lead screw and nut threads can be easily visualized by considering unrolled threads (see Figs. 2.1 and 2.2). This way, the rotation of lead screw is replaced by an equivalent translation. Assuming one thread pair to be in contact at any given instant, Fig. 2.7 shows the interaction of the lead screw and nut threads for both left-handed and right-handed screws. The sign conventions used for the contact force, \( N \), is shown in this figure. In the configurations shown, when the right-handed lead screw is rotated clockwise/moved up (rotated counterclockwise/moved down) the nut moves backward/right (forward/left). For the left-handed screw, the direction of motion of the nut is reversed. Also, when the nut threads are in contact with the leading (trailing) lead screw threads, the normal component of contact force, \( N \), is considered to be positive (negative).

The friction force is given by

\[ F_f = \mu |N| \text{sgn}(v_s), \]

(2.4)

where \( \mu \) is the coefficient of friction (possibly velocity dependent) and \( v_s \) is the relative sliding velocity. The friction force acts tangent to the contacting thread surfaces and always opposes the direction of motion but does not change direction when normal force, \( N \), changes direction.

---

5By properly orienting the \( x \)-axis, this relationship applies to both left-hand and right-hand threads.
2.4 Effect of Thread Angle

Before moving on to the dynamic models of lead screw systems, the effect of thread geometry on the contact forces is considered here. The force interaction shown in Fig. 2.7 is essentially correct for the square threads where the normal force is parallel to the lead screw axis. For Acme or other types of threads, a slight modification is needed to take into account the thread angle.

Figure 2.8 shows the thread semi-angles as measured on a section through the axis of a screw, $\psi_a$, and as measured on a section perpendicular to the helix, $\psi_n$.

Using the geometric relationship in Fig. 2.9, one can write [47]

$$\tan \psi_n = \frac{x_n}{y}, \quad \tan \psi_a = \frac{x_a}{y},$$

(2.5)

$$x_n = x_a \cos \lambda.$$  

(2.6)
Combining (2.5) and (2.6) gives

\[ \tan \psi_n = \tan \psi_a \cos \lambda. \]

Figure 2.10 shows a portion of a lead screw with localized contact force \( \hat{N} \) (perpendicular to the thread surface) and friction force \( F_f \). The \( X \)-axis of \( XYZ \) coordinate system is parallel to the lead screw axis. The \( x-Z \) plane is perpendicular to the helix. The projection of contact force on the \( x-y \) (or \( X-Y \)) plane is calculated as

\[ N = \hat{N} \cos \psi_n. \quad (2.7) \]

Since \( \hat{N} \) is the normal force, using (2.4) the friction force for trapezoid threads is calculated by \( F_f = \hat{\mu} \hat{N} \text{sgn}(\dot{\theta}) \), where \( \hat{\mu} \) is the true coefficient of friction. One can define the apparent coefficient of friction as
\[
\mu = \hat{\mu} \cos \psi_n = \frac{\mu}{\sqrt{\tan^2 \psi_n \cos^2 \lambda + 1}}.
\] (2.8)

Using (2.7) and (2.8), the friction force is written conveniently as
\[F_f = \mu|N|\text{sgn}(\dot{\theta}),\]
which is the same as (2.4) and will be used in the subsequent chapters.
Friction-Induced Vibration in Lead Screw Drives
Vahid-Araghi, O.; Golnaraghi, F.
2011, X, 214 p., Hardcover