Preface

The third edition differs from the second mainly in that parts of the text have been elaborated upon in more detail. Moreover, some new sections have been added, for instance a separate section on Horn formulas in Chapter 4, particularly interesting for logic programming. The book is aimed at students of mathematics, computer science, and linguistics. It may also be of interest to students of philosophy (with an adequate mathematical background) because of the epistemological applications of Gödel's incompleteness theorems, which are discussed in detail.

Although the book is primarily designed to accompany lectures on a graduate level, most of the first three chapters are also readable by undergraduates. The first hundred twenty pages cover sufficient material for an undergraduate course on mathematical logic, combined with a due portion of set theory. Only that part of set theory is included that is closely related to mathematical logic. Some sections of Chapter 3 are partly descriptive, providing a perspective on decision problems, on automated theorem proving, and on nonstandard models.

Using this book for independent and individual study depends less on the reader's mathematical background than on his (or her) ambition to master the technical details. Suitable examples accompany the theorems and new notions throughout. We always try to portray simple things simply and concisely and to avoid excessive notation, which could divert the reader's mind from the essentials. Line breaks in formulas have been avoided. To aid the student, the indexes have been prepared very carefully. Solution hints to most exercises are provided in an extra file ready for download from Springer's or the author's website.

Starting from Chapter 4, the demands on the reader begin to grow. The challenge can best be met by attempting to solve the exercises without recourse to the hints. The density of information in the text is rather high; a newcomer may need one hour for one page. Make sure to have paper and pencil at hand when reading the text. Apart from sufficient training in logical (or mathematical) deduction, additional prerequisites are assumed only for parts of Chapter 5, namely some knowledge of classical algebra, and at the very end of the last chapter some acquaintance with models of axiomatic set theory.

On top of the material for a one-semester lecture course on mathematical logic, basic material for a course in logic for computer scientists is included in Chapter **4** on logic programming. An effort has been made to capture some of the interesting aspects of this discipline's logical foundations. The resolution theorem is proved constructively. Since all recursive functions are computable in PROLOG, it is not hard to deduce the undecidability of the existence problem for successful resolutions.

Chapter **5** concerns applications of mathematical logic in mathematics itself. It presents various methods of model construction and contains the basic material for an introductory course on model theory. It contains in particular a model-theoretic proof of quantifier eliminability in the theory of real closed fields, which has a broad range of applications.

A special aspect of the book is the thorough treatment of Gödel's incompleteness theorems in Chapters **6** and **7**. Chapters **4** and **5** are not needed here. **6.1**[1] starts with basic recursion theory needed for the arithmetization of syntax in **6.2** as well as in solving questions about decidability and undecidability in **6.5**. Defining formulas for arithmetical predicates are classified early, to elucidate the close relationship between logic and recursion theory. Along these lines, in **6.5** we obtain in one sweep Gödel's first incompleteness theorem, the undecidability of the tautology problem by Church, and Tarski's result on the nondefinability of truth, all of which are based on certain diagonalization arguments. **6.6** includes among other things a sketch of the solution to Hilbert's tenth problem.

Chapter **7** is devoted mainly to Gödel's second incompleteness theorem and some of its generalizations. Of particular interest thereby is the fact that questions about self-referential arithmetical statements are algorithmically decidable due to Solovay's completeness theorem. Here and elsewhere, Peano arithmetic (PA) plays a key role, a basic theory for the foundations of mathematics and computer science, introduced already in **3.3**. The chapter includes some of the latest results in the area of self-reference not yet covered by other textbooks.

Remarks in small print refer occasionally to notions that are undefined and direct the reader to the bibliography, or will be introduced later. The bibliography can represent an incomplete selection only. It lists most

[1] This is to mean Section **6.1**, more precisely, Section **1** in Chapter **6**. All other boldface labels are to be read accordingly throughout the book.

English textbooks on mathematical logic and, in addition, some original papers mainly for historical reasons. It also contains some titles treating biographical, historical, and philosophical aspects of mathematical logic in more detail than this can be done in the limited size of our book. Some brief historical remarks are also made in the *Introduction.* Bibliographical entries are sorted alphabetically by author names. This order may slightly diverge from the alphabetic order of their citation labels.

The material contained in this book will remain with high probability the subject of lectures on mathematical logic in the future. Its streamlined presentation has allowed us to cover many different topics. Nonetheless, the book provides only a selection of results and can at most accentuate certain topics. This concerns above all Chapters **4**, **5**, **6**, and **7**, which go a step beyond the elementary. Philosophical and foundational problems of mathematics are not systematically discussed within the constraints of this book, but are to some extent considered when appropriate.

The seven chapters of the book consist of numbered sections. A reference like Theorem 5.4 is to mean Theorem 4 in Section **5** of a given chapter. In cross-referencing from another chapter, the chapter number will be adjoined. For instance, Theorem 6.5.4 means Theorem 5.4 in Chapter **6**. You may find additional information about the book or contact me on my website www.math.fu-berlin.de/~raut. Please contact me if you propose improved solutions to the exercises, which may afterward be included in the separate file *Solution Hints to the Exercises*.

I would like to thank the colleagues who offered me helpful criticism along the way. Useful for Chapter **7** were hints from Lev Beklemishev and Wilfried Buchholz. Thanks also to Peter Agricola for his help in parts of the contents and in technical matters, and to Michael Knoop and David Kramer for their thorough reading of the manuscript and finding a number of mistakes.

Wolfgang Rautenberg, June 2009

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