Chapter 2
Introduction to Part I Modeling: What Is It? Why Do It?

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At ICTMA-13, where the chapters in this book were first presented, a variety of views were expressed about an appropriate definition of the term *model* – and about appropriate ways to think about the nature of *modeling activities*. So, it is not surprising that some participants would consider this lack of consensus to be a priority problem that should be solved by a research community that claims to be investigating models and modeling.

We certainly agree that conceptual fuzziness is not a virtue in a research community – especially if it impedes communication among members of the community. Furthermore, we agree that increasing clarity about key constructs is an important goal of research. Nonetheless, we also believe that, especially at early stages of theory development, a certain amount of diversity in thinking is as healthy for research communities as it is for (for example) engineers who are at early "brainstorming" stages in the design of space shuttles, sky scrapers, or transportation systems. Furthermore, we believe that the mathematics education research community in particular has suffered from more than enough pressure for premature ideological orthodoxy.

The theme of ICTMA-13 was *modeling students' modeling competencies*; and, in many of the research methodologies that are described throughout this book, students develop models to describe or design "real life" artifacts or tools; teachers develop models to describe students' modeling competencies – or to design productive learning environments; and, researchers develop models of interactions among students, teachers, and learning environments. So, the goal of many of our most productive studies focus on the development of powerful, sharable, and re-useable models – which then, in turn, influence theory development. But, model

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R. Lesh et al. (eds.), *Modeling Students' Mathematical Modeling Competencies*,
DOI 10.1007/978-1-4419-0561-1_2, © Springer Science+Business Media, LLC 2010

development research is not the same as theory development research. For example, in theory-driven research, the theory determines what questions are appropriate to ask, what kind of evidence needs to be collected, how the information should be interpeted, analyzed, and assessed – and when the question has been answered or the issue resolved. But, in model-development research (or design research), the problem arises from a "real world" decision-making situation. Thus, this work is more like engineering than a "pure" science. As engineers (and other design scientists) often emphasize (see Zawojewski et al., 2009):

- Engineering is the science of solving "real life" problems where you don't have enough time, or money, or other resources – and when multiple stake holders often hold partly conflicting views about the nature success (low costs versus high quality, simplicity versus completeness, and so on).
- Realistic solutions to realistically complex problems usually need to integrate ideas and procedures drawn from more than a single discipline, or theory, or textbook topic area.
- When high stakes decision making issues arise in "real life" situations, it usually is important to design for success (power, sharability, re-usability) – not simply to test for it.
- In design research, many of the "things" that we most need to understand and explain are "things" that we ourselves are in the process of developing or designing. For example, in math/science education, these "things" range from students' conceptual systems to curriculum materials which include systems of activities for learning or assessment. And, in each of these cases, as soon as we come to better understand the "thing" being developed or designed, we tend to change them – so that another cycle of adaptation is needed. (Note: This is one reason why scientists do not speak of developing a single theory of space shuttles.)

In the book, *Beyond Constructivism: Models & Modeling Perspectives on Mathematics Problem Solving, Learning & Teaching* (Lesh and Doerr, 2003), we describe a variety of reasons why we consider MMP to be a "blue color" perspective which is like engineering more than it is like "pure" sciences such as physics, chemistry, or mathematics. In fact, for many of the same reasons why *Pragmatists* such as James, Pierce, or Dewey considered *Pragmatism* to be more of a framework for developing theories rather than being a theory in itself, we consider MMP to be a framework for developing models of students' modeling.

According to MMP, students, teachers, and researchers – all are in engaged in model development. Consequently, MMP research assumes that: (a) similar principles apply not only to the model development activities of students, but also to the model-development activities of teachers and researchers, and (b) researchers' early-iteration models are expected to be characterized by conceptual inadequacies similar to those that characterize the early-iteration interpretation systems of students or teachers.... MMP research recognizes that we – the researchers who study modeling – are humbly still in the early phases of *our own* model development. And, just as in the model development activities of the students and teachers that we study, we assume that we will only make progress if we go through multiple cycles of expressing > testing > revising our current ways of thinking. So just as Darwin emphasized in the development of other kinds of complex adaptive systems (1859), evolution of ways of thinking about teaching and students' learning is only likely to occur if provisions are made to encourage diversity, selection, communication, and accumulation (Sawyer, 2006).

In our own "blue collar" research on models and modeling, we take the following to be a useful first-iteration definition of a model. *A model is a system for describing (or explaining, or designing) another system(s) for some clearly specified purpose.*

The preceding definition seems simple and straightforward, but one thing that we like about it is that it has a clear history of pressing us toward important researchable questions. Furthermore, best of all, and unlike terms such as *cognitive structures* or *schemes* or other terms that are favored by cognitive scientists or constructivist philosophers, the preceding definition uses the term *model* in the same way that the term is used in mature science fields like physics or engineering. That is, a model is system that is used to describe (or interpret) another system of interest in a purposeful way. However, whereas physicists and engineers tend to focus on only the written symbolic aspects of the models they develop, when math/science educators investigate what it means to "understand" these models, it inevitably becomes apparent that the written and spoken embodiments that scientists emphasize scientists usually represent only something like the tip of an iceberg. For example, in order for scientific models to be useful, understanding them usually involves a variety of diagrams, concrete models, experience-based metaphors, and other expressive media – in addition to technical spoken language and symbol systems, each of which emphasize some aspects (but deemphasize others) for the "thing" that they are used to describe, or explain, or design. Furthermore, model development often involves dimensions of development such as intuition-to-formalization, concrete-to-abstract, situated-to-decontextualized, specific-to-general, implicit-to-explicit, global-to-analytic,[1] and so on. Furthermore, during early stages of development, models often function in ways that are more like "windows" that we look through rather than as "objects" that we look at; and, they often function in ways that are rather unstable. That is, when the model developer focuses on "forest-level" or "big picture" interpretations of the "thing" being described, they often neglect to keep in mind "tree-level" details about the "thing" being described. In other words, even for experienced scientists, but especially for young students, and especially during early stages of model development, a model developer's early interpretation of the "thing" being described is often far more situated, piecemeal, and non-analytic than most traditional theories of learning consider it to be.

Should early interpretations (such as those that function intuitively, informally, or in piecemeal or unstable ways) be referred to as models? We believe that such

[1] When we speak of these dimensions of development, we recognize that, in specific "real life" situations where model development is needed, the most useful model is not necessarily the one that is most complex, most formal, most abstract, or most decontextualized.

questions should be resolved through research – not through political consensus building. This point brings out a point that we especially like about the term "models and modeling". That is the term allows room for debates among researchers with opposing points of view. And, we believe that debates about the nature of various models have exhibited a clear history of leading to researchable questions, testable hypotheses, and a variety of model validation activities. This is why we say that Models and Modeling is not so much a view of how learning works as it is a methodological approach and framework for investigating learning.

Finally, we should not neglect to mention some other reasons why authors throughout this book view research on models and modeling to be especially important for mathematics education researchers. First, in nearly every field where researchers have investigated similarities and differences between experts and novices (or between successful learners or problem solvers versus those who are less successful), results have shown that expertise not only involves *doing* things differently (or better), it also involves *seeing* or *interpreting* things differently (or better). And, in mathematics and the natural sciences, interpretation development means model development. Furthermore, in students' lives beyond school, the ability to describe or explain things mathematically is one of the main factors that is needed in order for mathematics to be useful.

Throughout this volume, even though most of the authors generally support the ideas outlined above, conflicting views emerged – even during ICTMA-13. One reason why this happened is that the organizers of ICTMA-13 made special efforts to attract not only mathematics educators from more than 25 different countries, but equal efforts also were made to attract leading science educators, engineering educators, and mathematics educators focusing on mathematical and scientific thinking beyond school. Yet, even with such diversity, we know of no cases where differences in perspectives did not appear to be leading toward productive adaptations for the community as a whole.

This volume beings with chapters that aim to answer the fundamental question: what are models? Hestenes outlines important relationships among various forms of modeling and fields that apply modeling theory. In his plenary article, he distinguishes between uses of the term "model" to refer to a conceptual mental model and uses of "model" to refer to a publically shared model. By so doing, he sets the stage for a discussion that relates the act of modeling in professional and non-professional so-called "everyday life" to research in both science and mathematics education, thus giving a full view of modeling theory as describing the process by which a form of knowledge, modeling, is developed through lived acts that are both mental and social in nature. Niss continues this examination of the question of what models are and what modeling is with studies into how to model the learning of modeling itself. His study hints at perhaps the deepest and most philosophical of all questions related to the origin of modeling competencies and knowledge: What is the origin of understanding how to model? From his study of three different modeling problems, he suggests that the capability to model is a learned capacity. The teaching of modeling, he notes, can be approached through activities that encourage cognitive dissonance of a type that drives the emergent modeling of Gravemeijer –

or the model eliciting activities described by Zawojewski or Carmona, or the kind of scaffolded approaches described by Kaiser. Niss considers each as possible modes for teaching modeling. Larson and colleagues also address the question of what models are while also looking ahead to the future of modeling in the curriculum and examining ways modeling can meet the challenge to provide students opportunities to "create and refine ideas for themselves."

With a notion about what models are, discussion then proceeds to examinations of where models and the modelers who make them are found. Noss and Hoyles take research about the pedagogical aspects of modeling beyond the classroom into the workplace, performing an investigation as to what "Techno-mathematical Literacies" are needed to understand manufacturing processes. Cardella examines the modeling of engineering undergraduates and graduates who might not otherwise see the connection of the math they are learning in the classroom to the real world work they will perform. This study is followed by Alpers who examines the modeling of mechanical engineers outside of school contexts, looking at math and modeling performed on the job. Together, these studies show both the application and real-world relevance of modeling in studies beyond the school setting – while at the same time demonsrating as well as to in-school modeling studies have to studies in other disciplines, and in higher level mathematics course.

The modeling process is examined by Larson by employing one of Lesh's original modeling activities, "Summer Jobs", in a study of how students develop quantitative reasoning needed in real-world problems, supporting concluding that changes in the perception students have of relationships among quantities leads to the development of better quantitative reasoning ability along a progression of Piagetian-like stages. Using a related activity, "University Cafeteria", Mousoulides and Sriraman examine the progression in mathematical understandings made by middle school students over the course of their schooling.

The question of what is needed for modeling to occur is taken on by Galbraith, Stillman, and Brown, who provide insight into what has been recognized as one of the most critical aspects of setting up a modeling experience for learners – that is, creating a meaningful context in which it is hoped the modeling learners will engage in significant concepts. They explore this from a uniquely Australian perspective, investigating the response of Australian students to Australian themed modeling activities. Haines and Crouch further investigate this issue while exploring the intricacies of modeling cycles that distinguish modeling activities from other assessments, as they expand their discussion to include the assessment of modeling competencies in general. Amit and Jan then present an "extension of model-eliciting problems into model-eliciting environments which are designed to optimize the chances that significant modeling activities will occur." Speiser and Walter then investigate another aspect critical to modeling activities and models in general. That is, models are tools created and applied for a purpose. And, purposes are strongly influenced by communities and societies in which designers and modelers operate. Clark and colleagues then discuss lessons learned from a pilot course they designed to elicit systems thinking in which they employed and modified MEAs designed to create a need for modeling. Davis then concludes the section by exploring the need for tools that can help teachers make sense of data collected in generative activities.

The question of how models develop is examined by Riede who studies students as they explore and rediscover Weber's law, progressing through modeling cycles, during their activities. Amit and Neria then examine the express > test > revise cycles of students engaged in generalizing pictorial linear pattern problems, finding that many students applied recursive strategies despite the appropriateness of global strategies. Work in the study of conceptual and model development is also important in the work of Dominguez who uses modeling activities to reveal multiplicity in students' ways of thinking – even when one final answer is agreed upon.

Finally the question of what ways modeling is different from solving traditional textbook word problems is addressed starting with Zawojewski, who asks what research implications and distinctions between the two can be drawn. She is followed by Carmona and Greenstein who find that modeling is suggestive of a spiral curriculum where powerful and underlying themes are revisited constantly – not arrived at permanently. Jensen then shows and investigates distinctions between modeling and problem solving competencies. Greefrath concludes with the examination of students planning processes, noting the differences in strategies used between modeling and problem solving questions.

These chapters reveal many facets of both modeling activities and the modeling research being conducted around student modeling activities. Although this collection of contributions is not intended to provide a comprehensive overview of the work being done to study the nature of modeling, this collection certainly gives the reader a diverse introduction to some of the most important frontiers in research on modeling and applications.

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http://www.springer.com/978-1-4419-0560-4

Modeling Students' Mathematical Modeling
Competencies
ICTMA 13
Lesh, R.; Galbraith, P.L.; Haines, C.; Hurford, A. (Eds.)
2010, XIV, 650 p., Hardcover
ISBN: 978-1-4419-0560-4