This book has grown out of lectures and courses given at Linköping University, Sweden, over a period of 15 years. It gives an introductory treatment of problems and methods of structural optimization. The three basic classes of geometrical optimization problems of mechanical structures, i.e., size, shape and topology optimization, are treated. The focus is on concrete numerical solution methods for discrete and (finite element) discretized linear elastic structures. The style is explicit and practical: mathematical proofs are provided when arguments can be kept elementary but are otherwise only cited, while implementation details are frequently provided. Moreover, since the text has an emphasis on geometrical design problems, where the design is represented by continuously varying—frequently very many—variables, so-called first order methods are central to the treatment. These methods are based on sensitivity analysis, i.e., on establishing first order derivatives for objectives and constraints. The classical first order methods that we emphasize are CONLIN and MMA, which are based on explicit, convex and separable approximations. It should be remarked that the classical and frequently used so-called optimality criteria method is also of this kind. It may also be noted in this context that zero order methods such as response surface methods, surrogate models, neural networks, genetic algorithms, etc., essentially apply to different types of problems than the ones treated here and should be presented elsewhere. The numerical solutions that are presented are all obtained using in-house programs, some of which can be downloaded from the book’s homepage at www.mechanics.iei.liu.se/edu_ug/strop/. These programs should also be used for solving some of the more extensive exercises provided.

The text is written for students with a background in solid and structural mechanics with a basic knowledge of the finite element method, although in our experience such knowledge could be replaced by a certain mathematical maturity. Previous exposure to basic optimization theory and convex programming is helpful but not strictly necessary.

The first three chapters of the book represent an introductory and preparatory part. In Chap. 1 we introduce the basic idea of mathematical design optimization and indicate its place in the broader frame of product realization, as well as define basic concepts and terminology. Chapter 2 is devoted to a series of small-scale problems that, on the one hand, give familiarity with the type of problems encountered in structural optimization and, on the other hand, are used as model problems in upcoming chapters. Chapter 3 reviews basic concepts of convex analysis, and exemplifies these by means of concepts from structural mechanics. Chapter 4 is, from an algorithmic point of view, the core chapter of the book. It introduces the basic idea of sequential explicit convex approximations, and CONLIN and MMA are presented. In Chap. 5 the latter is applied to stiffness optimization of a truss. This is a classical
model problem of structural optimization which we investigate thoroughly. Chapter 6 concerns sensitivity analysis for finite element discretized structures. Sensitivities for shape changes are combined with two-dimensional shape representations such as Bézier and B-splines in Chap. 7, and this closes the treatment of shape optimization. Chapter 8 is essentially a preparation for the formulation of the problem of stiffness topology optimization. We review some classical results of the calculus of variations, and derive optimality conditions for stiffness optimization of distributed parameter systems. In Chap. 9 this problem is slightly extended and discretized, and it provides a gateway into the problem of topology optimization for continuous structures. We derive the optimality criteria method as a special case of the general explicit convex approximation method, discuss well-posedness and different types of regularization methods.

This being an introductory treatment, we have not made an effort to give a complete set of references, nor an historical account of structural optimization. For that we refer to existing monographs such as Haftka and Gürdal [18], Kirsch [22] and Bendsøe and Sigmund [4].

As mentioned, this book has its roots in several series of lectures at Linköping University, where the first one was given by the second author of this book in 1992. Following these, in the year 2000, a separate course in structural optimization was established, and Joakim Petersson was made responsible and defined its basic contents. After having taught the course on two occasions, Joakim very unexpectedly and sadly passed away in September 2002, [3]. The authors of this book then took over and shared responsibility for the course, initially teaching it in a way that was very close to the lecture notes of Joakim. Out of appreciation, we have continued to teach the course, and eventually written this book, closely following the spirit and style of Joakim, as we remember and understand it.

We like to extend a special thanks to Bo Torstenfelt and Thomas Borrvall for having provided some of the numerical solutions presented in the book. Torstenfelt’s easy-to-use finite element program TRINITAS may be downloaded from the book’s homepage, and should be used for two computer exercises on shape and topology optimization. A Java applet by Borrvall for performing topology optimization is also available on the homepage. For the permission to use their programs we are sincerely grateful.

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