WOLFGANG LENZEN

KNOWLEDGE, BELIEF, AND SUBJECTIVE PROBABILITY: OUTLINES OF A UNIFIED SYSTEM OF EPISTEMIC/DOXASTIC LOGIC

FOREWORD

The aims of this paper are (i) to summarize the *semantics* of (the propositional part of) a unified epistemic/doxastic logic as it has been developed at greater length in (48) and (ii) to use some of these principles for the development of a semi-formal *pragmatics* of epistemic sentences. While a semantic investigation of epistemic attitudes has to elaborate the truth-conditions for, and the analytically true relations between, the fundamental notions of belief, knowledge, and conviction, a pragmatic investigation instead has to analyse the specific conditions of rational utterance or utterability of epistemic sentences. Some people might think that both tasks coincide. According to Wittgenstein, e.g., the *meaning* of a word or a phrase is nothing else but its *use* (say, within a certain community of speakers). Therefore the pragmatic conditions of utterance of words or sentences are assumed to determine the meaning of the corresponding expressions. One point I wish to make here, however, is that one may elaborate the meaning of epistemic expressions in a way that is largely independent of – and, indeed, even partly incompatible with – the pragmatic conditions of utterability. Furthermore, the crucial differences between the pragmatics and the semantics of epistemic expressions can satisfactorily be explained by means of some general principles of *communication*. In the first three sections of this paper the logic (or semantics) of the epistemic attitudes *belief*, *knowledge*, and *conviction* will be sketched. In the fourth section the basic idea of a general pragmatics will be developed which can then be applied to epistemic utterances in particular.

1. THE LOGIC OF CONVICTION

Let '$C(a,p)$' abbreviate the fact that person a is firmly convinced that p, i.e. that a considers the proposition p (or, equivalently, the state of affairs expressed by that proposition) as absolutely certain; in other words, p has maximal likelihood or probability for a. Using 'Prob' as a symbol for subjective probability functions, this idea can be formalized by the requirement:

(PROB-C) $\quad C(a,p) \leftrightarrow \mathrm{Prob}(a,p) = 1$

Within the framework of standard possible-worlds semantics $\langle I, R, V \rangle$, $C(a,p)$ would have to be interpreted by the following condition:

(POSS-C) $\quad V(i, C(a,p)) = \mathrm{t} \leftrightarrow \forall j(iRj \rightarrow V(j,p) = \mathrm{t})$

V.F. Hendricks, K.F. Jørgensen and S.A. Pedersen (eds.), Knowledge Contributors, 17–31.

Here I is a non-empty set of (indices of) possible worlds; R is a binary relation on I such that iRj holds iff, in world i, a considers world j as possible; and V is a valuation-function assigning to each proposition p relative to each world i a truth-value $V(i,p) \in \{\mathrm{t},\mathrm{f}\}$. Thus $C(a,p)$ is true (in world $i \in I$) iff p itself is true in every possible world j which is considered by a as possible (relative to i).

The probabilistic 'definition' **POSS-C** together with some elementary theorems of the theory of subjective probability immediately entails the validity of the subsequent laws of conjunction and non-contradiction. If a is convinced both of p and of q, then a must also be convinced that p and q:

(C1) $C(a,p) \wedge C(a,q) \rightarrow C(a,p \wedge q)$

For if both $\mathrm{Prob}(a,p)$ and $\mathrm{Prob}(a,q)$ are equal to 1, then it follows that $\mathrm{Prob}(a,p \wedge q) = 1$, too. Furthermore, if a is convinced that p (is true), a cannot be convinced that $\neg p$, i.e. that p is false:

(C2) $C(a,p) \rightarrow \neg C(a,\neg p)$

For if $\mathrm{Prob}(a,p) = 1$, then $\mathrm{Prob}(a,\neg p) = 0$, and hence $\mathrm{Prob}(a,\neg p) \neq 1$. Just like the alethic modal operators of possibility, $\Diamond$, and necessity, $\Box$, are linked by the relation $\Diamond p \leftrightarrow \neg\Box\neg p$, so also the doxastic modalities of thinking p to be possible – formally: $P(a,p)$ – and of being convinced that p, $C(a,p)$, satisfy the relation

(Def. P) $P(a,p) \leftrightarrow \neg C(a,\neg p)$

Thus, from the probabilistic point of view, $P(a,p)$ holds iff a assigns to the proposition p (or to the event expressed by that proposition) a likelihood greater than 0:

(PROB-P) $V(P(a,p)) = \mathrm{t} \leftrightarrow \mathrm{Prob}(a,p) > 0$

Within the framework of possible-worlds semantics, one obtains the following condition:

(POSS-P) $V(i,P(a,p)) = \mathrm{t} \leftrightarrow \exists j(iRj \wedge V(j,p) = \mathrm{t})$,

according to which $P(a,p)$ is true in world i iff there is at least one possible world j – i.e. a world j accessible from i – in which p is true.

In view of **Def P**, the former principle of consistency, **C2**, can be paraphrased by saying that whenever a is firmly convinced that p, a will a fortiori consider p as possible. However, considering p as possible does not conversely entail being convinced that p. In general there will be many propositions p such that a considers both p and $\neg p$ as possible. Such a situation, where $P(a,p) \wedge P(a,\neg p)$, makes clear that unlike the operator C, P does not satisfy a principle of conjunction analogous to **C1**. However, the converse entailment

(C3) $P(a,p \wedge q) \rightarrow P(a,p) \wedge P(a,q)$

and its counterpart

(C4) $\quad C(a, p \wedge q) \rightarrow C(a,p) \wedge C(a,q)$

clearly are valid, because the probabilities of the single propositions p or q always are at least as great as the probability of the conjunction $(p \wedge q)$. Similarly, since the probability of a disjunction $(p \vee q)$ is always at least as great as the probabilities of the single disjuncts p and q, it follows that both operators C and P satisfy the following principles of disjunction:

(C5) $\quad C(a,p) \vee C(a,q) \rightarrow C(a, p \vee q)$

(C6) $\quad P(a,p) \vee P(a,q) \rightarrow P(a, p \vee q)$

Now the probabilistic 'proofs' of such principles are not without problems. Since its early foundations by De Finetti (38), the theory of subjective probability has always been formulated in terms of *events,* while in the framework of philosophical logic, attitudes like $C(a,p)$ are traditionally formulated in terms of *sentences.* So if one wants to apply the laws of the theory of subjective probability to the fields of cognitive attitudes, one has to presuppose (i) that for every event X there corresponds exactly one proposition p, and (ii) that the cognitive attitudes really are 'propositional' attitudes in the sense that their truth is independent of the specific linguistic representation of the event X. That is, whenever two sentences p and q are logically equivalent and thus describe one and the same event X, then $C(a,p)$ holds iff $C(a,q)$ holds as well. This requirement can be formalized by the following rule:

(C7) $\quad p \leftrightarrow q \vdash C(a,p) \leftrightarrow C(a,q)$

This principle further entails that everybody must be convinced of everything that logically follows from his own convictions:

(C8) $\quad p \rightarrow q \vdash C(a,p) \rightarrow C(a,q)$

For if p logically implies q, then p is logically equivalent to $p \wedge q$; thus $C(a,p)$ entails $C(a, p \wedge q)$ (by **C7**) which in turn entails $C(a,q)$ by **C4**.

There has been a long discussion (still going on in the literature) whether and to which extent the cognitive attitudes of real subjects actually are *deductively closed.* In view of man's almost unlimited fallibility in matters of logic, some authors have come to argue that **C8** should be restricted to very elementary instances like **C4** or **C5** or to some other so-called 'surface tautologies.'[1] Which option one favours will strongly depend on the methodological role that one wants to assign to epistemic logic. If epistemic logic is conceived of as a *descriptive system* of people's factual beliefs, then not even the validity of elementary principles like **C4** seems warranted. If, however, epistemic logic is viewed as a *normative* system of *rational* belief, then even the strong condition of full deductive closure, **C8**, appears perfectly acceptable. Incidentally, if one presupposes

Springer

http://www.springer.com/978-1-4020-1747-6

Knowledge Contributors
Hendricks, V.F.; Jørgensen, K.F.; Pedersen, S.A. (Eds.)
2003, IX, 203 p., Hardcover
ISBN: 978-1-4020-1747-6