

In this year of celebrations of ‘First-Order Logic 75’ and ‘Trends in Logic – 50 Years of *Studia Logica*’, two phenomena stand out. One is the rapid expansion of themes in modern logic, the other the remarkable constancy of the basic methods encapsulated in first-order logic. The resulting light piece may be read as a companion to Ryszard Wojcicki’s survey of logic in Poland. Our main purpose is to show some patterns by means of aerial photography.

## 1. Agendas and Methods

Historically, the agenda of logic as the study of reasoning has changed over time. Topics like patterns of scientific explanation are now considered part of the philosophy of science, but the border-line is vague. Likewise, to take a modern example of such blurring, AI research into default rules evidently addresses a major aspect of reasoning as well. Here is one way of thinking about the subject matter. Reasoning is about handling propositions, and these lie mid-way in a hierarchy of data structures, which runs from words and phrases to complete sentences, and then upward to texts, and whole theories. Each of these levels is associated with activities that show logical structures, such as – from low to high – determining meanings of expressions, characterizing speech acts, understanding discourse and proof, or analyzing theory structure. All these levels have occurred to some extent in mathematical and philosophical logic, though the emphasis varies at times. Of course, there is a proliferation of logical systems in modern research on these topics, and logical pluralism is an undeniable fact of life. But it is also true to say that one system has shown a remarkable constancy throughout the period, viz. first-order predicate logic – so much so, that we still teach it as the gateway to the field. One obvious reason for this choice is that predicate logic sets the methodological standards. But more than that, it also serves as a sort of conceptual laboratory for implementing and testing new ideas, throughout the past 50 years. In particular, we will look at how predicate logic has fared when confronted with realities of language and reasoning,

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leading up to the second main theme of this paper. The above hierarchy of logical levels is at the same time a hierarchy of activities with their own logical structure. Our discussion will be light, pointing merely at the broad ideas at stake. Technical details are left to the references.

## 2. Truth and Meaning

**The basic format** The familiar semantics of first-order logic explains the notion of truth of a formula  $\phi$  in a model  $M$  under assignment  $s$ . It works via obvious recursive clauses, such as  $M, s \models \exists x\phi$  iff there exists an object  $d$  in  $M$  with  $M, s[x := d] \models \phi$ . Simple as it is, this truth definition embodies two major ideas. The first is *compositional interpretation* of formulas following their stepwise construction in an appropriate perspicuous logical syntax. The second is the selection of an appropriate *context of evaluation* making the compositional recursion possible. In first-order logic, the assignment functions do the job.

**Natural language semantics** Both ideas have guided all further work in the formal semantics of natural language, where they have been extended to deal with a wide spectrum of further expressions (van Benthem and ter Meulen, eds., 1997). A typical example is the semantics of generalized quantifiers such as “all”, “most”, “few”, where compositional interpretation of their common format  $Qx \bullet \phi(x)$  becomes possible as soon as one determines the appropriate type of denotation, taking a quantifier  $Q$  to denote a property of sets. At the same time, the quantifier repertoire of natural languages is much richer than that of first-order logic, and more complex ‘branching’ constructions such as “Ten firms owned a thousand jets (together)” raise doubts about compositionality in its straightforward form (Keenan and Westerståhl 1997). But this deviation and the ensuing discussion just show how the logical theory serves as a search-light in Popper’s sense for finding interesting phenomena that high-light peculiarities of language.

**Philosophical modalities** The same style of semantic analysis is found in the study of various linguistic concepts for philosophical purposes. For instance, consider modalities, such as the epistemic operator ‘agent  $j$  knows that  $\phi$ ’:  $K_j\phi$ . Here the relevant contexts are possible worlds structured by an accessibility relation of epistemic indistinguishability – and the compositional semantic clause says that  $K_j\phi$  is true in world  $s$  iff  $\phi$  is true in all worlds accessible for  $j$  from  $s$ . Likewise, a standard account of conditionals  $\phi \Rightarrow \psi$  says that  $\psi$  is true in all those  $\phi$ -worlds which are closest to the current world  $s$ . In this way, the compositional methodology has led

to the discovery of important general parameters in organizing logical contexts, viz. *accessibility* and *minimization* along an ordering. Incidentally, first-order logic does play a role even here, though notions of modality are not part of its repertoire of Boolean operations and standard quantifiers. Most well-behaved modal truth definitions can be *translated* into first-order terms, and this allows for a well-known transfer of insights about predicate logic to modal logic in so-called correspondence theory (cf. Blackburn, de Rijke and Venema 2001).

**Dynamic predicate logic** The above expansion of ideas from first-order semantics to semantics in general is well-known. What may be less known is that predicate logic itself can also serve to investigate *new mechanisms* of interpretation. One modern example was motivated by the anaphoric behaviour of natural language, which differs systematically from binding in predicate logic. E.g., an existential quantifier can bind across sentence boundaries, as in  $\exists xAx.Bx$ , (“A man came in. He smiled.”) and also across operator boundaries as in  $\exists xAx \rightarrow Bx$  (“If a man comes in, he smiles”). It turns out that such phenomena can be explained very naturally if we encode more dynamics of the evaluation process into the truth definition (Groenendijk and Stokhof 1991). This leads to a new format

$M, s_{in}, s_{out} \models \phi$ : the assignment transition  $(s_{in}, s_{out})$  records a successful evaluation of  $\phi$ .

In particular, an existential quantifier  $\exists x$  is now an action resetting the current assignment  $s$  to any  $s[x := d]$ , while an atomic formula  $Px$  is a test whether the value  $s(x)$  of the current assignment  $s$  has the property  $P^M$ . This can be done perfectly compositionally, with e.g., logical conjunction becoming composition of binary transition relations. In a sense, this format merely brings out the process intuitions behind ordinary first-order semantics. Predicate logic is a system for two kinds of semantic action: testing a fact, and changing a state. The latter happens either by ‘random assignments’ triggered by formulas  $\exists x\phi$ , or by definite assignments  $x := t$  triggered by substitution operators  $[t/x]\phi$ . On this view, predicate logic and natural language are more like imperative programming languages, whose programs denote actions that change states of a computational device. Thus, *processes* become a major logical topic in their own right. This is our first logical story line which naturally runs into computer science. More are to come.

**Evaluation games** A more drastic process view of first-order evaluation was proposed in Hintikka 1973. In each model  $M$ , statements  $\phi$  define an evaluation game between two players called Verifier (**V**) and Falsifier



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