CHAPTER III

NEWTON'S METHOD OF FLUXIONS

> "I consider time as flowing or increasing by continual flux & other quantities as increasing continually in time & from the fluxion of time I give the name of fluxions to the velocities with which all other quantities increase." (Newton, *The Mathematical Papers of Isaac Newton*, vol. 3, p. 17).

Newton and Leibniz invented the calculus at an early stage of their respective careers, Newton at his home in Woolsthorpe during his *annus mirabilis* (1666, when he was only twenty four years old) and Leibniz toward the end of his stay in Paris (1675, at the age of twenty nine).[1] Both of them developed a sophisticated natural philosophy and became presidents of great scientific academies in their old age. However, their careers followed very different paths. Newton became one of the most influential scientific figures in England, his success and fame reaching international levels, whereas Leibniz even at his greatest moments remained unappreciated by many of his German contemporaries and the wider natural philosophical community. The bitter dispute that arose between them regarding the priority of the calculus in many ways reflects their personal careers. Though Leibniz published his results twenty years before Newton, and developed a more sophisticated algorithm for calculation than Newton did, he was the one accused of plagiarism by the natural philosophical community. The verdict in the dispute was given by a committee of scientists from the Royal Society, of which Newton was president. Leibniz, though president of the Berlin Academy and a foreign member of the Royal Society, was not only accused of plagiarism but abandoned in Hanover by his Duke, afterwards George I, King of England.[2]

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[1] Leibniz's paper was published in the *Acta Eruditorum*, October, 1684, titled: "Nova Methodus pro Maximis et Minimis." Newton appended his "Tractatus de Quadratura Curvaturum," and "Enumeratio Linearum Tertii Ordinis" in the *Opticks* (1704).

[2] For a more detailed biographical account of the differences between the two men, see: H. G. Alexander (ed.), *The Leibniz-Clarke Correspondence* (New York, 1970), introduction; Ernest Cassirer, "Newton and Leibniz," *Philosophical Review*, vol. 52 (1943), pp. 366-91; Gideon Freudenthal, *Atom and Individual in the Age of Newton: On the Genesis of the Mechanistic World View* (Dordrecht, 1986); Rupert Hall, *Philosophers at War: The Quarrel between Newton and Leibniz* (Cambridge, 1980); J.E. Hoffman, *Leibniz in Paris 1672-76: His Growth to Mathematical Maturity* (Cambridge, 1974); Frank Manuel, *A Portrait of Isaac Newton* (New York, 1968), pp. 321-349; Steven Shapin, "On Gods and Kings: Natural Philosophy and Politics in the Leibniz-Clarke Correspondence," *Isis*, vol. 72 (1981), pp. 187-215; Richard Westfall, *Never at Rest* (Cambridge, 1980), pp. 698-781.

I. Prologue: The History of the Calculus

Many historians of mathematics have written on the calculus, each emphasizing different aspects of its development.[3] "The invention of the calculus, - namely of a method of finding tangents and quadratures which identifies the reciprocity between these two operations –", writes Domenico Bertoloni-Meli, "was the culmination of a process involving several important advances. The establishment of a new, highly abstract, and general form of algebra, the formulation of analytical geometry, and the creation of a variety of techniques for finding maxima, minima, and tangents, [all of these] paved the way to the great inventions by Newton and Leibniz."[4] Indeed, an immense amount of knowledge of the calculus had accumulated before Newton and Leibniz made their syntheses. According to Morris Kline, their calculi were created primarily to treat four types of mathematical problems with which seventeenth century scientists were struggling. The first was, "given the formula for the distance a body covers as a function of time, to find the velocity and acceleration at any instant; and conversely, given the formula describing the acceleration of a body as a function of the time, to find the velocity and the distance traveled." The second type of problem was to find the tangent to a curve. The third was to find the maximum or minimum value of a function, and finally, the fourth was "finding the length of curves" (for example, the distance covered by a planet in a given period of time; the areas; and centers of gravity of bodies).[5] These four problems were considered as distinct before Newton and Leibniz developed their calculi and recognized the generality underlying the separate categories.

Indeed, regarding the category of treating geometrical entities in terms of motion, Newton was the mathematician who made the first significant treatment of general rate problems. He gave a general method for finding the instantaneous rate of change of one variable with respect to another and also showed that the area can be obtained by reversing the process of finding a rate of change. In his mathematical calculations, "motion and time are constantly at work: curves are trajectories traversed by moving bodies; on these curves, points approach each other; the curves themselves are bent, and so forth."[6] Many mathematicians before Newton, such as Galileo Galilei (1564-1642), Marin Mersenne (1588-1648), Gilles Personne de Roberval (1602-1675) and

[3] Florian Cajori, *A History of Mathematics* (New York, 1919); C. Boyer, *History of the Calculus* (New York, 1949); M.E. Baron, *The Origins of the Infinitesimal Calculus* (Oxford, 1969); D.T. Whiteside, "Patterns of Mathematical Thought in the Late Seventeenth Century," *Archive for History of Exact Science*, vol. 1 (1961), pp. 179-338; Morris Kline, *Mathematical Thought from Ancient to Modern Times* (New York, 1972).

[4] Domenico Bertoloni-Meli, *Equivalence and priority: Newton versus Leibniz* (Oxford, 1993), p. 56.

[5] Kline, *Mathematical Thought*, pp. 342-43.

[6] De Gandt, *Force and Geometry*, p. 202.

René Descartes (1596-1650), had already analyzed geometrical curves in motion. However, Newton was the first to analyze figures in motion with respect to finite ultimate ratios.[7] In his calculus, "fluxions express the speed of change of a variable and are finite. They result from variables flowing continuously, almost always with respect to time. Hence kinematics is part of the foundation of the Newtonian calculus."[8]

Regarding the second problem, of finding the tangent to the curve, mathematicians were searching for ways to free the definition of the tangent from physical concepts. Descartes' method was purely algebraic and did not involve the concept of the limit, whereas Pierre Fermat's (1601-1665) method had "the form of the now-standard method of the differential calculus, though it begs entirely the difficult theory of limits."[9] Isaac Barrow's (1633-1677) geometrical methods shortened calculations tremendously by employing the characteristic triangle (Blaise Pascal (1623-1662) had used it even earlier in connection with finding areas), which Leibniz further developed and generalized. Leibniz's main contribution to the definition of tangent was his ability to conceive curves as infinitangular polygons consisting of incomparably many rectilinear segments, while tangents were considered as the prolongation of these segments. Influenced by Pascal's work on harmonic series,[10] Leibniz discovered the existence of a structural resemblance between differences and sums of infinite series and that of tangents and quadratures of geometrical curves. Analyzing the resemblance of the two different realms helped him to develop a coherent theory of differentials and summations. In contrast to Newton's geometrical and kinematic insights, which strictly employ only finite ultimate ratios; Leibniz's thought penetrated deeper into the infinitely small. His calculus did not rely upon geometrical imagination, but upon a mechanical algorithm for calculating differentials and summations.

The problem of the infinitely small grew out of the fourth type of problem, that of finding areas, volumes, and length of curves. The identification of curvilinear areas and volumes with the sum of an infinite number of infinitesimal elements is the essence of Johann Kepler's (1571-1630) method. Galileo also conceived of areas in a manner similar to Kepler. In treating the problem of uniformly accelerated motions, Galileo pointed out that the area under the time-velocity curve is the distance. Bonaventura Cavalieri (1598-1647) further developed Kepler's and Galileo's work into a coherent geometrical method. Cavalieri regarded an area as made up of an indefinite number of equidistant parallel line segments and a volume as

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[7] Ultimate ratios will be discussed further on in the chapter.

[8] Bertoloni-Meli, *Equivalence and priority*, p.72.

[9] Kline, *Mathematical Thought*, p. 345.

[10] Actually Leibniz was interested in summations and differences already in his dissertation, *Dissertatio de arte combinatoria*. In this dissertation he was trying to work out an alphabet of human thought. See: E.J. Aiton, *A Biography*, (Bristol & Boston, 1985), pp. 18-21, and H.J.M. Bos, "Fundamental Concepts of the Leibnizian Calculus," *Studia Leibnitianna*, Sonderheft 14 (1986).

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