INTRODUCTION

“Today we really learnt something!” Mary exclaimed after she, together with Adam, had concentrated for almost two hours on setting up a spreadsheet. Something significant seems to have happened for Mary, something that should be considered when theorising about the learning of mathematics. In this study we are going to meet with Mary and Adam and many other students in the mathematics classroom. The main purpose of this meeting is to gather empirical resources to gain a better understanding of the role of communication in learning mathematics.

The initial idea that guides our investigations can be condensed in the following hypothesis: The qualities of communication in the classroom influence the qualities of learning mathematics. This is not a very original statement and certainly very general. If the statement is to be provided with meaning it is important to clarify at least the two expressions: ‘qualities of communication’ and ‘qualities of learning mathematics’. In this introduction, as well as during the rest of this book, we are going to struggle with clarifying in what sense communication and learning can be connected, and how to conceptualise this connection.

QUALITIES OF COMMUNICATION

In many different contexts, both inside and outside school, special attention is paid to communication. Thus, companies organise workshops and courses on communication in order to improve the way they operate (see, for instance, Isaacs, 1999a; Kristiansen and Bloch-Poulsen, 2000). The improvement of communication is expected not only to have an influence on the atmosphere of the workplace, but also on the way the company operates in terms of business, as expressed in figures and budgets. Communication becomes related to the idea of the ‘learning organisation’.

Qualities of communication can be expressed in terms of interpersonal relationships. Learning is rooted in the act of communicating itself, not
just in the information conveyed from one party to another. Thus, communication takes on a deeper meaning. In *Freedom to Learn*, first published in 1969, Carl Rogers (1994) considers interpersonal relationships as the crucial point in the facilitation of learning. Learning is personal, but it takes place in the social contexts of interpersonal relationships. Accordingly, the facilitation of learning depends on the quality of contact in the interpersonal relationship that emerges from the communication between the participants. In other words, the context in which people communicate affects what is learned by both parties.

This brings forward the idea that some ‘qualities of communication’ could be clarified in terms of *dialogue*. The word ‘dialogue’ has many everyday descriptive references but the important factor common to all is that they involve at least two parties. For instance, it is possible to talk about the dialogue between East and West and about the breakdown of the dialogue between Palestine and Israel. Such references to dialogue are not strictly part of our concern. In philosophical contexts the notion of dialogue occurs in many places. Plato presented his ideas as dialogues; in 1632 Galileo Galilei wrote *The Dialogue Concerning the two Chief World Systems* (which brought him close to the Inquisition), and Imre Lakatos (1976) presented his investigation of the logic of mathematical discovery in the form of a dialogue taking place in an imaginary classroom. Such uses of ‘dialogue’ refer first of all to analytical forms and presentations of inquiries and of ‘getting to know’. As soon as we enter the field of ‘getting to know’, dialogue becomes relevant to epistemology. However, although our concept of dialogue is also related to epistemology in this way, it will diverge from the traditional philosophical use of the term by being related to ‘real’ dialogues and not to in-principle dialogues. We use the word ‘dialogue’ for a conversation with certain *qualities*, and the specification of ‘dialogue’ is one of the tasks awaiting for us as part of this study.

In talking of qualities related to conversation, we recognise that the notion of quality may have a double meaning. On the one hand, quality may refer to properties of a certain entity. Thus, we can talk (almost in Aristotelian terms) about the quality of a cup as being different from the quality of a glass. In this sense quality refers to descriptive aspects of an entity. However, quality may also contain a normative element. Thus, we can talk about one glass being of a better quality that another glass. Maintaining the distinction between descriptive and normative references to quality is not simple. For instance, we may prefer the quality of a glass to the quality of a cup when drinking wine. In a similar way, we may prefer a dialogue when we think of certain forms of learning, bearing in mind that dialogue refers to certain properties of an interaction.
Paulo Freire (1972) emphasises the importance of interpersonal relationships in terms of dialogue. To Freire dialogue is not just any conversation. Dialogue is fundamental for the freedom to learn. The notion of dialogue is integral to concepts like ‘empowerment’ and ‘emancipation’, and from this perspective Freire makes a connection between the quality of what is happening between people and the possibility of pursuing political actions. He defines dialogue as a meeting between people in order to ‘name the world’, which means talking about events and the possibility of changing these. In this way dialogue is seen as existential. Dialogue cannot exist without love (respect) for the world and for other people, and it cannot exist in relations of dominance (Freire, 1972, 77ff.). Further, taking part in a dialogue presupposes some kind of humility. You cannot enter a dialogic relationship being self-sufficient. The participants have to believe in each other and to be open-minded towards each other in order to create an equal and faithful relationship. As the dialogue is directed by the hope of change, it cannot exist without the engagement of the partners in critical thinking (Freire, 1972, 80ff.). To Freire the co-operation of the participants is a central parameter of dialogic communication. In co-operation the participants throw light on the world that surrounds them and the problems that connect and challenge them. Freire points out the importance of co-operation between action and reflection (Freire, 1972, 75ff.). Hand and head have to go together. Acting without reflecting would end up in pure activism and reflection without action would result in verbalism. However, in a dialogue, reflection and action can enrich each other. According to Freire, the educational dialogue is supposed to examine the universe of the people – its thematic universe – which announces emancipation through education. Freire’s program was originally aimed at illiterate people, and it has to be remembered that only in May 1985 did illiterate people in Brazil get the right to vote.\textsuperscript{1} To Freire, dialogue clearly refers to a form of interaction with many specific qualities.

In classical philosophy, dialogue first of all refers to a presentation (and confrontation) of two or more different (and contradictory) points of view, with the aim of identifying a conclusion that can be agreed on.

\textsuperscript{1} Freire makes the revolutionary leaders responsible for the communication remaining dialogic. They should not invade the perspectives of the people and inform or instruct them, nor should they just adapt to the expectations of the people. They should learn about the people’s world together with the people – by naming the world (Freire, 1972, 161). The relationship between the revolutionary leaders and the people, as suggested by Freire, can be interpreted in terms of the relationship between teacher and students – an asymmetrical relationship.
Freire and Rogers, however, also viewed dialogue as encompassing interpersonal relationships, where listening and accepting on the part of the participants is fundamental. Dialogue is not just a mode of analysis, but also a mode of interaction. In the following clarification of the notion of dialogue we shall maintain this combination of epistemic and relational aspects of dialogue.

Rogers and Freire have much in common although they work from different historical positions. This is perhaps not surprising as they both relate to the German philosopher Martin Buber (1957), who emphasises the relationship, ‘the interhuman’, in the dialogue as a certain way of meeting the other with unconditional acceptance. Rogers calls his approach to learning ‘person-centered’ as opposed to the ‘traditional mode’, and he describes the two approaches as opposite poles of a continuum (Rogers, 1994, 209f.). He argues that the person-centered mode prepares the students for democracy, whereas the traditional mode socialises the students to obey power and control. In the traditional mode, he argues, “the teacher is the possessor of knowledge and power,” and “rule by authority is accepted policy in the classroom”. Students are expected to be recipients of knowledge, and examinations are used to measure their receptivity. Rogers emphasises that “trust is at a minimum,” and “democratic values are ignored and scorned in practice”. In the person-centred mode, he argues, the environment is trustful and the responsibility for the learning processes is shared. “The facilitator provides learning resources,” and “the students develop their program of learning alone and in co-operation with others”. The main principle is learning how to learn, and self-discipline and self-evaluation guarantee a continuing process of learning. This growth-promoting climate not only facilitates learning processes but also stimulates the students’ responsibility and other competencies for democratic citizenship: “I have slowly come to realize that it is in its politics that a person-centred approach to learning is most threatening. The teacher or administrator who considers using such an approach must face up to the fearful aspects of sharing of power and control. Who knows whether students or teachers can be trusted, whether a process can be trusted? One can only take the risk, and risk is frightening.” (Rogers, 1994, 214)

Freire contrasts his dialogic approach with ‘banking education’, where the teacher makes an investment, and where the students are considered boxes and are supposed to preserve what is invested. To both Rogers and Freire, dialogue represents certain forms of interaction fundamental to processes of learning, which, in Freire’s terms, can ensure empowerment, and which in Rogers’ terms can ensure person-centered learning and students’ responsibility. In this sense they find that qualities of
communication can turn into qualities of learning, referring to both descriptive and normative elements. When we talk about qualities of communication and qualities of learning, we also have in mind both descriptive and normative elements. We want to locate certain aspects of communication which may support certain aspects of learning, and at the same time it becomes important to support these aspects of learning.

Many studies of communication concentrate on classrooms that are situated in the school mathematics tradition. Here, we refer to a tradition where the textbook plays a predominant role, where the teacher explains the new mathematical topics, where students solve exercises within the subject, and where correction of solutions and mistakes characterise the overall structure of a lesson. We have observed classrooms from a school mathematics tradition where there is a nice atmosphere, and where the teacher-student communication appears friendly. So, by the school mathematics tradition we do not simply refer to the non-attractive features of the mathematics classroom, where a never-smiling teacher dominates the students. However, within the school mathematics tradition we can locate characteristic patterns of communication which have certain qualities, but we are not tempted to refer to these patterns as dialogue.

The form of communication depends on the context of communication, and, like many others, we find that the school mathematics tradition frames the communication between students and teacher in a particular way. In the first chapter of this study we will summarise a few of our observations and analyses of this phenomenon, but in the rest of the book we primarily undertake our investigations in classrooms outside the school mathematics tradition. We are interested in situations where the students become involved in more complex and also unpredictable processes of inquiry. This opens a new space for communication, where new qualities can emerge.

In many cases the mathematics classroom has undergone radical changes. Thematic approaches and project work challenge tradition in such a way that the distinction between learning mathematics and learning something else is not always sharply maintained.

With the exception of Chapters 1 and 2, we describe projects where the planning of the subject matter was a shared process between the teachers and us. Then, when it came to the classroom practice, the teachers were in charge. One reason for this division of labour is simply that the teacher's professionalism in real-life classrooms is much higher than ours. We discussed the interpretations of the observations with the teachers, and we have included their suggestions for possible interpretations. In some cases, we also interviewed the students about
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