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Inconsistent Reasoning Toward Consistent Theories

One of the most fascinating aspects of scientific creativity is how consistent theories can sometimes emerge from inconsistent premises or inappropriate data. Using cases from the history of science I will explore this process and try to draw some general lessons concerning how science is done and, more particularly, about the concept of scientific progress.

I will examine three important sources of inconsistencies:

1. Reasoning from incorrect, or unknowingly restrictive, experimental data.
2. Reasoning from incorrectly interpreted premises.
3. Reasoning on the basis of concepts that are later jettisoned.

We will see how recognising and then eliminating inconsistencies that arise from sources of the sorts in statements 1–3 can lead to a better understanding of scientific creativity.

1. Reasoning from Incorrect, or Unknowingly Restrictive, Experimental Data

This can also go under the heading “Unwanted Precision”. Sometimes less accurate data are more informative. Examples from Galileo’s research are particularly relevant here.

A thought experiment on which Galileo spent a great deal of time in his 1632 Dialogue on the Two Chief World Systems involves someone who, standing on the mast of a moving ship, drops a stone (Galileo 1632/1967, 148-150). Galileo pondered where it will land? Aristotelian argue that the ship moves out from under the stone and so it does not fall directly under the person who dropped it. This is the result expected from our intuition which, in an untutored state, is essentially Aristotelian. Galileo, on the other hand, argues that the stone will land directly beneath the person who dropped it.

Imagine, however, that Galileo had extremely precise measuring instruments, or that he lived on a planet that spun on its axis orders of magnitude faster than ours. He would have found that the stone does not fall directly under the person who

dropped it. Its deviant course is due to a force whose origin is in the Earth's rotation about its axis: the Coriolis force. The effects of this force depend on where you are on the rotating Earth and how fast you are moving. The Coriolis force is of key importance in the motion of large air masses such as cyclones, tornadoes, and trade winds. But for falling bodies on the Earth, its effects are small. In Galileo's experiment the deviation from vertical fall due to the Coriolis force for an object dropped from a mast of height 30 feet at a latitude of 45° is only .02 inches. Luckily, Galileo did not have highly sensitive measurement devices. Any measurements revealing this deviation would only have confused matters because a first step in understanding how objects fall is to formulate a theory of motion in which the Coriolis force is not present. This is what Galileo did by restricting all considerations to inertial reference systems and free fall through vacuum.

There is yet another force arising from the Earth's rotation called the centrifugal force, which depends only on where you are on the rotating Earth. One of its effects is to produce a variation in the acceleration of free fall depending on your latitude. Any corrections are in the third decimal place.

Similarly, Galileo's experiments with pendula may well be flawed because he pulled the pendulum bob far enough from the vertical so that the motion was not simple harmonic. Better clocks for timing purposes would have been deleterious to his goal of using pendula to convince himself that all bodies fall with the same acceleration in vacuum regardless of their weight. This hypothesis is central to Galileo's theory of motion.

We can say that Galileo reasoned toward his theory of motion with data that were inexact and sometimes with fallacious logic. Besides his unbridled enthusiasm, we can conclude only that most of these experiments served merely as checks on already performed thought experiments and already accepted universal laws.

It is interesting to take note of the fact that a concept of key importance in the theories of Einstein, Galileo and Newton, is the inertial reference system. This is a platform that moves at a constant velocity in a straight line forever. We assume the Earth is an inertial reference system. Yet the Earth hurtles around the sun at the rate of one revolution per year at a mean speed of 67,000 miles/hour, while revolving on its axis once every 24 hours at a mean rate of 1,040 miles/hour at the equator. Despite these accelerated motions, engineers and scientists can still do the vast majority of their calculations using Newton's laws as if the Earth were moving in a straight line at a constant velocity. We have already noted corrections due to the Coriolis and centrifugal forces. Any effects of the Earth's orbital acceleration on laboratory experiments performed on the Earth depend on terms in Newton's gravitational law containing ratios of the Earth's radius to that of the Earth's distance from the sun. Detecting such effects requires accuracy to at least six decimal places. All of this is fortuitous, because if it were not the case then it would have been very difficult, indeed, for Galileo to have formulated a theory of motion alternative to Aristotle's.

Since Newtonian science is based on inertial reference systems, we would expect to see examples of them. Yet there are none in practice, only in principle, that is, in the mind of the thought experimenter. We are faced with a situation in which the very basis of Newtonian mechanics doesn't exist. Is this not inconsistent? We say
that Newton’s theory is approximate, and don’t blame the collapse of bridges on there being no real inertial reference systems. Einstein’s special theory of relativity is also based on the inertial reference system in the sense that it deals only with measurements made in such systems, which Einstein himself took to be its “logical weakness” (Einstein 1923, 480). And so special relativity is also approximate. Einstein repaired this situation in 1915 with his general theory of relativity in which measurements can be made in accelerating reference systems.

This brings me to the ether-drift experiments of the latter part of the nineteenth century. These were state of the art experiments performed by some of the greatest high precision experimenters of the day. They sought to detect effects of the Earth’s motion through the ether on measurements of the velocity of light. The effects sought were of the order of \((v/c) = 10^{-4}\) and \((v/c)^2 = 10^{-8}\) where, for the sake of determining an upper limit, \(v\) is taken as the Earth’s orbital velocity about the sun (30 km/sec) and \(c\) is the velocity of light as measured in the free ether, that is, vacuum, and is \(3 \times 10^8 \text{ m/sec}\). Yet are not the very premises of the \((v/c)\) experiments flawed? The reason is that this effect will be swamped by effects due to the Earth’s not being an inertial reference system, which can also affect second order results.

For example, in their famous 1887 ether-drift experiment, Michelson and Morley took account only of the Earth’s orbital motion about the sun. Their corrections, however, were less than precise because they had to be folded into the effects due to the motion of the solar system about which, in Michelson and Morley’s words, “but little is known with certainty” (Michelson and Morley 1887, 281). Moreover, had these experiments or the subsequent more precise ones performed in the 1920’s succeeded, they would have been evidence neither for an ether nor against Einstein’s special theory of relativity. Rather, they would simply be indicators of the “noninertiality” of the Earth as a moving platform.

Another example of unknowingly reasoning from restricted data is the Dutch physicist H. A. Lorentz’s successful explanation in 1896 of his colleague Pieter Zeeman’s data on the splitting of certain spectral lines of sodium in an externally imposed magnetic field. Fortuitously, most of Zeeman’s experiments occurred in large enough magnetic fields so that, to use quantum mechanical terminology, the spin and orbital angular momenta decouple. In this regime Lorentz’s classical electrodynamics sufficed for counting line splittings and approximately fitting line spacings. Although in 1898 the so-called anomalous Zeeman effect began to be noticed, in 1902, Lorentz and Zeeman were awarded the Nobel Prize in physics for explaining Zeeman’s high field data.

We turn next to a deeper level of inconsistencies.

2. REASONING FROM INCORRECTLY INTERPRETED PREMISES

Consider the wave-particle duality of light. Einstein’s hypothesis of a light quantum in 1905 was avoided essentially until after the formulation of quantum mechanics in 1925 by Werner Heisenberg. The reasons had nothing at all to do with empirical data, they were conceptual. As Max Planck put it in 1910, no visualisable model could be formulated with light quanta to explain optical interference, whereas one had been in existence for 300 years for waves—Huygens’s wavelets (Planck 1910).
Apropos to paraconsistent logics is a quote from the American physicist O. W. Richardson who expressed the conceptual situation regarding the nature of light in 1916 (Richardson 1916, 507-508):

The same energy of radiation behaves as though it possessed at the same time the opposite properties of extension and localisation. At present there seems no obvious escape from the conclusion that the ordinary formulation of the geometrical propagation involves a logical contradiction.

What to do? The first step physicists took was to exclude light quanta from atomic theories. This had been Niels Bohr’s tack in 1913 and he continued it as long as possible into the 1920’s (see Miller 1986). Bohr’s reason was the same as Planck’s—the “image of light quanta precludes explaining optical interference” (Bohr 1921, 241-242). In the face of data favouring light quanta taken in 1923 by A. H. Compton, Bohr offered a desperate attempt to exclude them by proposing in 1924, a version of his atomic theory that violated (inconsistent with) energy and momentum conservation. Although it failed empirically, one of its fundamental assumptions—virtual oscillators—turned out to be of great importance in Heisenberg’s discovery of the quantum or matrix mechanics in 1925 (see Miller 1994, 4-8).

By 1927 the wave-particle duality moved from being inconsistent to paradoxical. It seemed to be like relating apples and fishes. Consider a light quantum. Its energy $E$ and momentum $p$ are related to its frequency $\nu$ and wavelength $\lambda$ through Planck’s constant $h$ as

$$ E = h \nu \quad (1) $$

$$ p = h/\lambda \quad (2) $$

Eqs. (1) and (2) relate the light quantum’s “particle” properties ($E$ and $p$) with its “wave” properties ($\nu$ and $\lambda$). Is this not inconsistent, that is, paradoxical. Bohr’s approach to paradox busting was unique: instead of reconciliation, or rejecting one horn of the dilemma, he realised that both horns had to be embraced. As Bohr put it in his complementarity principle of September 1927, taking serious account of Planck’s constant—which is the connector of the light quantum’s “wave” and “particle” modes [see Eqs. (1) and (2)]—means that we must extend our intuitivity into a domain where, in fact, entities are “wave” and “particle” simultaneously (Bohr 1928). In this regime the terms “wave” and “particle” do not have the same connotation as they do in the world of sense perceptions. This is why I placed them in quotes. In Bohr’s own words (Bohr 1928, 590; Miller 1996, 65-68):

Indeed we find ourselves here on the very path taken by Einstein of adapting our modes of perception borrowed from the sensations to the gradually deepening knowledge of the laws of nature. The hindrances met on this path originate above all in the fact that, so to say, every word in the language refers to our ordinary perception.

Consequently, it turns out that Richardson’s “logical contradiction” and the subsequent misunderstandings in interpreting the wave/particle duality of light were
rooted in attempting to interpret this phenomenon using arguments based on sense perceptions.

I next move to a version of inconsistency that is not unconnected to the previous one.

3. REASONING ON THE BASIS OF CONCEPTS THAT ARE LATER JETTISONED

Although this is a *posteriori* regarding scientific research, it throws light on the concept of scientific progress. I begin with the earliest attempts in modern science to formulate theories of heat and electricity based on so-called subtle fluids such as caloric. Subtle fluids possess no mass and, in the case of caloric, are composed of particles that repel one another while attracting ordinary matter. We recall that the modus operandi for this approach was to keep as close touch as possible to our everyday intuition (of, for example, heat flowing) and yet get a clue as to how to mathematise thermodynamics and electricity in analogy with Newtonian fluid mechanics. The rejection of caloric by the 1850’s came about as a result of severe inconsistencies with experiments of Count Rumford and James Joule, among others, as well as with Sadi Carnot’s results on the second law of thermodynamics. While caloric disappeared, the electrical fluids were superseded by the ether which was supposed to be the carrier of electromagnetic disturbances. At bottom, the argument for an ether was anthropocentric: something is needed to transport electromagnetic disturbances. Is this not what one’s intuition would expect?

In 1900, Henri Poincaré pointed out what he took to be an important inconsistency in the principal electromagnetic theory of the late nineteenth century—Lorentz’s theory: Lorentz’s ether violated Newton’s law of action and reaction (Poincaré 1900). To which Lorentz replied laconically in a letter to Poincaré of 20 January 1901, “must we in truth worry ourselves about it” (Miller 1981, 44)? Lorentz simply declared this to be a fact of life, sacrificed in order that he could deduce other results essential to explaining optical data.

The principal inconsistency of all ether-based theories of light and electromagnetism was their disagreement with the results of ether-drift experiments. The inconsistency here, or tension, between theory and experiment is as follows:

\[ c' = c + \nu \quad \text{(predicted by theory)} \]  
\[ c' = c \quad \text{(measured to order } (v/c)^2 = 10^{-8}) \]  

where \( c' \) is the velocity of light measured on the Earth (assumed to be an inertial reference system), \( c \) is the velocity of light in the free ether as measured by someone in a reference system fixed in the ether, and \( \nu \) is the relative velocity between the Earth and ether. Eqs. (3) and (4) are inconsistent with each other. Devilishly clever hypotheses were invented to explain away disagreement between data and theory. Amongst them was the Lorentz Contraction Hypothesis. Essentially physicists took \( \nu \) to be a causative agent that, for example, caused a body to contract in the direction of its motion, owing to the interaction between the moving body’s constituent
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