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CONCERNING SOME PHILOSOPHICAL REASONS FOR THE RECOURSE TO MATHEMATICS IN THE STUDY OF PHYSICAL PHENOMENA IN THE THOUGHT OF NEWTON AND LEIBNIZ

Considering the physics of Newton and of Leibniz, we are confronted with two different ways of explaining physical phenomena and with different kinds of concepts. Whether we speak about space, time, force, matter, vortices etc., their conceptions diverge.

In this paper, we are not principally concerned with a comparison of their respective theories of physics considered as a whole or as fully accomplished. Thus, a straight comparison of their respective concepts and of their world systems will not be a main issue. And, if we touch upon some theological or metaphysical arguments in favour of their physics, we will not give an account on how these are discussed in later years when their systems enter in conflict with each other.

Today, our interest in Newton's and Leibniz's physics is more focused on what happens before their theories find their final expression. We will pay attention to how different concepts are articulated during this period preceding their "accomplishment". For, these concepts change their meaning and signification as a result of the authors' reflection on them and/or as a consequence of their meditation on others. And it seems to us that the question of the role and place attributed to mathematics within the theory is of great relevance in this context.

By studying their thought as a theory "en devenir" or as a "work in progress", we believe being not only able to examine the evolution of central concepts in their systems, but also to have a closer look on how Newton and Leibniz consider the problem of mathematisation[1]. For, both inherit the belief from Descartes and Galileo that mathematics are appropriate to explain physical phenomena, and they do both have the intention of elaborating a physical theory where mathematics play an essential role. But in doing that, they are both led to clarify how a mathematical theory is possible or acceptable, i.e. on which grounds the possibility of having recourse to mathematics in the study of physics (mechanics) is laid. And further on, this process sometimes implies conceptual consequences for the theories themselves. "Work in progress" certainly also means work of deepening and clarifying.

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TOWARDS AN ABSOLUTE SPACE. NEWTON AND THE PROBLEM OF MOTION

Then, concerning Newton, we can consider that, even though he produced many of his ideas of physics in the 1660s, it is principally in the period from the *On Motion* [2] of 1684 to the *Principia* [3] of 1687 that his ideas became a part of a process of maturation leading to their final expression in a coherent theory. And, it was particularly in this process that Newton encountered the problem of mathematisation of physics.

The story is well known. When Edmund Halley came on visit to Newton in August 1684, he asked him what trajectory a planet should follow if one supposed an inverse square law for the central force acting upon it. According to John Conduitt, Newton answered that it should be an ellipse [4]. And when Halley asked him why he thought so, he said that he had calculated it. But as he couldn't find it, he promised to send him the solution later. Some months later then, Halley received a short tract entitled *On Motion* which constitutes "the point of departure" of what was to become the celebrated *Principia.*

In this tract, Newton elaborates a mathematical representation of planetary motion. Following a synthetic scheme, he presents – like in the *Principia,* a set of definitions, hypothesis (laws in the *Principia*) and lemmas before demonstrating eleven theorems and problems. However, this set differs slightly from the one in the *Principia.* In *On Motion,* we don't find any definitions of quantity of matter, quantity of motion nor is the force defined in exactly the same manner. And there are no trace – at least not in this part of the tract – of what was to become the second and third laws of motion. Nor does he mention absolute space and time.

It is interesting to note that, while the initial propositions don't change essentially from manuscript to manuscript, the set of definitions, hypothesis and lemmas as well as the scholia following some of the propositions *do.* These changes show how Newton is confronted with two kinds of requirements; one concerning the clarity and the pertinence of fundamental concepts on which the truth of the following propositions largely depends; the other involves the relation between theory and real world. However, these two requirements are not necessarily independent. Reflecting on the relation between theory and real world may modify the set of fundamental concepts, qualitatively or quantitatively. And vice versa, a modification of these concepts may have a consequence for the apprehension of the relation theory – real world.

In the first revision of *On Motion* entitled *On Motion of Spherical Bodies in Fluidis* [5], Newton takes these aspects into account. He largely modifies the hypothesis and lemmas. The 'hypothesis' become 'laws' (*leges*). He introduces a first assertion of the second law of motion as well as two new laws inspired by the principle of inertia. He also makes a first step towards the method of first and last ratios.

However, we will concentrate on a problem encountered in an addition to the scholium of the fourth theorem.[6] In *On Motion*, he had only considered the motion of a body revolving around a force centre considered – not as a body – but as a simple mathematical point. He never envisaged possible actions of other bodies. In the new scholium, on the contrary, he introduces a "whole space of the planetary heavens"[7] and defines a centre of gravity in it. Leaning on one of the new laws (law 4) he states that this space is either at rest or in uniform rectilinear motion[8]. And from the other new law (law 3) he infers that all motions between planets are the same whether this space is at rest or in uniform rectilinear motion[9]. Consequently, he considers this space to be immobile, and hence the centre of gravity will be immobile as well.

Now, defining a centre of gravity implies that he takes the motion of all planets into account. On the other hand, if the conception of a common space being always equal in relation to the planets' motions is true, then it has important consequences for his mathematical explanation of planetary motion as it appears in the *On Motion*. Newton is well aware of it. In fact, it means that the centre of force doesn't coincide with the centre of gravity, and hence will no longer be immobile as presupposed in his mathematical theory. At this stage, though, Newton accepts that his theory is a simplification of the real world – it would indeed "exceed the force of human wit"[10] to consider all these motions at the same time. Nevertheless, he explains, the mathematical theory still has its power of explanation – for by taking the mean values of observations, one should still obtain an ellipse near the calculated one.

So, in this revision Newton considers the complexity of the real world and evaluates its consequences for his mathematical theory of planetary motions. What allowed him to do this, though, was the extension of the principle of inertia to the system of planets, postulating an immobile space. But, it is not yet an absolute space, and we have to move further to the *On Gravitation*[11] to discover it.

In this manuscript it is not the particular question of planetary motion that is the main problem, even though it constitutes an important element in the argumentation, *but* rather the general question of how we can possibly speak of motion mathematically.

After having defined place, body, rest and motion, he is led to argue against the natural philosophy of Descartes. He attacks his conception of motion arguing that it leads to absurdities and contradictions. He is refuting the relativity of motion the Cartesian conception implies, stating that it is impossible to ever know which body is truly in motion and which is truly at rest. The reason for this is, according to Newton, that Descartes did not distinguish between matter and extension (*étendue*) as independent of body and matter.

Furthermore, Newton states that the Cartesian point of view implies that it is impossible to clearly define places (*locus*) in relation to what the displacement of a body and thus its motion can be identified. In his opinion the Cartesian motion is not a motion at all because we cannot assign any fixed places, and hence no displacement nor any speed. Consequently, he is led to assert the necessity of relating places and local motion to something immobile.

This something immobile is the space itself and Newton characterises it as infinite, eternal and divisible. It is all over uniform and has no capacity whatsoever of resistance or of provoking change in motion. It is not a substance nor an accident, but an emanating effect of God and a certain affection of all being. It is something without which nothing could ever exist. This space is called absolute in the manuscript *On Motion of Bodies in Uniformly Yielding Media*[12]. It has then acquired the necessary characteristics (infinite, eternal, divisible, non-corporeal) in order to constitute a reference frame for a mathematical treatment of motion. It allows Newton to identify real motion and to distinguish it from the relative. Force becomes a causal principle that enables him to that; real motion or rest "are never changed except by force impressed on the body moved or at rest, and are always changed after [the action of] such force, while relative motion can be changed by forces impressed on other bodies"[13].

TOWARDS AN ABSOLUTE FORCE. LEIBNIZ AND THE PROBLEM OF MOTION

The case of Leibniz' physics seems more complicated, because he never wrote a treatise like the *Principia* and because his physics involves assembling of many different manuscripts and articles that cover different sides of it. Nevertheless, we can identify an evolution in his thought with respect to the problem of mathematisation of physics.

In a letter of July 1676 to Edmonde Mariotte, Leibniz expresses his belief that "there exist physical effects of which it is possible to find the last cause"[14]. And this is the case, according to Leibniz, "when a truth of physics depends on a truth of metaphysics or geometry". He is probably referring to his discovery in the *On the Secret of Reducing Motion and Mechanics to pure Geometry*[15] written that same summer. In it, he takes up the problem of mathematisation of physics, stating that the solution should be found in a principle which allows such a "reduction of mechanics to geometry". Being inspired by the equilibrium law of Archimedes which allows a mathematical explanation of statics, Leibniz states that by this principle, one should be able to establish equations of relations in mechanics. However, it is the geometry that offers him the key to a solution:

> As the principle of calculation usually is laid down from the equation between the whole and all its parts, the principle in all mechanics depend on the equation between the full cause and the whole effect.[16]

This is, in fact, the first known statement of the principle of equivalence between the full cause and the whole effect, and it implies that something should be conserved. It is a principle of conservation that enables Leibniz to treat physical phenomena mathematically. But the question then is: what is to be conserved? Leibniz speaks about a power. "Of the full cause and whole effect, he says, the power (*potentia*) is the same" [17]. However, what is exactly meant by this "power" or "*potentia*" doesn't become clear until February 1678 when Leibniz accomplishes a genuine "research project" entitled *On Collisions of Bodies* [18]. Intent on finding some fundamental laws of mechanics based on this principle of conservation, he considers the case of collisions. Without going into detail, we should nevertheless mention that he supposes at first this "*potentia*" to be the Cartesian absolute force (or quantity of motion)[19]. After several calculations he obtains some numerical results that he compares with those obtained from an experiment done on a double pendulum system. Now, these results diverge considerably, and Leibniz continues his work. He then restates the cited principle adding that a "force" still has to be conserved. However, he doesn't reconfirm that this "force" is the Cartesian quantity of motion, but rather:

> one should not estimate the force in the body by its speed and magnitude, but by the height from which it descends. [20]

This statement along with Galilee's law for falling bodies allow Leibniz to establish a new measure for force: mv^2 – the product of the body's magnitude or quantity and the square of its velocity. By adopting this measure for the absolute force, he is now able to establish what is today known as the conservation principle of quantity of motion which is essentially different from the Cartesian as this last one doesn't take the direction into account, notably because it was considered absolute.

In *On Collisions of Bodies*, Leibniz expresses an occasionalist point of view asserting that a "very wise cause" maintains the bodies in motion. But a year later, he starts revising this conception. In two short manuscripts published by Loemker under the title *On the Elements of Natural Science* [21], he indicates the direction that his thoughts on the absolute force make it necessary to link it more strongly to a metaphysical conception. He states – as before – that the laws of motion "depend upon the mathematical principle of the equality of cause en effect", and that they are among those things that "cannot be explained from the necessity of matter alone" [22]. But he goes even further:

> In fact, he says, I have contended that the reasons for physical motion cannot be found in mathematical rules alone but that metaphysical propositions must necessarily be added. [23]

However, although he asserts the necessity of including metaphysical explanations in order to fully understand mechanics, he doesn't yet make a clear step in any specific direction. But, in a letter to the Duke Johann-Friedrich, he writes that he had "re-established demonstratively the substantial forms that the Cartesians pretend to have exterminated" [24].

We then understand the significance that the concept of force takes in the *Discourse on Metaphysics* [25] of 1686 and in the following correspondence with Antoine Arnauld [26].

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