The term "law" appears in different contexts with different meanings. We are used to speaking of natural laws, legal laws, moral laws, aesthetic laws, historical laws. Such a linguistic convention has represented a constant phenomenon through the history of civilization. Is there any deep common root among all these different uses and meanings?

I. LAWS OF NATURE

The expression "νόμοι τῆς φύσεως" (laws of nature) can be found in Democritus, the Sophists, Plato, Aristotle. Its main meaning seems to be: "something that cannot be escaped", a kind of ἀνάγκη (fate). Sometimes "νόμοι τῆς φύσεως" were opposed to "human laws", which are possibly violated.

In the wake of the modern scientific revolution, the term "law of nature" was currently used by Kepler, Galileo, Newton, Leibniz. In a letter to Benedetto Castelli, Galileo writes:

La natura inesorabile e immutabile non trasgredisce mai i termini delle leggi impostele.
(Nature, which is inexorable and immutable, never violates the laws that have been imposed to her).

We find here again a juxtaposition of human laws (which are possibly violated) and natural laws (which cannot be escaped).

What is a natural law? Galileo proposes what seems to our eyes a "quite modern" definition:

A natural law represents a constant mathematical relation among varying values of physical quantities, in the framework of a class of similar phenomena.

Sometimes Galileo speaks of constant proportions among phenomena, which have been associated to numerical values by means of measurements. The celebrated law of falling bodies

\[ d = \frac{1}{2}gt^2 \]
represents an example of this kind. The law asserts that a constant mathematical relation holds between the values of two physical quantities: 1) the distance \( d \) covered by a falling body; 2) the time \( t \) used to cover such a distance (in a vacuum-situation).

In a sense, a natural law represents a *quid facti* against a *quid iuris* that may be imposed by an authority. However, natural laws are far from being mere descriptions of pure facts because of the essential role played by mathematics. According to Galileo, "le certe e necessarie dimostrazioni" (the certain and necessary proofs) are not less important than "le sensate esperienze" (the sense experiences):

L'esperienza serve a piegare quelli alla mente dei quali non arrivano le necessità delle dimostrazioni geometriche.
(Experience serves to bend those whose mind is unable to grasp the necessity of geometrical proofs).

A quite similar position has been defended by Newton: natural laws represent certainties that are derived by means of mathematical arguments and that are verified by observations. A translation into a contemporary language might be:

*a natural law is a proposition that can be proved in a theory verified by experiments.*

What is still missing in Galileo and in Newton? Three basic ideas of contemporary science:

1) Any measurement is accompanied by a characteristic precision.
2) Any scientific law is associated to a domain of validity.
3) Generally, a law is a probabilistic assertion.

In a contemporary perspective, natural laws can be formally characterized in the framework of what has been called *the semantic approach to empirical sciences* ([5], [8], [9], [4]).

According to van Fraassen, the basic slogan of the semantic approach can be sketched as follows:

*presenting a theory means presenting a class of models.*

But how can models be characterized [4]?

Take a physical theory \( T \) (say classical or relativistic or quantum mechanics). A model \( M \) of \( T \) can be described as a *structure* consisting of three parts:

1) a mathematical part;
2) an experimental part;
3) a translation, that gives a mathematical interpretation to all elements of the experimental part.

We can write:

\[
M = \langle \text{Math, Exp, Transl} \rangle,
\]
where:

1) the mathematical part Math corresponds to a mathematical model, in the standard sense of logical semantics;

2) the experimental part Exp consists of a set of physical systems, accompanied by an appropriate set of physical quantities, which are supposed to be operationally defined;

3) Transl represents the mathematical interpretation.

In this framework one can define the concept of empirical truth with respect to a given model $M$. For instance, Galileo's law of falling bodies

$$d = (1/2)gt^2$$

turns out to be empirically true with respect to an appropriate class of models of classical mechanics.

Roughly, the empirical-truth definition can be sketched as follows. Consider a sentence $A$ asserting that a given mathematical relation holds among the values of some physical quantities (say, the law of falling bodies):

$A$ will be empirically true with respect to a model $M$ if, and only if, whenever we perform a measurement of the physical quantities (which $A$ refers to) in any physical system (of $M$), we obtain some results that satisfy the mathematical relation expressed by $A$, up to a given precision (where the precision depends on the operational definitions of the quantities in question).

Such a semantics permits us to formally analyze precisions and approximations, domains of validity and probabilistic results. On this basis one can understand how it is possible that two rival theories are at the same time empirically true with respect to a certain domain of phenomena, even if their logical conjunction turns out to be contradictory!

Let us think, for instance, of the case of classical mechanics and quantum theory. Such puzzling situations can be hardly described in the framework of classical semantics, where the non-contradiction principle (a sentence and its negation cannot be at the same time true) is supposed to be always valid.

According to van Fraassen ([8]), the semantic approach leads to a strong criticism of the very notion of law of nature, which is based on a metaphysical "pre-kantian" conception of absolute truth.

Van Fraassen claims:

there are no laws of Nature, there are only laws of models!

Is such an "extremistic conclusion" justified? It seems to us that the intuitive idea of law of nature (which is currently used by the working scientists) can be preserved and justified even in the framework of a semantic (non-metaphysical) approach. Laws of nature can be regarded as sentences that are empirically true with respect to all the models of an accepted theory. In fact, most physicists
usually speak of "laws of nature" just in this sense, and not according to a metaphysical commitment, which seems to be somewhat far from a scientific attitude.

II. ALETHIC AND DEONTIC MODALITIES

What can be said about the relationship between natural laws and legal norms? A main difference can be explained in terms of logical modalities.

Let us first distinguish two kinds of modal operators:

- the operator **physically necessary** (or according to a physical law);
- the operator **legally obligatory** (or according to a legal norm).

We will simply write: **necessary** and **obligatory**, respectively. **necessary** represents an alethic modality, while **obligatory** corresponds to a deontic modality. The basic difference between the two cases is illustrated by the following logical relations:

- **Necessary** $A$ implies $A$.

In other words: all that is necessary does happen. Physical laws cannot be escaped!

However:

- **Obligatory** $A$ does not imply $A$.

Legal norms are possibly violated!

The distinction between alethic and deontic modalities is clearly reflected in some natural languages by means of appropriate verbs. In English, the verb "ought" mainly corresponds to the modality **obligatory**, while: "must" mainly corresponds to **necessary**. Similarly in German we have the distinction between: "sollen" and "müssen".

Also the dual modalities are linguistically distinguished both in English and in German. Consider the modal operators: **permitted** and **possible**. Clearly, **permitted** means: **not obligatory** not; while **possible** means: **not necessary** not. In English, the verb "may" mainly corresponds to the modality **permitted**, while "can" mainly corresponds to **possible**. Similarly in German we have the distinction between "dürfen" and "können".

The result of a confusion between deontic and alethic modalities has been ironically commented in Busch’s **Max und Moritz**:

**ES KANN NICHT SEIN WAS NICHT SEIN DARF!**

In other words:

**IF A IS NOT PERMITTED, THEN A IS NOT POSSIBLE!**
Strangely enough such modal distinctions are not precisely reflected either in the Latin languages (Latin, Italian, French, Spanish, ...) or in most Slavish languages (Russian, Polish, ...).

Logicians have proposed a natural semantic analysis of the behaviour of different modal operators. The conceptual tool that has been successfully used is represented by the Leibnizian notion of possible world.

Possible worlds are not necessarily bound to a metaphysical interpretation. As shown by the development of contemporary semantics, the concept of possible world represents a quite flexible notion, which can be used for different applications. For instance, in the sense of:

- context or circumstance;
- time-instant or time-interval;
- physical situation or physical system in a given state;
- information system.

Suppose a class of possible worlds \( i, j, k, \ldots \). Two worlds \( i \) and \( j \) may be accessible. We will say the the world \( j \) is accessible to the world \( i \), whenever \( j \) represents a possible alternative for \( i \).

On this basis one can give a truth definition for modal sentences:

- **Necessary** \( A \) is true in the world \( i \) if and only if
  - \( A \) is true in all the worlds that are accessible to \( i \).

Similarly:

- **Obligatory** \( A \) is true in the world \( i \) if and only if
  - \( A \) is true in all the worlds that are accessible to \( i \).

In this framework, it is useful to distinguish two semantic hypotheses:

I) Any world is accessible to itself
   (in other words, the accessibility relation is reflexive).

II) A world might be inaccessible to itself
    (the accessibility relation is not generally reflexive).

Our first hypothesis gives rise to an alethic modality:

**Necessary** \( A \) implies \( A \).

If \( A \) is true in all the worlds that are accessible to \( i \), then \( A \) must be true also in \( i \) (because the accessibility relation is reflexive).

Our second hypothesis, instead, gives rise to a deontic modality:

**Obligatory** \( A \) does not imply \( A \).

It may happen:

- \( A \) is true in all the worlds that are accessible to \( i \).
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