II. LAYING THE FOUNDATIONS OF PHENOMENOLOGY

ON HUSSERL'S MATHEMATICAL APPRENTICESHIP AND PHILOSOPHY OF MATHEMATICS

Insight into the formative role that Edmund Husserl's early training in mathematics played in the development of his ideas is fundamental to understanding his philosophy as a whole. Besides shedding light on the genesis of phenomenology, which began to take shape in Husserl's reflections on the inability of the logic, psychology, mathematics and philosophy of his time to respond to certain onerous questions raised by his earliest attempts to secure radical foundations for arithmetic, understanding Husserl's ideas about mathematics sheds needed light on a number of other dimensions of his thought that have puzzled and challenged philosophers in this century. For example, this is precisely where many of the clues are to be found that are needed to answer questions of a controversial nature about seemingly enigmatic aspects of his thought, among them questions regarding the nature and evolution of his views on psychologism, on Platonism, on realism, and the relationship between his formal and his transcendental logic.

Moreover, this is the only way there is to situate and evaluate Husserl's philosophy in relation to the ideas and innovations of the most eminent and influential mathematicians of his time, notably Karl Weierstrass, Georg Cantor, David Hilbert, and Kurt Gödel, or Gottlob Frege and Bertrand Russell, men who often shared Husserl's desire to discover secure, scientific foundations for mathematics and the theory of knowledge, his concern to reform logic, his intent to fight against psychologism, his desire to develop a theory of meaning, his questions as to the philosophical significance of the latest developments in mathematics, and so on.

Understanding the evolution of Husserl's views on mathematics is therefore essential to establishing Husserl's proper place in 20th century philosophy of logic and mathematics, a field with deep roots in Austro-German ideas about mathematics, logic and philosophy, which flowered in English-speaking countries in the twentieth century, but into which Husserl's ideas have never been properly integrated. Given the preeminent role that philosophy of logic and mathematics has played in shaping the way philosophy was done in English-speaking countries in the twentieth century, investigations into Husserl's work in this area thus also supply the material essential for the building of any possible bridge between phenomenology and its principal rival, analytic philosophy. And such investigations afford the best possible explanation as to why so many of Husserl's ideas seem so close to those of that antagonistic school, while others remain so plainly diametrically opposed to it.

UNDER THE INFLUENCE OF WEIERSTRASS

Husserl came to the decision to pursue mathematics as a career during his student years in Berlin, where he enthusiastically threw himself into the study of that most rigorous of disciplines. It was there that from 1887–1881 he attended the courses of the great mathematician Karl Weierstrass (Schuhmann, 7; M. Husserl; Osborn, 12–14).

Weierstrass' thoroughgoing, systematic treatment, *ab initio*, of the theory of analytic functions had led him to profound investigations into the principles of arithmetic. His scrupulous manner of submitting the foundations of analytic functions to close scrutiny awoke in Husserl an interest in seeking radical foundations for mathematics. "I came to understand", Husserl recalled, "the pains he was taking to transform analysis from the mixture of reason and irrational instincts and know-how it was at the time into a pure rational theory. His aim was to expose its original roots, its elementary concepts and axioms on the basis of which the whole system of analysis might be deduced in a completely rigorous, perspicuous way" (Schuhmann, 7; Jourdain, 295–96).

In reaction to the Kantian psychologization of mathematics popular among his contemporaries, Weierstrass was preaching the arithmetization of analysis, the rigorous founding of analysis purely on the basis of the positive whole numbers. Weierstrass was famous for teaching that once one had thus admitted the notion of whole number, arithmetic needed no further postulate, but then could be built up in a purely logical fashion. This would have the effect of depsychologizing and degeometrizing analysis, of liberating it from the insidious appeals to intuitions of space and time that had been imported into it since Kant had proclaimed that mathematical propositions were synthetic a priori (Coffa; Demopoulos, 1994).

Husserl's encounter with Weierstrass had a deep and lasting effect on the future founder of the phenomenological movement. It was from Weierstrass, Husserl would say, that he acquired the ethos of his intellectual endeavors (Schuhmann, 7). Late in his career he would even say that he had sought to do for philosophy what Weierstrass had done for mathematics (Becker, 40–42; Schuhmann, 34). As Andrew Osborn, who actually consulted with Husserl about this, explained: "Through Weierstrass especially, too, the Berlin school placed enormous importance on the rigor of demonstration, a practice that seized hold on Husserl's imagination so that when later he turned to philosophy he sought to find there a strict science similar to that on which Weierstrass insisted, along with the certainty that follows from such strictness and such rigorous proof" (Osborn, 12).

Indeed, closely inspecting the course of Husserl's intellectual career one continually finds him reworking themes present in Weierstrass' work and striving to apply the very principles that underpinned the mathematician's efforts to rigorize analysis. This is, for example, evident not only in Husserl's early espousal of Weierstrass' conviction that the cardinal number was "the first and most underivative domain, the sole foundation of all remaining domains of numbers" (Husserl, 1994, 2), but also in Husserl's struggles with psychologism, his lifelong search for radical foundations for knowledge, his striving to lay bare the original roots, the most primitive concepts and principles of knowledge, to uncover the fundamental building blocks on the basis of which his whole system of philosophy might rest, his ideas about phenomenology as a strict science, his efforts to extend the notion of the analyticity, and so on. The nature of his attraction to Weierstrass' work also explains much about the nature of Husserl's attraction to the work of Franz Brentano, Georg Cantor, Bernard Bolzano, David Hilbert, and even Gottlob Frege.

Husserl was, of course, not alone in being decisively influenced by Weierstrass' thoroughness and systematic approach. What we know of Husserl's reaction before Weierstrass' efforts to rigorize analysis is consonant with the impression that he left on much of the mathematical world of his time. "Mathematicians under the influence of Weierstrass", Bertrand Russell once noted, "have shown in modern times a care for accuracy, and an aversion to slipshod reasoning, such as had not been known among them previously since the time of the Greeks" (Russell, 1917, 94).

In Berlin, Husserl was also influenced by Leopold Kronecker who also believed that: "Sometime we shall succeed in 'arithmetizing',—that is to say, in founding alone on the number-concept in the narrowest sense, and therefore in stripping away again all the modifications and extensions of this concept, which have mostly been caused by the applications to geometry and mechanics,—the whole of arithmetic" (cited Jourdain, 5). Osborn credited Kronecker with having "sown the first seeds of philosophical understanding" in Husserl "and fostering the interest so aroused". "Husserl found in him," Osborn recounted, "a depth of understanding that stirred an echo in his own nature. Kronecker's special field was the philosophy of mathematics and it was through contact with him accordingly that Husserl first came to any appreciation of the philosophic point of view. Reflective by nature, Husserl found a ready interest in the philosophy of mathematics which was for him, as it proved, a very big step in the direction of an interest in pure philosophy". Osborn speculates that Husserl's interest in Descartes may have first been awakened by Kronecker (Osborn, 12).

As happy as Husserl was in Berlin, acting upon his father's wishes, he left for Vienna to prepare his doctoral thesis on the calculus of variations. Summoned by Weierstrass to serve as his assistant, Husserl later returned to Berlin. However, he quickly took advantage of an opportunity to return to Vienna to indulge a growing interest in philosophy (M. Husserl; Osborne, 15).

HUSSERL MAKES PHILOSOPHY HIS LIFE'S WORK

Although Husserl manifested little interest in philosophy during his time in Berlin, it became the minor subject for his doctorate in mathematics in Vienna. During that time, when his interest in philosophy was growing and he was wondering whether to make mathematics or philosophy his life's work, Husserl began attending the courses of the philosopher Franz Brentano. At first he did so merely out of curiosity, but these courses finally proved to be the decisive factor encouraging him to dedicate himself entirely to philosophy. But for Brentano, Husserl would say, he would not have become a philosopher (Husserl, 1919, 342; M. Husserl; Brück).

The specific reasons for admiring Brentano that Husserl gave actually quite resemble his reasons for admiring Weierstrass. The man in whom Weierstrass had awakened an interest in seeking radical foundations for knowledge was impressed by Brentano's clear, rigorous, insightful, objective, and precise philosophical analyses and ability to transform unclear beginnings into clear thoughts and insights, his "finely dialectical measuring of various possible arguments, his clarifying of equivocations, and retracing of every philosophical concept to its original intuitive sources". "Brentano relatively quickly moved from intuition to theory, to the delimitation of sharp concepts, to theoretical formulation of working problems", Husserl recalled. For Husserl, Brentano was someone entirely devoted to the austere ideal of a strict philosophical science, someone completely certain of his method who believed that his sharply polished concepts, his strongly constructed and systematically ordered theories, and his all round aporetic refutation of alternative interpretations, captured final truths. He "strove constantly to satisfy the highest claims of an almost mathematical strictness". "Sometimes it was the subject matter which overcame me," Husserl recalled, "other times the quite singular clearness and dialectical sharpness of his expositions, the cataleptic power as it were of his way of developing problems and of his theories". It was from Brentano, Husserl acknowledged, that he acquired the conviction that philosophy "was a serious discipline which could and must be dealt with in the spirit of the strictest science" (Husserl, 1919, 343–44).

GEORG CANTOR AND HUSSERL'S PHILOSOPHY OF ARITHMETIC

Having attended Brentano's lectures for two years, Husserl's next career move was to the University of Halle, to prepare his *Habilitationsschrift* under the direction of Carl Stumpf, a member of Brentano's circle (Smith, 21–24) convinced of the great need for cooperation between mathematicians or scientists and philosophers in the area of logic (Frege, 1980a, 171).

Husserl would reside in Halle from 1886 to 1901. These were years during which his ideas were particularly malleable and changed considerably and definitively. In 1887, he completed *On the Concept of Number. The Philosophy of Arithmetic* was published in 1891. The better part of the subsequent years was spent in the throes of an intellectual struggle in the course of which he abandoned some of the main lessons he had learned from Weierstrass and Brentano and came to write the groundbreaking *Logical Investigations*, in which he began laying the foundations of the phenomenological movement that went on to shape the course of 20th century philosophy in Continental Europe.

Georg Cantor, the creator of set theory, taught at the University of Halle during those years and served on the *Habilitationskommittee* that judged Husserl's *On the Concept of Number* (Gerlach). The two became close friends. At the height of his creative powers in the 1880s and 1890s, Cantor had studied in Berlin from 1863 to 1869, where he too had come under the influence of Weierstrass, a fact which explains much of the initial intellectual kinship between Husserl and Cantor, whose ideas overlapped and crisscrossed in a number of respects (Hill, 1997a; Hill, 1999).

During Husserl's time in Halle, Cantor was particularly seeking philosophical justification for his theories. He wanted to show how his entire transfinite set theory rested upon sound principles and how the transfinite numbers might be regarded as consistent extensions of the finite reals. He had begun his *Mannigfaltigkeitslehre* explaining to his readers that he had come to a point of realizing that further work on set theory would require extending the concept of real whole numbers beyond previously set bounds and in a direction which as far as he knew no one had searched yet, and he offered this a justification or an excuse for introducing apparently strange ideas (Cantor, 1883).

Cantor was one of the few mathematicians of his time intent upon wedding mathematics and philosophy. Over the years he had grown increasingly interested in philosophy and by the time of Husserl's arrival in Halle was primed to abandon mathematics for philosophy. In 1894 Cantor would write to the French mathematician Charles Hermite that "in the realm of the spirit" mathematics had no longer been "the essential love" of his soul for more than twenty years. Metaphysics and theology, Cantor "openly confessed", had so taken possession of his soul as to leave him relatively little time for, his "first flame", i.e., mathematics. He was now serving God better, he told Hermite, than, owing to his "apparently meager mathematical talents", he might have done through exclusively pursuing mathematics (Cantor, 1991, 350).

Although older, and far less in a position to change course than Husserl was, this did not prevent Cantor from trying to teach philosophy (Cantor, 1991, 210, 218) and from seasoning his writings with philosophical reflections and references. In 1883, Cantor had published the *Grundlagen einer allgemeine Mannigfaltigkeitslehre*, a work which, according to its original 1882 foreword, had been "written with two groups of readers in mind—philosophers who have followed the developments in mathematics up to the present time, and mathematicians who are familiar with the most important older and newer publications in philosophy" (Hallett, 6–7). During Husserl's early years in Halle, Cantor published his theories in the *Zeitschrift für Philosophie und philosophische Kritik* because, as he said, he had grown disgusted with mathematical journals. He was in fact trying to integrate philosophy into his mathematical work to such an extent that colleagues warned him that this was liable to harm his reputation (Dauben, 139, 336 n. 29).

During Husserl's years in Halle, Cantor persisted in clothing his theories about numbers in a metaphysical garb. And he left no doubts as to where his philosophical sympathies lie. In the *Mannigfaltigkeitslehre* he had emphasized that the idealist foundations of his reflections were essentially in agreement with the basic principles of Platonism according to which only conceptual knowledge in Plato's sense afforded true knowledge (Cantor, 1883, 181, 206 n. 6). His own idealism being related to the Aristotelian-Platonic kind, Cantor wrote in an 1888 letter, he was just as much a realist as an idealist (Cantor, 1991, 323). "I conceive of numbers", he informed Giuseppe Peano, "as 'forms' or 'species' (general concepts) of sets. In essentials this is the conception of the ancient geometry of Plato, Aristotle, Euclid etc." (Cantor, 1991, 365). To Hermite he wrote that "the whole numbers both separately and in their actual infinite totality exist in that highest kind of reality as eternal ideas in the Divine Intellect" (cited Hallett, 149). Cantor considered his transfinite numbers to be but a special form of Plato's *arithmoi noetoi* or *eidetikoi*, which he thought probably even fully coincided with the whole real numbers (Cantor, 1884, 84; Cantor, 1887/8, 420). By "manifold" or a "set" he explained in the *Mannigfaltigkeitslehre*, he was defining something related to the Platonic *eidos* or *idea*, as also to what Plato called a *mikton* (Cantor, 1883, 204 n. 1). For Cantor, the transfinite "presented a rich, ever growing field of ideal research" (Cantor, 1887/8, 406).

Cantor considered that his technique for abstracting numbers from reality provided the only possible foundations for his Platonic conception of numbers (Cantor, 1991, 363, 365; Cantor, 1887/8, 380, 411). Abstraction was to show the way to that new, abstract realm of ideal mathematical objects that could not be directly perceived or intuited. It was a way of producing purely abstract arithmetical definitions, a properly arithmetical process as opposed to a geometrical one with appeals to intuitions of space and time (Cantor, 1883, 191–92). He envisioned it as a technique for focussing on pure, abstract arithmetical properties and concepts which divorced them from any sensory apprehension of the particular characteristics of the objects figuring in the sets and freed mathematics from psychologism, empiricism, Kantianism and insidious appeals to intuitions of space and time to engage in strictly arithmetical forms of concept formation (ex. Cantor, 1883, 191–92; Cantor, 1885; Cantor, 1887/8, 381 n. 1; Eccarius, 1985, 19–20; Couturat, 325–41).

With his theory of abstraction Cantor believed that he was laying bare the roots from which the organism of transfinite numbers developed with logical necessity. In the "*Mitteilungen*," written during the late 1880s, the embattled mathematician was particularly intent upon proving that his theorems about transfinite numbers were firmly secured "through the logical power of proofs" which, proceeding from his definitions which were "neither arbitrary nor artificial, but originate naturally out of abstraction, have, with the help of syllogisms, attained their goal" (Cantor, 1887/8, 418). Inspired by Weierstrass' famous theory to that effect, he was hard at work demonstrating that the positive whole numbers formed the basis of all other mathematical conceptual formations.

All this was part of his greater strategy aimed at providing his "strange" new transfinite numbers with secure foundations by demonstrating precisely how the transfinite number system might be built from the bottom up (Dauben, 1979, Chapter 6). In so doing, he was acting on a conviction, spelled out in an 1884 letter to Gösta Mittag-Leffler, that the only correct way to proceed was "to go from what is most simple to that which is composite, to go from what already exists and is well-founded to what is more general and new by continually proceeding by way of transparent considerations, step by step without making any leaps" (Cantor, 1991, 208).

HUSSERL'S FIRST FORAYS INTO PHILOSOPHY

Impressed by Karl Weierstrass' work to arithmetize analysis and armed with analytical tools learned from Brentano, Husserl embarked on a project to help supply radical foundations for mathematics by submitting the concept of number itself to closer scrutiny. *On the Concept of Number* and *The Philosophy of Arithmetic* were the result.

Husserl began *On the Concept of Number* writing of the need to examine the logic of the concepts and methods that mathematicians were introducing and using and for a logical clarification, precise analysis, and rigorous deduction of all of mathematics from the least number of self-evident principles. The definitive removal of the real and imaginary difficulties on the borderline between mathematics and philosophy, he deemed, would only come about by first analyzing the concepts and relations which were in themselves simpler and logically prior, and then analyzing the more complicated and more derivative ones (Husserl, 1887, 92–95).

The natural and necessary starting point of any philosophy of mathematics, Husserl still believed, was the analysis of the concept of whole number (Husserl, 1887, 94–95). He was confident that: "a rigorous and thoroughgoing development of higher analysis . . . would have to emanate from elementary arithmetic alone in which analysis is grounded. But this elementary arithmetic has . . . its sole foundation . . . in that never-ending series of concepts which mathematicians call 'positive whole numbers'. All of the more complicated and artificial forms which are likewise called numbers the fractional and irrational, and negative and complex numbers have their origin and basis in the elementary number concepts and their interrelations" (Husserl, 1887, 95).

As he undertook his project to provide a more detailed analysis of the concepts of arithmetic and a deeper foundation for its theorems, the still faithful student of Brentano also considered that psychology was the indispensable tool for analyzing the concept of number (Husserl, 1913, 33; see Husserl, 1891, 16). However, although the psychological analyses of *On the Concept of Number* were almost entirely incorporated into the first four chapters of *The Philosophy of Arithmetic*, the enthusiastic espousal of psychologism found in the earlier work is absent from the later one. And Husserl, who had not initially considered Brentano's teachings to be empirical and psychological in a pernicious sense, later confessed that there had been "connections in which such a psychological foundation never came to satisfy" him, that it could bring "no true continuity and unity", that he had grown "more and more disquieted by doubts of principle, as to how to reconcile the objectivity of mathematics, and of all of science in general, with a psychological foundation for logic" (Husserl, 1900–01, 42; Husserl, 1975, 34).

Husserl also soon abandoned Weierstrass' teaching on the primacy of the cardinal number. In a letter to Stumpf, written in 1890 or 1891, Husserl revealed that the theory that the concept of cardinal number forms the foundation of general arithmetic that he had tried to develop in *On the Concept of Number* had soon proved to be false. By no clever devices, he explained, "can one derive negative, rational, irrational, and the various sorts of complex numbers from the concept of cardinal number. The same is true of the ordinal concepts, of the concepts of magnitude, and so on. And these concepts themselves are not logical particularizations of the cardinal concept" (Husserl, 1994, 13). Husserl's tergiversation in this regard also becomes apparent through a comparison of the foreword and the introduction to *The Philosophy of Arithmetic* (Husserl 1891, viii, 5 and note; Hill, 1991, 81–85).

The lessons learned from his revered mentors had left him in the lurch. Husserl felt forced to embark upon an independent path. Ten years of hard, lonely work and struggle ensued. He felt that his efforts had brought him "close to the most obscure parts of the theory of knowledge", and that he was standing before "great unsolved puzzles" concerning the very possibility of knowledge in general. He described himself as having been "powerfully... gripped by deep, and by the deepest, problems" (Husserl, 1975, 16–17; Husserl, 1994, 167, 492–93). His search for answers that he did not believe his early training could provide eventually led him to adopt metaphysical and epistemological views that he had learned to consider odious and despicable (Hill, 1998).

FROM BOLZANO THE MATHEMATICIAN TO BOLZANO THE PHILOSOPHER

By his own account, Husserl had always been well positioned to appreciate the work of Bernard Bolzano who, as a mathematician, had already come to his attention as a student of Weierstrass. Husserl had become further acquainted with Bolzano's ideas through Brentano's critical discussions of the paradoxes of infinity in his lectures, and then through Georg Cantor (Husserl, 1975, 37).

Bolzano was a forerunner of the movement to rigorize analysis that would gain momentum later in the 19th century. His pioneering work to rebuild intuitively accepted proofs of theorems in a rigorous way solely on the basis of arithmetical and logical concepts prepared the way for much that Weierstrass would later advocate and undertake. And, as Weierstrass himself acknowledged, Bolzano actually developed much of the theory of real functions in much the same form that, inspired by him, Weierstrass would teach it in his inspiring courses forty years later (Sebestik, 17, 107 and note; Kline, 948, 950–55; Jourdain, 297; Føllesdal, 7–10; Coffa).

With so many of his mentors impressed by Bolzano's work, Husserl should have been primed to appreciate it. This was not, however, immediately the case. Once acquainted with Bolzano's thought, Husserl recalled, he had "made a point of looking through the long-forgotten *Wissenschaftslehre* of 1837 and of making use of it from time to time with the help of its copious index", but he originally misinterpreted Bolzano's original thoughts about ideas, propositions and truths in themselves as being about mythical entities, suspended somewhere between being and non-being (Husserl, 1975, 37; Husserl, 1994, 201–02).

This particular reaction on Husserl's part is understandable. For Brentano inculcated in his students a model of philosophy based on the natural sciences and trained them to despise metaphysical idealism. So, it is easy to see how Husserl, so completely under Brentano's influence in the beginning, might not have quickly warmed to philosophical ideas that Brentano taught his students to disdain (Husserl, 1919, 344–45). It was only after having grown disillusioned with Brentano's empirical psychology that Husserl became receptive to Bolzano's idealism.

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