The hardest thing to do with this book was to decide what to call it. The original working title was ‘Mathematics and the environment’, and my aspiration was, and is, to provide a blueprint for the application of mathematical models to problems in the environment which involve the use of differential equations.

The environment is becoming fashionable in applied mathematics, but it often means different things to different people. It may mean oceans and atmospheres, and numerical modelling; it may mean groundwater flow and related pollution problems, for example involving remediation of hydrocarbons or dispersal of phosphates and nitrates in the soil; or it might be the application of statistical methods in the assessment of risk and uncertainty in, for example, hydrological forecasting.

No doubt these subjects concern the environment, but they are particular topics. This book is about general scientific problems concerning phenomena in the world around us. In the sense that ‘mathematical biology’ is the mathematical study of living things, the logical title for this book would be ‘Mathematical Geology’, the mathematical study of processes on (or in) the Earth. Unfortunately, Geology is a subject which tends to carry the narrower meaning of the study of rocks, and it is partly to get away from this that university departments have increasingly rechristened themselves as departments of Geology and Geophysics, or of Earth Science, or (most recently) of Earth System Science.

So, this book is not just about mathematical geology: it concerns much more than the study of rocks. Nor is it mathematical geophysics, although it contains a good deal of this also. It is mathematics and the environment, but where the word ‘environment’ is used in a much wider sense than the narrower uses alluded to above.

The two books which are closest to this in theme and subject matter are Andrew Goudie’s ‘The Nature of the Environment’, and Arthur Holmes’s masterful ‘Principles of Physical Geology’. The latter book could almost provide the contents list for the present one. The difference of course is that my concern here is in providing mathematical models which can explain some of the physical phenomena which are described in these two books.

Writing about recent theories for subglacial landforms, Clarke (2005) said that ‘the work has a daunting mathematical level, uncertain relevance, but potentially
interesting implications.’ For an applied mathematician working in seriously interdisciplinary subjects, perhaps this slightly barbed comment is as good as it gets. This book is, I expect, daunting. It is not necessary that hard scientific problems beget hard mathematical problems when they are done properly, but it ought to be what you expect. Decent science does not come cheap.

I personally hope that most of this book is relevant, but that is ultimately a matter for the scientific community. Relevance is promoted by a kind of cultural acceptance, and it needs to be argued through, almost religiously.

This book in its earliest form consisted of written course notes for a sixteen lecture final year undergraduate course at Oxford. I have taught a similar twenty-four lecture course at masters’ level at Limerick. For such courses, I select four or five chapters, and selectively teach material from them. For example, the current Oxford and Limerick courses take material from Chaps. 2, 4, 5 and 10. Of course the chapters contain much more material than one could cover in four or six lectures; one could in fact take an entire course from a single chapter. But my purpose here is to allow a freedom for selection, and also to elaborate the material to the point where it becomes of research interest. In writing the book, I have been stimulated to question accepted wisdom, and to explore new ideas, and some of the material has even been written up in the form of research papers after the fact.

There is a danger in trying to write an encompassing book about mathematical geoscience, of which I am only too aware. Most obviously, there are many subjects which have been left out, and for those which are included, there is no space for a comprehensive exposition. A glance at the reference list will show that I have largely followed my own personal view of the subject matter. References are given at the end of each chapter, but do not aim to give a complete review; rather the intention is to provide pointers for those interested, with the hope that others will engage with some of the problems. Geoscience is full of extraordinarily interesting problems.

The audience for this book is largely what is called the GFD community, brought up on fluid mechanics in the oceans and atmosphere, but which has now branched out into many of the subjects dealt with here. It is my hope that applied mathematicians may chance on the material, and be stimulated to explore some of the models which are discussed. It is also my hope that geoscientists will find some of the phenomena and ideas interesting, even if some of the technical detail becomes at times too threatening.

A large number of people have been of considerable assistance and help in the something like ten years it has taken to finally produce this book. Firstly, I should thank my publishers at Springer, who have been very patient over the years: Karen Borthwick, and more recently, Lauren Stoney. I am grateful to Felix Ng, who rapidly and expertly produced early drafts of some of the figures for Chaps. 2, 4, 7 and 11. Ian Hewitt produced Fig. 10.14 and Fig. 9.15. Christine Butler unearthed a copy of Fig. 11.12 from the vaults of the International Glaciological Society. Bill Shilts, Christian Zdanowicz and Brian Moorman were very helpful concerning the image in Fig. 10.22; Gary Parker, Norm Smith and Terence McCarthy were equally helpful concerning Fig. 5.1. Thanks also to Emanuele Schiavi, Stephen O’Brien, Thomas Vitolo, Dave Cocks, Rachel Zammett, Geoff Evatt, Rob Style, Sarah Mitchell, Chris
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I solicited comments on individual chapters from many people, and these have been of great use. Firstly, my thanks to Garry Clarke and Chris Clark, who provided images (of Trapridge Glacier and ribbed moraine in Northern Ireland) for the front cover; sadly they could not be used because it took me so long to finish the book that in the meantime Springer changed the series design! Bruce Malamud spent a year in Oxford, and was no end of help in the minutiae of computer technology. I have received useful critical comments from Tom Witelski, Stephen O’Brien, Eric Wolff, Richard Alley, Henry Winstanley, Slava Solomatov, Alison Rust, Ian Hewitt, Garry Clarke, Janet Elliott and Don Drew.

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For a book such as this, it would be remiss not to mention with gratitude the annual GFD summer school at Woods Hole, where I have variously spent long periods of time, most recently in 2010, and where I have benefitted from the experience and wisdom of that excellent community of scholars, in particular Joe Keller, Lou Howard, George Veronis, and Ed Spiegel. Those who have spent time on the porch or in the classroom at Walsh Cottage will know what a privilege it is to be there, in the presence of one of the brightest and wittiest seminar audiences on the planet.

The University of Limerick has supported me through my appointment there as an adjunct Professor and subsequently, through an award by Science Foundation Ireland, as Stokes Professor. The funds they have generously provided have enabled me to maintain a research presence at conferences and workshops, as well as purchasing two of the laptops on which this book was written. They have provided a pleasant and stimulating working environment, not to mention easy access to the best countryside in the world.

This book is dedicated to Jim Murray and his wife Sheila. I first met Jim on a cold, dark December evening in 1970, when I ascended staircase 10 in Corpus Christi College, Oxford, to be interviewed for a place as an undergraduate. We peered at each other in the ancient, wood-panelled room by candlelight (these were the days of miners’ strikes and power cuts). Ever since then, Jim has been the torchbearer for my path in applied mathematics, yielding to no man in his quest for the practical and useful.
My view of science, and the act of doing science, is that at best it is like driving a car on an icy road. You know the car works, the road is flat, but actually, you do not really know what you are doing. You try out a few things and they more or less work. You might hit a slippery bit, but if you are lucky you get there somehow. And if you are not lucky, you end up in the ditch. What you have to avoid is the idea that, if you end up in the ditch, it is the right place to be. Do not get stuck in the ditch. Get out of the car and back on the road.

It was Kolumban Hutter who said: you do not finish a book, you abandon it. He was so right. It is like bringing up a child. You love it, change its nappies, feed it, nurture it, but by the time it is an adult, it is time to go. Be gone!

Limerick, Ireland

A.C. Fowler