Phenomenal progress in nonlinear systems theory has been made during the last decades. It has been reflected in two aspects. On the one hand, internal and external global stability notions have been studied intensely for uncertain nonlinear systems. On the other hand, the applications of these advanced stability results to control engineering systems have led to numerous novel methodologies for the design of nonlinear feedback controllers. It is fair to say that input-to-state stability (ISS), a notion invented by E.D. Sontag in the late 1980s, plays an influential role in the work of many researchers including the authors of this book. ISS has bridged the gap which previously existed between the input–output and the state-space methods, two popular approaches within the control systems community. Roughly speaking, the importance of ISS for the study of nonlinear systems is reflected by the intriguing fact that it captures two main stability notions: Lyapunov stability (i.e., the behavior of the zero-input response with respect to nonzero initial conditions) and input–output stability (i.e., the behavior of the zero-state response with respect to nonzero external inputs).

Nonlinear systems are encountered frequently in almost all branches of science and engineering. In fact, in engineering, physics, economics, and biology, nonlinearity is the rule, and linear systems are rare (which almost exclusively exist only in our computer programs). Despite the importance of nonlinear system theory, graduate students or researchers in mathematics, engineering, physics, economics, and biology often have difficulties in taking advantage of recent advances in mathematical systems and control theories. There are several excellent textbooks that provide nice introductions to nonlinear systems theory, but many recent stability results are scattered in the vast literature. Motivated by this observation, we set our hands to write this monograph about a year and half ago. The specific objectives of this book are described in the following.

The first aim of the book is to provide the basic knowledge needed for a graduate student in order to be able to understand the current research in nonlinear stability theory and nonlinear control theory. A relatively high level of mathematical background is assumed: the reader is required of having basic knowledge in differential equations, calculus, and real analysis. Measure theory is not needed (although
measurable functions are met even in the first pages of the book): the reader can replace “measurable” functions by “piecewise continuous” functions. The book is self-contained in the sense that all results are proved in detail by using basic mathematical knowledge and other results presented in the book. Only global stability notions are studied in the present book. It should be mentioned that there are many important results about local or regional stability in the literature.

The second aim of the book is to give a perspective of nonlinear stability theory and nonlinear control theory that is not frequently encountered in the literature. The idea can be stated in the following (informal) way:

“For every method of proving global stability, there is a corresponding method of nonlinear feedback design.”

Therefore, the book is designed to help the reader to understand this one-to-one correspondence. In the first five chapters the reader is introduced to internal and external stability notions and characterizations. Necessary and sufficient conditions for each stability notion are provided, and a description of the various methods for proving stability is presented. Finally, in Chaps. 6 and 7 of the book the reader is introduced to the various methods of nonlinear feedback design. Each method aims to design a feedback law such that the resulting closed-loop system is “stable.” The proof of the stability properties of the closed-loop system is performed by using one particular method of proving stability. We believe that this perspective can help the reader to understand the proposed feedback design methodologies and can inspire the reader to suggest new ones. However, for want of space, some methods of proving stability and feedback design are only briefly mentioned (e.g., the method of using Matrosov’s theorem).

The third aim of the book is to show that the same mathematical tools, up to minor modifications, can be used for all kinds of systems. Working within an abstract system-theoretic framework, one can see that systems described by Ordinary Differential Equations (ODEs), systems described by retarded functional differential equations (RFDEs), systems described by coupled retarded functional difference equations and retarded functional differential equations, and sampled-data systems can be analyzed and studied using (almost) the same tools. We believe that this feature is important: many recent contributions to nonlinear systems theory are developed for complex dynamical systems other than those described by ODEs. Furthermore, it has been recognized that feedback laws of new kinds (e.g., feedback laws with delays, hybrid/switching feedback laws) can give rise to features for the closed-loop system that cannot be encountered in systems described by ODEs. In such cases, the closed-loop system in question becomes a system with different mathematical description from the original open-loop system. Time-varying systems are not excluded: the proposed system-theoretic framework can capture all features of time-varying systems (e.g., nonuniform stability phenomena).

As the fourth and last aim of the book, it is the authors’ view that nonlinear systems theory has reached a level of maturity which can provide interesting contributions to other areas of applied mathematics. There are many results and examples in the book that illustrate the use of nonlinear systems theory to game theory,
fixed-point theory, numerical analysis, and (only superficially) mathematical biology. Some open problems are listed in the last Chap. 8 with a unique objective to entice the reader, in particular graduate students, to develop their novel ideas and techniques, which will contribute to the further development of modern mathematical control theory.

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