Preface

The central notion in this book is that of a local principle. Local principles provide an abstract frame for the natural and extremely useful idea of localization, i.e. to divide a global problem into a family of local problems. The local principles the reader will encounter in this text are formulated in the language of Banach algebras and can be characterized as non-commutative Gelfand theories. They now form an integral part of the theory of Banach and C^*-algebras, and they provide an indispensable tool to study concrete problems in operator theory and numerical analysis.

More than thirty years ago, Douglas derived the Gohberg-Krupnik symbol calculus for singular integral operators via a combination of a local principle (which now bears his name) and Halmos' two projections theorem. Around the same time Kozak proved the equivalence between the stability of an operator sequence and the invertibility of a related element in a certain Banach algebra which he then studied by Simonenko's local principle. Since that time there have appeared dozens of papers where the idea of localization has been used, further developed, and applied in several contexts. As the outcome of this development, we now have a powerful, rich and beautiful theory of algebraic localization, the principles and results of which are widely scattered in the literature. The lack of a general context, and the use of different notation from paper to paper make it difficult for the researcher and the graduate student to familiarize themselves with the theory behind local principles and to make use of these results to study their own problems.

It is this defect that the present book seeks to solve. It started as a much simpler task: an updated re-edition of an out-of-print report [168], back in 1998. The changing objectives and professional obligations of the authors kept on increasing the scope and delaying the work. After more than ten years, we are finally able to present it. We think that the delay has been worth it, and the reader has a readable and useful text in his hands.

It is our intention that this work be a basic but complete introduction to local principles, formulated in the language of Banach algebras, that allows the reader to get a general view of the area and enables him to read more specialized works. Many results that appeared in periodicals or reports, and can be hard to find, are presented, streamlined and contextualized here, and the relations with other results

are made clear. Some results which were in complete form available only in the Russian literature, like the local principle by Simonenko, are included. And finally, a few existing gaps in the theory are filled in with full proofs, which appear here for the first time.

The text starts with a chapter on the relevant notions for local principles of Banach algebra theory. As such, the first part can serve as a textbook for a one semester graduate course on Banach algebras with emphasis on local principles. Exercises and examples are given throughout the text. We focus on applications to singular integral operators and convolution type operators on weighted Lebesgue spaces. The choice of applications is the result both of our particular interests as researchers and of the genetic inheritance of the text, which was born as a report on algebras of convolution type operators.

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Chemnitz, Darmstadt, Lisboa   *Steffen Roch*
December 2009   *Pedro A. Santos*
                                                                             Bernd Silbermann

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