

Contents

Preface	vii
Preface to the First Edition	ix
Notation	xvii
1 Complex Numbers in Algebraic Form	1
1.1 Algebraic Representation of Complex Numbers	1
1.1.1 Definition of Complex Numbers	1
1.1.2 Properties Concerning Addition	2
1.1.3 Properties Concerning Multiplication	3
1.1.4 Complex Numbers in Algebraic Form	5
1.1.5 Powers of the Number i	7
1.1.6 Conjugate of a Complex Number	8
1.1.7 The Modulus of a Complex Number	10
1.1.8 Solving Quadratic Equations	16
1.1.9 Problems	19
1.2 Geometric Interpretation of the Algebraic Operations	23
1.2.1 Geometric Interpretation of a Complex Number	23
1.2.2 Geometric Interpretation of the Modulus	24
1.2.3 Geometric Interpretation of the Algebraic Operations	25
1.2.4 Problems	29
2 Complex Numbers in Trigonometric Form	31
2.1 Polar Representation of Complex Numbers	31
2.1.1 Polar Coordinates in the Plane	31
2.1.2 Polar Representation of a Complex Number	33
2.1.3 Operations with Complex Numbers in Polar Representation	38

- 2.1.4 Geometric Interpretation of Multiplication 41
- 2.1.5 Problems 42
- 2.2 The n th Roots of Unity 44
 - 2.2.1 Defining the n th Roots of a Complex Number 44
 - 2.2.2 The n th Roots of Unity 46
 - 2.2.3 Binomial Equations 54
 - 2.2.4 Problems 55
- 3 Complex Numbers and Geometry 57**
 - 3.1 Some Simple Geometric Notions and Properties 57
 - 3.1.1 The Distance Between Two Points 57
 - 3.1.2 Segments, Rays, and Lines 58
 - 3.1.3 Dividing a Segment into a Given Ratio 61
 - 3.1.4 Measure of an Angle 62
 - 3.1.5 Angle Between Two Lines 65
 - 3.1.6 Rotation of a Point 65
 - 3.2 Conditions for Collinearity, Orthogonality, and Concyclicity . . 70
 - 3.3 Similar Triangles 73
 - 3.4 Equilateral Triangles 76
 - 3.5 Some Analytic Geometry in the Complex Plane 82
 - 3.5.1 Equation of a Line 82
 - 3.5.2 Equation of a Line Determined by Two Points 84
 - 3.5.3 The Area of a Triangle 85
 - 3.5.4 Equation of a Line Determined by a Point
and a Direction 88
 - 3.5.5 The Foot of a Perpendicular from a Point to a Line . . . 89
 - 3.5.6 Distance from a Point to a Line 90
 - 3.6 The Circle 90
 - 3.6.1 Equation of a Circle 90
 - 3.6.2 The Power of a Point with Respect to a Circle 92
 - 3.6.3 Angle Between Two Circles 92
- 4 More on Complex Numbers and Geometry 97**
 - 4.1 The Real Product of Two Complex Numbers 97
 - 4.2 The Complex Product of Two Complex Numbers 104
 - 4.3 The Area of a Convex Polygon 109
 - 4.4 Intersecting Cevians and Some Important Points
in a Triangle 112
 - 4.5 The Nine-Point Circle of Euler 115
 - 4.6 Some Important Distances in a Triangle 120
 - 4.6.1 Fundamental Invariants of a Triangle 120
 - 4.6.2 The Distance OI 121
 - 4.6.3 The Distance ON 122
 - 4.6.4 The Distance OH 124
 - 4.6.5 Blundon’s Inequalities 125

- 4.7 Distance Between Two Points in the Plane of a Triangle 128
 - 4.7.1 Barycentric Coordinates 128
 - 4.7.2 Distance Between Two Points in Barycentric Coordinates 129
- 4.8 The Area of a Triangle in Barycentric Coordinates 132
- 4.9 Orthopolar Triangles 138
 - 4.9.1 The Simson–Wallace Line and the Pedal Triangle 138
 - 4.9.2 Necessary and Sufficient Conditions for Orthopolarity 145
- 4.10 Area of the Antipedal Triangle 149
- 4.11 Lagrange’s Theorem and Applications 153
- 4.12 Euler’s Center of an Inscribed Polygon 162
- 4.13 Some Geometric Transformations of the Complex Plane 164
 - 4.13.1 Translation 164
 - 4.13.2 Reflection in the Real Axis 165
 - 4.13.3 Reflection in a Point 165
 - 4.13.4 Rotation 166
 - 4.13.5 Isometric Transformation of the Complex Plane 167
 - 4.13.6 Morley’s Theorem 169
 - 4.13.7 Homothety 172
 - 4.13.8 Problems 174
- 5 Olympiad-Caliber Problems 175**
 - 5.1 Problems Involving Moduli and Conjugates 175
 - 5.2 Algebraic Equations and Polynomials 192
 - 5.3 From Algebraic Identities to Geometric Properties 198
 - 5.4 Solving Geometric Problems 207
 - 5.5 Solving Trigonometric Problems 231
 - 5.6 More on the n th Roots of Unity 239
 - 5.7 Problems Involving Polygons 249
 - 5.8 Complex Numbers and Combinatorics 258
 - 5.9 Miscellaneous Problems 267
- 6 Answers, Hints, and Solutions to Proposed Problems 277**
 - 6.1 Answers, Hints, and Solutions to Routine Problems 277
 - 6.1.1 Problems (p. 19) from Section 1.1: Algebraic Representation of Complex Numbers 277
 - 6.1.2 Problems (p. 29) from Section 1.2: Geometric Interpretation of the Algebraic Operations 282
 - 6.1.3 Polar Representation of Complex Numbers 283
 - 6.1.4 The n th Roots of Unity 286
 - 6.1.5 Some Geometric Transformations of the Complex Plane 291
 - 6.2 Solutions to the Olympiad-Caliber Problems 292
 - 6.2.1 Problems Involving Moduli and Conjugates 292

6.2.2	Algebraic Equations and Polynomials	305
6.2.3	From Algebraic Identities to Geometric Properties	313
6.2.4	Solving Geometric Problems	317
6.2.5	Solving Trigonometric Problems	334
6.2.6	More on the n th Roots of Unity	343
6.2.7	Problems Involving Polygons	352
6.2.8	Complex Numbers and Combinatorics	359
6.2.9	Miscellaneous Problems	365
Glossary		377
References		383
Author Index		387
Subject Index		389



<http://www.springer.com/978-0-8176-8414-3>

Complex Numbers from A to ... Z

Andreescu, T.; Andrica, D.

2014, XVII, 391 p. 83 illus., Softcover

ISBN: 978-0-8176-8414-3

A product of Birkhäuser Basel