Preface

The present volume is the second volume of the book “Singularities of Differentiable Maps” by V. I. Arnold, A. N. Varchenko and S. M. Gusein-Zade. The first volume, subtitled “Classification of critical points, caustics and wave fronts”, was published by Moscow, “Nauka”, in 1982. It will be referred to in this text simply as “Volume 1”.

Whilst the first volume contained the zoology of differentiable maps, that is it was devoted to a description of what, where and how singularities could be encountered, this volume contains the elements of the anatomy and physiology of singularities of differentiable functions. This means that the questions considered in it are about the structure of singularities and how they function.

Another distinctive feature of the present volume is that we take a hard look at questions for which it is important to work in the complex domain, where the first volume was devoted to themes for which, on the whole, it was not important which field (real or complex) we were considering. Such topics as, for example, decomposition of singularities, the connection between singularities and Lie algebras and the asymptotic behaviour of different integrals depending on parameters become clearer in the complex domain.

The book consists of three parts. In the first part we consider the topological structure of isolated critical points of holomorphic functions. We describe the fundamental topological characteristics of such critical points: vanishing cycles, distinguished bases, intersection matrices, monodromy groups, the variation operator and their interconnections and method of calculation.

The second part is devoted to the study of the asymptotic behaviour of integrals of the method of stationary phase, which is widely met with in applications. We give an account of the methods of calculating asymptotics, we discuss the connection between asymptotics and various characteristics of critical points of the phases of integrals (resolution of singularities, Newton polyhedra), we give tables of the orders of asymptotics for critical points of the phase which were classified in Volume 1 of this book (in particular for simple, unimodal and bimodal singularities).

The third part is devoted to integrals evaluated over level manifolds in a neighbourhood of the critical point of a holomorphic function. In it we shall consider integrals of holomorphic forms, given in a neighbourhood of a critical point, over cycles, lying on level hypersurfaces of the function. Integral of a holomorphic form over a cycle changes holomorphically under continuous deformation of the cycle from one level hypersurface to another. In this way there arise many-valued holomorphic functions, given on the complex line in a
neighbourhood of a critical value of the function. We show that the asymptotic behaviour of these functions (that is the asymptotic behaviour of the integrals) as the level tends to the critical one is connected with a variety of characteristics of the initial critical point of the holomorphic function.

The theory of singularities is a vast and rapidly developing area of mathematics, and we have not sought to touch on all aspects of it.

The bibliography contains works which are directly connected with the text (although not always cited in it) and also works connected with volume 1 but for some or other reason not contained in its bibliography.

References in the text to volume 1 refer to the above-mentioned book “Singularities of Differentiable Maps”.

The authors offer their thanks to the participants in the seminar on singularity theory at Moscow State University, in particular A. M. Gabrielov, A. B. Givental, A. G. Kushnirenko, D. B. Fuks, A. G. Khovanski and S. V. Chmutov. The authors also wish to thank V. S. Varchenko and T. V. Ogorodnikova for rendering inestimable help in preparing the manuscript for publication.

The authors.
Singularity of Differentiable Maps, Volume 2
Monodromy and Asymptotics of Integrals
Arnold, E.; Gusein-Zade, S.M.; Varchenko, A.N.
2012, X, 492 p. 83 illus., Softcover
A product of Birkhäuser Basel