Preface

The aim of this book is to present in an elementary manner the basic notions related with differentiable manifolds and some of their applications, especially in physics. The book is aimed at advanced undergraduate and graduate students in physics and mathematics, assuming a working knowledge of calculus in several variables, linear algebra, and differential equations. For the last chapter, which deals with Hamiltonian mechanics, it is useful to have some previous knowledge of analytical mechanics. Most of the applications of the formalism considered here are related to differential equations, differential geometry, and Hamiltonian mechanics, which may serve as an introduction to specialized treatises on these subjects.

One of the aims of this book is to emphasize the connections among the areas of mathematics and physics where the formalism of differentiable manifolds is applied. The themes treated in the book are somewhat standard, but the examples developed here go beyond the elementary ones, trying to show how the formalism works in actual calculations. Some results not previously presented in book form are also included, most of them related to the Hamiltonian formalism of classical mechanics. Whenever possible, coordinate-free definitions or calculations are presented; however, when it is convenient or necessary, computations using bases or coordinates are given, not underestimating their importance.

Throughout the work there is a collection of exercises, of various degrees of difficulty, which form an essential part of the book. It is advisable that the reader attempt to solve them and to fill in the details of the computations presented in the book.

The basic formalism is presented in Chaps. 1 and 3 (differentiable manifolds, differentiable mappings, tangent vectors, vector fields, and differential forms), after which the reader, if interested in applications to differential geometry and general relativity, can continue with Chaps. 5 and 6 (even though in the definitions of a Killing vector field and of the divergence of a vector field given in Chap. 6, the definition of the Lie derivative, presented in Chap. 2, is required). Chapter 7 deals with Lie groups and makes use of concepts and results presented in Chap. 2 (one-parameter groups and Lie derivatives). Chapters 2 and 4 are related with differential equations and can be read in an independent form, after Chaps. 1 and 3. Finally,
for Chap. 8, which deals with Hamiltonian mechanics, the material of Chaps. 1, 2, and 3, is necessary and, for some sections, Chaps. 6 and 7 are also required.

Some of the subjects not treated here are the integration of differential forms, cohomology theory, fiber bundles, complex manifolds, manifolds with boundary, and infinite-dimensional manifolds.

This book has been gradually developed starting from a first version in Spanish (with the title Notas sobre variedades diferenciables) written around 1981, at the Centro de Investigación y de Estudios Avanzados del IPN, in Mexico, D.F. The previous versions of the book have been used by the author and some colleagues in courses addressed to advanced undergraduate and graduate students in physics and mathematics.

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