Preface

The primary audience for this book, as I see it, is teachers of mathematics. The book may also be of interest to mathematicians desiring a historical viewpoint on a number of the subject's basic topics. And it may prove useful to those teaching or studying the subject's history.

The book comprises five parts. The first three (A–C) contain ten historical essays on important topics: number theory, calculus/analysis, and proof, respectively. (The choice of topics is dictated by my interests and is based on articles I have published over the past twenty-five years.) Part D deals with four historically oriented courses, and Part E provides biographies of five mathematicians who played major roles in the historical events related in Parts A–D.

Each of the first three parts – on number theory, calculus/analysis, and proof – begins with a survey of the respective subject (Chaps. 1, 4, and 7), which is followed in more depth by specialized themes. In number theory these themes deal with Fermat as the founder of modern number theory (Chap. 2) and with Fermat's Last Theorem from Fermat to Wiles (Chap. 3). In calculus/analysis, the special topics describe various aspects of the history of the function concept, which was intimately related to developments in calculus/analysis (Chaps. 5 and 6). The themes on proof discuss paradoxes (Chap. 8) and the principle of continuity (Chap. 9), and offer a historical perspective on a very interesting debate about proof initiated in a 1993 article by Jaffe and Quinn (Chap. 10).

The four chapters in Part D (Chaps. 11–14) describe courses showing how a teacher can benefit from the historical point of view. More specifically, each of Chaps. 11–14 describes a mathematics course inspired by history. Chapters 11 and 12 are about numbers as a source of ideas in teaching. Chapters 13 and 14 deal, respectively, with great quotations and with famous problems. Moreover, Chaps. 4 and 6 (on analysis and on functions) contain explicit suggestions for teachers, while such suggestions are implicit throughout the book.

Mathematics was discovered/invented by mathematicians. In each of the first 14 chapters the creators of the relevant mathematics are mentioned prominently, but because of space constraints are given shorter shrift than they deserve. I have therefore found it useful to set aside a chapter that will give a much fuller account of five mathematicians who have played important roles in the developments that I am recounting in the book. They are Dedekind, Euler, Gauss, Hilbert, and Weierstrass (Chap. 15). I hope these mini-biographies will prove to be instructive and inspiring. (My choice is limited to five by space considerations, but other mathematicians could justifiably have been picked.)

There is considerable repetition among the various chapters. This should make possible independent reading of each chapter. The book has many references, placed at the end of each of its fifteen chapters (in the case of Chap. 15, at the end of each of the five biographies). The references are mainly to secondary sources. These are, as a rule, easier to comprehend than primary sources, and more readily accessible. (Many of the secondary sources contain references to primary sources, which are often in German or French.)

I had two main goals in writing this book:

(a) To arouse mathematics teachers' interest in the history of mathematics.
(b) To encourage mathematics teachers with at least some knowledge of the history of mathematics to offer courses with a strong historical component.

Let me explain why I view these as important goals.

I come to the history of mathematics from the perspective of a mathematician rather than of a historian of mathematics. The two perspectives are, in general, not the same. My longstanding interest in the history of mathematics stems largely from trying to improve my teaching of mathematics.

Early in my teaching career I became dissatisfied with the exclusive focus on the formal theorem-proof mode of instruction. I admired the elegance of the logical structure of our subject, but over time I did not find it sufficient to sustain my enthusiasm in the classroom, perhaps because most of my students did not sustain theirs.

In due course I found that the history of mathematics helped boost my enthusiasm for teaching by providing me with perspective, insight, and motivation – surely important ingredients in the making of a good teacher. For example, when I taught calculus I was able to understand where the derivative came from, and how it evolved into the form we see in today's textbooks; and when I taught abstract algebra, I was able to understand how and why the concepts of ring and ideal came into being, and the source of Lagrange's theorem about the order of subgroups of finite groups.

Such examples could be multiplied endlessly. They supplied insight and added a new dimension to my appreciation of mathematics. I came to realize that while it is important to have technical knowledge of mathematical concepts, results, and theories, it is also important to know where they came from and why they were studied. The following quotation from the preface to C. H. Edwards' *The Historical Development of the Calculus* is apt:

> Although the study of the history of mathematics has an intrinsic appeal of its own, its chief *raison d'être* is surely the illumination of mathematics itself. For example, the gradual unfolding of the integral concept – from the volume computations of Archimedes to the intuitive integrals of Newton and Leibniz and finally the definitions of Cauchy, Riemann,

and Lebesgue – cannot fail to promote a more mature appreciation of modern theories of integration.

I hope to achieve my first goal – to arouse mathematics teachers' interest in the history of mathematics – by focusing in this book on two important areas, number theory and analysis, and on the fundamental notion of proof, a perennially hot topic of discussion among mathematics teachers (Chaps. 1–10). I trust that the five biographies will also capture the reader's interest (Chap. 15).

I have found *historical digressions* to be a useful device in teaching mathematics courses. For example, when introducing infinite sets I will give a brief history, starting with Zeno and culminating with Cantor, of how and why they came to be studied; when discussing Pythagorean triples in a first course in number theory, I will comment on Fermat's note in the margin of Diophantus' *Arithmetica* about the "marvelous proof" he had of what came to be known as Fermat's Last Theorem; and when appropriate I will briefly recount interesting stories of mathematicians (Archimedes and Galois come to mind). Such historical departures from standard teaching practice should convince students that mathematics is a human endeavor, that its history is interesting, and that it can give them some insight into the grandeur of the subject.

I have taught upper-level undergraduate mathematics courses with a strong historical orientation, dealing broadly with "mathematical culture," and a graduate course in the history of mathematics – a required course in an In-Service Master's Program for high school teachers of mathematics. (I was fortunate that my colleagues recognized – not without a "battle" – that these courses could be given in a *department of mathematics*, and that they formed a desirable component in the education of budding mathematicians.) Chapters 11–14, on numbers, great quotations, and famous problems, describe courses of the above types. Suitable material from Chaps. 1–10 can be used in these courses when appropriate. For example, the theme "Algebraic numbers and diophantine equations" in Sect. 11.3.4 will benefit from material in Chap. 3; the theme "Changing standards of rigor in the evolution of mathematics," Sect. 14.2.3, can usefully draw upon Chaps. 7 and 10; and Sect. 14.4, (e) and (g), will find Chap. 2 useful.

But a question presents itself: Why should we teach such courses in a mathematics department (or for that matter, why make historical digressions)? The answer depends on how we view the education of mathematics students. These courses (or digressions) may not make students into better researchers or theorem provers, but they *can* help make them "mathematically civilized."

The last phrase is the title of a note by Professor O. Shisha in the *Notices of the American Mathematical Society* (vol. 30, 1983, p. 603). In it he briefly discusses what it means for students to be mathematically civilized (or cultured). Among other desiderata, such students will have "good mathematical taste and judgment," and will know "how to express mathematical ideas, orally and in writing, correctly, rigorously, and clearly." We can encourage mathematical culture, according to Professor Shisha, by (among other things) "constantly pointing out in our courses the historical development of the subjects, their goals and relations with other subjects in and outside mathematics," and by "requiring students to take courses in the history of mathematics."

The implementation of the second important goal of this book – to encourage teachers to offer mathematics courses with a strong historical component (or courses in history with a strong mathematics component) – should result in mathematically cultured students. Such students might be able to discuss, for example, whether there are revolutions in mathematics (what is such a revolution anyway?), and what to make of Cantor's dictum that "the essence of mathematics lies in its freedom."

The following quotation, from an editorial about mathematics teaching by the then editors of *The Mathematical Intelligencer*, B. Chandler and H. Edwards, is a fitting conclusion to these comments (vol. 1, 1978, p. 125):

> Do let us try to teach the general public more of the sort of mathematics that they can use in everyday life, but let us not allow them to think – and certainly let us not slip into thinking – that this is an essential quality of mathematics.
> There is a great cultural tradition to be preserved and enhanced. Each new generation must learn the tradition anew. Let us take care not to educate a generation that will be deaf to the melodies that are the substance of our great mathematical culture.

I want to express heartfelt gratitude to my friend and colleague Hardy Grant for his kindness, support, and assistance over the past 40 years, and, in particular, for his help with this work. Of course all remaining errors (of omission or commission) are solely mine; I would be grateful if they were brought to my attention. Finally, I want to thank Tom Grasso, Katherine Ghezzi, and Jessica Belanger of Birkhäuser for their outstanding cooperation in seeing this book to completion.

http://www.springer.com/978-0-8176-8267-5

Excursions in the History of Mathematics
Kleiner, I.
2012, XXI, 347 p. 36 illus., Hardcover
ISBN: 978-0-8176-8267-5
A product of Birkhäuser Basel