Preface

This book deals with two subjects. The first subject is the geometric theory of compact Riemann surfaces of genus greater than one, the second subject is the Laplace operator and its relationship with the geometry of compact Riemann surfaces.

The book grew out of the idea, a long time ago, to publish a Habilitationsschrift, a thesis, in which I studied Bers' pants decomposition theorem and its applications to the spectrum of a compact Riemann surface. A basic tool in the thesis was cutting and pasting in connection with the trigonometry of hyperbolic geodesic polygons. As this approach to the geometry of a compact Riemann surface did not exist in book form, I took this book as an occasion to carry out the geometry in detail, and so it grew by several chapters. Also, while I was writing things up there was much progress in the field, and some of the new results were too challenging to be left out of the book. For instance, Sunada's construction of isospectral manifolds was fascinating, and I got hooked on constructing examples for quite a while. So time went on and the book kept growing. Fortunately, the interest in existence proofs also kept growing. The editor, for instance, was interested, and so was my family. And so the book finally assumed its present form. Many of the proofs given here are new, and there are also results which appear for the first time in print.

Introductory remarks and some history about the individual subjects are given at the beginning of each chapter. I shall therefore use this place to add a few global remarks.

The book has two parts. The first part consists of Chapters 1 through 6 and is an introduction to the geometry of compact Riemann surfaces based on hyperbolic geometry and on cutting and pasting. This part is in textbook form at about graduate level. The prerequisites are kept to a minimum, but I assume that the reader has a background either in differential geometry or in complex Riemann surface theory. Consequently, the standard introductory material which belongs to the intersection of these fields is not treated here. In particular, the fundamental group, the universal covering and the topological classification of compact surfaces are assumed to be known. The theorems about isotopies of curves on surfaces, on the other hand, are less standard. Since they are basic for Teichmüller theory, they are treated in the Appendix.

Chapter 1 deals with the general properties of surfaces which are obtainable by pasting together geodesic polygons from the hyperbolic plane. Chapter 2 is an account of hyperbolic trigonometry, the basic computational tool in this book. This chapter also contains an account of two less familiar models of the hyperbolic plane, the hyperboloid model and the quaternion model. The reader may skip this chapter, though, as only the formulae will be needed later on.

Chapters 3 and 6 describe the construction of compact Riemann surfaces based on the pasting of geodesic hexagons and lead to the Fenchel-Nielsen model of Teichmüller space. The chapters may be read in this order. Chapter 6 is organized in such a way that it may also be used as a starting point for further reading in Teichmüller theory. Chapters 4 and 5 contain the basic qualitative geometric results about Riemann surfaces: the collar theorem and Bers' theorem on length controlled pants decompositions. In these chapters we also briefly consider surfaces of variable curvature.

The second part of the book starts with a fairly self-contained introduction to the spectrum of the Laplace operator based on the heat kernel. This approach is particularly suitable in the case of a Riemann surface because the heat kernel of the hyperbolic plane is known explicitly. After a brief look at isoperimetric techniques and the famous small eigenvalues in Chapter 8 we devote the rest of the book to the question of how far and to what extent the geometry of a compact Riemann surface is reflected in the spectrum of the Laplacian.

Many years ago Huber [2] proved that two compact Riemann surfaces have the same sequence of eigenvalues of the Laplace operator if and only if they have the same sequence of lengths of the closed geodesics. This theorem does not only show that the eigenvalues contain a great deal of geometric information, it also indicates that spectral problems may be approached by geometric methods such as those developed in the first part of the book. This is important because the computation of the individual eigenfunctions and eigenvalues is a very difficult matter and practically unsolved, whereas the computation of the closed geodesics can be carried out explicitly and we may focus our attention right away on the global problems. These so-called inverse spectral problems have been studied quite successfully in recent years (not only in the case of Riemann surfaces, of course), and we are now in a position to present a number of global results within a common framework. This will be carried out in Chapters 10 through 14. Huber's theorem and related results will be proved in Chapter 9 where we shall use trace formula techniques.

During the course of the years I was writing this book up I profited from innumerable discussions with friends and colleagues to whom I should like to express my warmest thanks. I am particularly indebted to Philippe Anker, Colette Anné, Pierre Bérard, Gérard Besson, Leesa Brieger, Isaac Chavel, Bruno Colbois, Gilles Courtois, Jozef Dodziuk, Patrick Eberlein, Edgar Feldman, Burton Randol, Paul Schmutz and Klaus-Dieter Semmler for their advice and encouragement, for teaching me special subjects, for reading drafts and lecturing on various chapters, and, last but not least, for their efforts to make this text look more English. My thanks also go to Françoise Achermann for typesetting the first version of the book.

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