Preface

Practitioners and statisticians are often faced with incomplete or censored data. In life testing, censored samples are present whenever the experimenter does not observe the failure times of all units placed on the life test. This may happen intentionally and unintentionally and may be caused, e.g., by time constraints on the test duration like in Type-I censoring, by requirements on the minimum number of observed failures, or by the structure of a technical system. Naturally, the probabilistic structure of the resulting incomplete data depends heavily on the censoring mechanism and so suitable inferential procedures become necessary.

Progressive censoring can be described as a censoring method where units under test are removed from the life test at some prefixed or random inspection times. It allows for both failure and time censoring. Many modifications of the standard model have been developed, but the basic idea can be easily described by progressive Type-II censoring which can also be considered as the most popular model. Under this scheme of censoring, from a total of n units placed simultaneously on a life test, only m are completely observed until failure. Then, given a censoring plan $\mathscr{R} = (R_1, \ldots, R_m)$:

- At the time $x_{1:m:n}$ of the first failure, R_1 of the $n-1$ surviving units are randomly withdrawn (or censored) from the life-testing experiment.
- At the time $x_{2:m:n}$ of the next failure, R_2 of the $n-2-R_1$ surviving units are censored, and so on.
- Finally, at the time $x_{m:m:n}$ of the mth failure, all the remaining $R_m = n-m-R_1-\cdots-R_{m-1}$ surviving units are censored.

Note that censoring takes place here progressively in m stages. This scenario is illustrated in Fig. 1 which may be one of the most reproduced figures in the literature on progressive censoring. Clearly, this scheme includes as special cases the complete sample situation and the conventional Type-II right censoring scenario. The ordered failure times $X^{\mathscr{R}}_{1:m:n} \leq \cdots \leq X^{\mathscr{R}}_{m:m:n}$ arising from such a progressively

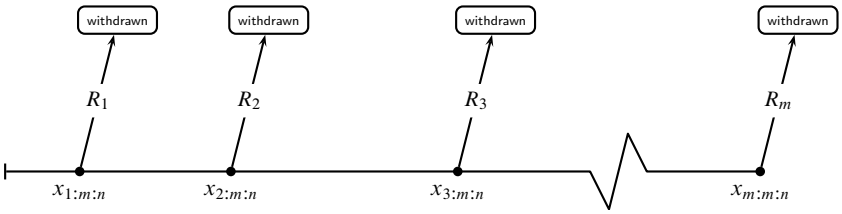

Figure 1 Illustration of progressive Type-II censoring.

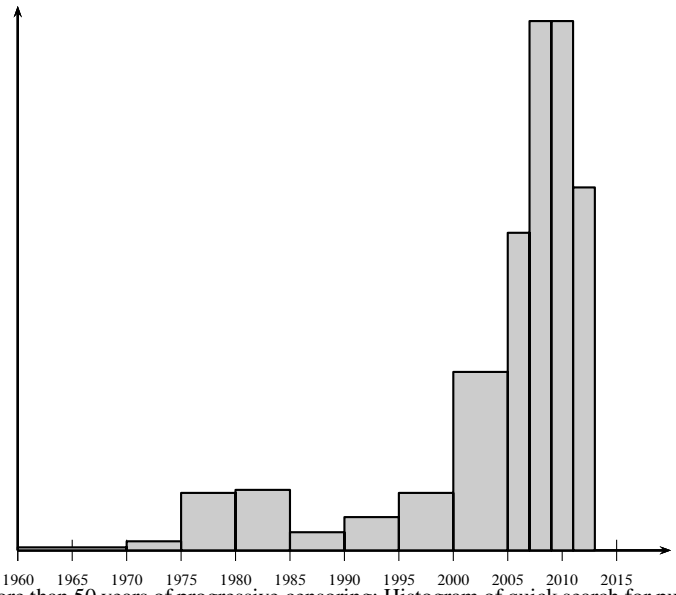

Figure 2 More than 50 years of progressive censoring: Histogram of quick search for publications (zbMATH) from 1961–2013.

Type-II right censored sample are called progressively Type-II censored order statistics. These are natural generalizations of the usual order statistics that have been studied quite extensively during the past century.

Progressive censoring has been termed a *relatively unexplored idea* in the monograph of Balakrishnan and Aggarwala [86] that appeared in 2000. Based on the outcome of a quick search in the zbMATH[1] database using the keyword `progressive* censor*`, we constructed a histogram on the number of published papers presented in Fig. 2.

It readily reveals that this statement was indeed true in the year 2000. But, the histogram also shows that the number of publications since this time has grown very fast and that it is still growing with about 200 papers in the last five years.

[1] http://zbmath.org.

Moreover, the topics addressed have by now become quite diverse. They range from distribution theory and various approaches in inference to modifications of the model, etc. All these new developments pertaining to progressive censoring, since the publication of the previous book by Balakrishnan and Aggarwala [86], have been carefully and systematically analyzed in the present book. Thus, it provides an up-to-date account on the state of the art on progressive censoring!

As mentioned above, the research on progressive censoring has grown fast in the recent past and great progress has been made with regard to distribution theory as well as in developing inferential procedures. In this book, we review the relevant literature and present a comprehensive, detailed, and unified account of the material. Due to the burgeoning literature on progressive censoring and many different developments that have taken place, we have presented here a detailed coverage of the following key topics which also reflect the structure of the book:

- **Distribution Theory and Models**
  After introducing the basic notion and models of progressive censoring, we present a comprehensive treatment of distributional properties of progressively censored order statistics. Even though the major part is devoted to progressive Type-II censoring, we give details on various other models like progressive Type-I censoring, progressive hybrid censoring, adaptive progressive censoring, and progressive censoring for nonidentical distributions and dependent variates. The material not only includes general results on joint, marginal, and conditional distributions and the dependence structure of the failure times, but also focuses on life distributions that are most important in applications (e.g., exponential and Weibull distributions). Further topics are moments, recurrence relations, characterizations, stochastic ordering, extreme value theory, simulation, and information measures like Fisher information and Shannon entropy.
- **Inference**
  The inferential topics cover linear, likelihood, and Bayesian inference in various models of progressive censoring. We discuss point and interval estimation for many life distributions as well as prediction problems. The discussion is completed by nonparametric inferential approaches and statistical tests including goodness-of-fit and precedence-type tests.
- **Applications in Survival Analysis and Reliability**
  Finally, applications in survival analysis and reliability are provided. The presentation ranges from acceptance sampling, accelerated life testing including step-stress testing, stress-strength models, and competing risks to optimal experimental design. These ideas also provide testimony to the usefulness and efficiency of progressive Type-II censoring as compared to conventional Type-II censoring.

The book provides an elaborate discussion on progressive censoring, with a special emphasis on Type-II right censoring. Even though we provide proofs of the results in most cases, it was not possible to include each detail especially when the derivations become quite technical. Further, we illustrate the methods and

procedures by several plots and diagrams just to reveal the censoring mechanism and the differences between the models. All the inferential results are illustrated with several numerical examples and many tables are also provided. In this regard, it needs to be mentioned here that many of the tables of best linear unbiased estimators and optimal progressive censoring schemes that are in the book of Balakrishnan and Aggarwala [86] will still continue to be useful for practitioners. For illustrative purposes and the support of future research, we have included a great number of progressively censored data sets which have been analyzed in the literature and used to illustrate the inferential methods. Many generalizations to some other related censoring schemes like generalized order statistics and sequential order statistics and their applications are also highlighted. An extensive up-to-date bibliography on progressive censoring has also been included which reflects the current state of research. All these aspects of this book will make it a valuable resource for researchers and graduate students interested in the area of life testing and reliability and also serve as an important reference guide for reliability practitioners.

We have written this book with the sincere hope that more practitioners will recognize the versatility of progressive censoring and be tempted to employ in their work this type of censoring/sampling scheme and the methodologies based on it. We also hope that the mathematical ideas and results presented in this book will motivate the aspiring researchers (among the statistical and the engineering communities) to explore further into the theoretical aspects of progressive censoring.

The book has been written in a self-contained manner and, therefore, will be quite suitable either as a text for a graduate topic course, a text for a directed-reading course, or as a handbook on progressive censoring. Though a one-year mathematical statistics course at the undergraduate level will provide an adequate background to go over the introductory chapters of this book, a basic exposition to order statistics (such as the one based on the book

A First Course in Order Statistics

by Arnold, Balakrishnan, and Nagaraja [58]) will make the journey through this book a lot more pleasant! In order to show different roadmaps through the book, we have added two flow charts in Figs. 3 and 4. They illustrate several possibilities to go through the material. However, you may leave the path at any crossing, just to explore! Some advanced topics included in the book require a deeper knowledge of mathematics and statistics and may be mainly of interest to researchers and advanced practitioners. In this direction, the book serves as a comprehensive compendium on progressive censoring providing the background for research and applications of progressive censoring.

We express our sincere thanks to Allen Mann, Mitch Moulton, and Kristin Purdy (all of Birkhäuser, Boston) for their enthusiasm and keen interest in this project from the very beginning. Our thanks also go to Dharmaraj Raja (Project Manager at SPi Technologies India Pvt. Ltd) for helping us with the final production of the volume. Our final appreciation goes to the Natural Sciences and Engineering Research Council of Canada for providing research grants which certainly facilitated our many meetings during the course of this project, thus enabling the work to progress

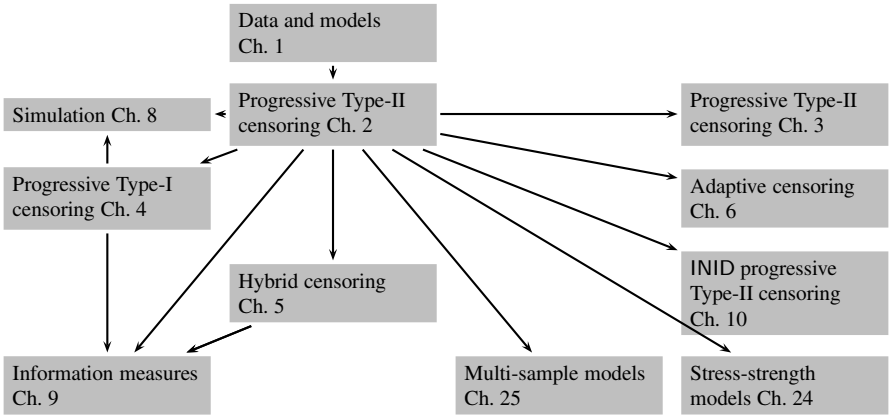

Figure 3 Journey through the book: the probability and model path.

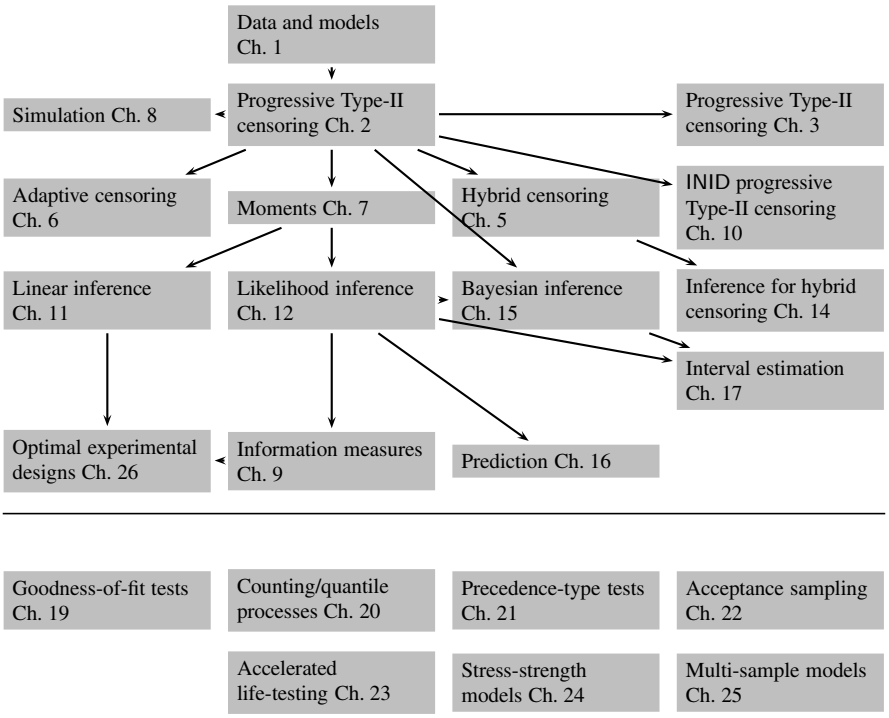

Figure 4 Journey through the book: The Type-II path. The chapters in the lower part of the chart may be chosen as a continuation.

smoothly. This book has been a true labor of love for us and it is our sincere hope and wish that it will serve as a valuable guide for researchers in the years to come and stimulate much more research activity in this interesting and useful area of research!

Section and Equation Numbering and Referencing

Throughout this book, chapters, sections, and subsections are labeled consecutively by Arabic numbers. Theorems, definitions, remarks, examples, etc., are jointly labeled as chapter no.section no.X, where X is restarted by a new section (for instance, Theorem 1.2.3). Equations are referenced by (chapter no. X) where the counter X is restarted by a new chapter (for example, (3.5) refers to the fifth numbered equation in Chap. 3).

Further, theorems, definitions, remarks, examples, etc., are set in sans serif font. The end of a proof is marked by □.

References are organized in the bibliography in alphabetical order by the first author and consecutively numbered. They are referred to in an author–number style like Cramer and Balakrishnan [292]. For three and more authors, the reference has the form Balakrishnan et al. [129].

Hamilton, ON, Canada — N. Balakrishnan
Aachen, Germany — Erhard Cramer
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