CHAPTER 3
SHARING MATHEMATICAL PERSPECTIVES

As I pointed out in the last chapter Saussure saw language as having the dual function of articulating meaning (process) and communication (product). However, Saussure and also his modern day advocates in post-structuralism see both process and product as contingent on each other. As educators we are often concerned not so much with the learning itself as with giving an account of what learning has taken place (cf. Garfinkel, 1967, pp. 24-31, who discusses the difficulties of moving between what is said and what is being spoken about). We need to decide what has been learnt by a student and how this has been demonstrated through tangible product, whether this be the application of a method, a reproduction of a famous result, or some verbal explanation of work completed, etc. Learning by the student, however, continuously evolves, oscillating between understanding and explanation; that is, between an on-going learning process and statements generated within this process, which become as if frozen in time. Further, this learning encompasses concerns beyond the frame which maybe anticipated by the teacher.

Questions were raised in earlier chapters concerned with the problematic relationship between mathematical ideas and the symbols which represent them. Such ideas, it has been suggested, cannot be transferred “ready-made” but, rather, are susceptible to interpretive modification as they move between people and through time. Hermeneutical views of mathematical learning emphasise the individuality and time dependency in our understanding of specific mathematical ideas. In assessing mathematical work we are thus faced with a task of finding an adequate way of locating mathematical knowledge - with seeing how it is held between the perspectives of teacher and student. Further, if we see mathematical work as linguistic activity, our view of developing understanding is dependent on the way in which we see language functioning in relation to reality and also on the way in which we see mathematics relating to its symbolic and physical embodiments. This chapter commences by briefly reviewing how the range of views of language offered by the various hermeneutical schools provide a framework for understanding mathematical learning and how they condition what mathematics is. This is followed by an examination of the task teachers and students face in sharing mathematical perspectives. The chapter concludes with an example of a lesson
featuring students capturing their mathematical understanding in words, diagrams and pictures.

DISCOURSE OR REALITY?

In his book, “Post-Modernism, or the Cultural Logic of Late Capitalism”, Jameson (1971, pp. 6-10) makes an interesting distinction between Modernism and Post-Modernism. He associates hermeneutical depth interpretation with the former and the textuality of post-structuralism with the latter. He illustrates this by contrasting two paintings; “A Pair of Boots” by van Gogh and “Diamond Dust Shoes” by Warhol. He suggests that van Gogh’s painting of a pair of peasants boots gives rise to the possibility of various interpretations. He offers the magnificence of the bucolic landscapes we might normally associate with the paintings of van Gogh or alternatively, the stark peasant lifestyle suggested by such clothing. Either view can be developed as a fairly full account of what the painting might be seen as evoking. Warhol’s effort, however, a dark, sparse, shadowy affair that may have been produced with the help of an X-Ray machine seems to defy any such generation of stories. It seems to be all in the surface - it begins and ends with the painting.

With this as our analogy in mind how might we distinguish between Modernist and Post-Modernist presentation of mathematics? Interpretation of mathematics might be seen as a relatively new idea, unless you are talking about statistics or mechanics. As suggested in previous chapters there needs to be an emphasis on the activity of mathematics before interpretation seems tenable. It is only when we consider people choosing to use some piece of mathematics or other that alternatives present themselves. This choosing however, has only recently been reintroduced into the vocabulary of school mathematics. The emphasis has generally been on mathematics where the choices have already been made. The students have typically been delivered to the already formed ideas and told to work with them.

In another reflection on post-modernism, Zizek (1989, p. 96) explores the Coca Cola advert which declares “This is it”. What is “it”? he asks. He suggests that “it” is nothing other than America itself and the associated glossy lifestyle of which Coke is supposedly part. I suggest that “mathematics” has pulled off a similar coup d’état. Mathematics is ordered, it is logical, obeys strict methods, is
fully decidable... or so they say! After carrying out mathematics for years we have looked back on it and claimed certain features as being “it”. These features however, seem completely devoid of the humans and their struggle that brought them into existence. Mathematics which has been derived from the activity that has given rise to it, is, it might be claimed, all in the surface.

Another feature which Jameson sees as distinguishing Modernism from Post-Modernism is that styles of the former have become icons of the latter - for example, a process of iteration becomes a button on a calculator. Things shrink as the field they are in expands into ever greater complexity. Interpretation comes firmly into play as we are forced to choose between an ever increasing number of things. The activity of choosing at all levels forces a permanent oscillation between interpretable mathematical activity and making statements as if free from the situation which gave rise to them. It may be that this manifestation of the hermeneutic circle asserts the dynamics which prevent mathematics seen as a discipline standing still long enough to be defined in either way.

This echoes a distinction drawn earlier between the hermeneutics of Gadamer and Habermas. In defining the scope of language we oscillate between seeing it as something of which we are part to seeing it as something we can operate on. I suggested Gadamer uses the former as his home base while Habermas uses the latter. Different breeds of hermeneuticians position themselves at varying points around this spectrum.

Post-structuralists analyse texts as individual performances of language (parole) but without any detailed investigation of any external reality to which they may refer. Their focus is exclusively on the usage of language rather than on any externally defined meaning of language. Meanwhile, Gadamer prefers to emphasise how text resonates with the reader’s experience. Language is seen as being in a dialectical relationship with reality, albeit a reality conditioned by language. Habermas prefers to assume a critical distance from language, seeking to understand how it functions in relation to its creators’ intentions. A conservative view would neutralise language to the more limited function of labelling reality as in the work of Russell (1914, pp. 63-97) or the early Wittgenstein (1961). Nevertheless in all of these various positions we are concerned with how experience gets mapped into language and vice versa. If we take mathematics as a language we similarly move between seeing it as a dimension of human activity and as something as if free of human intervention - between seeing
mathematics as discourse and seeing it as transcending human experience.

LOCATING MATHEMATICAL KNOWLEDGE

In assessing mathematics we seem at first to be caught between on the one hand, working with a style of mathematics where we assert a field of symbols as if devoid of humans and, on the other, speaking of mathematics as a depth interpretation of a certain style of human activity. This however, is not a satisfactory dichotomy, if only because we never have a choice of one over-arching symbolic framework. Mathematics can assume a multitude of linguistic styles, which can sometimes meet and intersect, but which very often conflict, or get confused. The choice is perhaps more accurately between on the one hand, depth interpretation of activity and on the other, composing fragments from alternative discourses. Both of these have an implicit interpretive dimension since choice is central in each.

If we emphasise mathematics as generative discourse we downplay the supposed permanence of its attributes. In the absence of hard mathematical knowledge which can be transported around intact we are faced with a difficulty in assessing the results of mathematical activity. If mathematics is seen as being interpretable it becomes more malleable and susceptible to variation according to who is presenting it. This causes problems for assessment of mathematical achievement. It might be suggested that the supposed transferability of mathematical topics influences the prominence they are given in the school curriculum. That is, those areas of mathematics more easily describable in clearly defined linguistic categories are more robust since they are more easily accounted for. Gattegno (1988, pp. 118-119), for example, argues that school education in general is mainly verbal and that many areas of mathematics sit uneasily in such a curriculum. As an example, he sees this as having led to a widespread deficiency in geometrical intuition - an area which needs to be taught yet does not lend itself to easy description. In Gattegno's view, school geometry is generally algebraic in nature and is mainly about categorising geometrical phenomena into discrete notions, a partitioning which frustrates intuition. Geometrical understanding is not fully classifiable in language and as soon as it becomes framed in language it is reduced into an algebraic style of thinking. Such an
emphasis allows it to be mechanised, made repeatable, so that it is more manageable in a school setting. Geometrical intuition is harder to account for since any attempt to share it with others requires translation into algebra with the cost to “geometrical” experience that entails. Intuition thus becomes a “spin-off” of teaching rather than something easily targeted in didactic presentations, or described in curricula.

We are then faced with a question of how much of any mathematical experience can be held in the language which describes it. We are also concerned with how we might witness others attempting to capture their experience in language. Clearly, whatever view you take, mathematical expressions themselves do not mean the same to all people; individuals see expressions in the context of their own experience, cultural perspective and current intentions. Their intended meaning depends both on the way in which the individual perceives their task and on their familiarity with such expressions. Nevertheless, there remains an issue of how far we see such activity gravitating around “correct” or culturally specific meanings. In the teaching of mathematics it is the norm to assume that the teacher’s task has something to do with drawing the student to a particular point of view. The teacher needs to find ways of enabling his students to share a perspective. Insofar as developing understanding is seen hermeneutically, however, learning is not necessarily about the reproduction of the teacher’s knowledge in the mind of the student, but rather can be seen as a transforming of both positions. Knowledge is not a fully constituted object being confronted by a fully constituted student, rather, both change through a time-dependent process. Learning is not just about adding to knowledge, rather knowledge, or at least our state of knowing, can be transformed in many ways; one subtracts from it as well as adds to it, forgetting as well as remembering, one reorganises so that known “things” get new meanings - and knowing is not just about things.

In his early days I sometimes tried pointing out things to my baby son Elliot, but, it seemed, he just looked at my pointing hand. For pupils in classrooms there is a frequent conflict between attending to the teacher’s understanding and attending to the object of that understanding. Should the child pay attention to the pointing hand or to the thing being pointed at? For the traditionalist anchored by some sort of positivistic reality, the object itself arbitrates. However, the further we move away from this towards views of language that see it constructing reality we are faced with more complex decisions as to the supposed location of knowledge.