Index

A priori, a posteriori probability, 123
Absorbing state, 271
Absorption probability, 301
Absorption time, 256
Allocation models, 54
Almost surely, 244
American put, 324
Aperiodic class, 299
Arbitrage
  meaning of, 362
  opportunity, portfolio, 362
Arbitrage-free market, 363
Area, 20, 41
Arithmetical density, 38
Artin, 175
Asset
  see “financial instrument”, 323
Asset return
  see “return”, 325
Asset return distribution, 340
  continuous compounding, 341
  logarithmic scale, 341
  with fat tails, 341
Asset risk, see risk
Asymptotically equal, 218
Axioms for probability, 24
Banach’s match problem, 72
Bayes’ theorem, 123
Bernoulli’s formula, 38
Bernoulli, J., 235
Bernoullian random variable, 93, 175, 187
Bertrand’s paradox, 100
Binomial coefficient, 51
  generalized, 135
  properties, 60, 197
  properties, 57
Binomial distribution, 93
Birth-and-death process, 295
Birthday problem, 65
Black–Scholes formula, 361
Bond, 324
  maturity date, 324
  par value, 324
  zero-coupon, 324
Boole’s inequality, 31
Borel, 99
Borel field, 103
Borel’s theorem, 240
Branching process, 305
Brownian motion, 259
Buffon’s needle problem, 160
Call option, 353
Capital asset pricing model, 339
Card shuffling, 314
Cardano’s paradox, 172
Cauchy distribution, 341, 346
Cauchy functional equation, 160
Cauchy–Schwarz inequality, 174
Central limit theorem, 229
Certificate of deposit, 324
Chapman–Kolmogorov equations, 266
Characteristic function, 190
  see stable distribution, 344
Chebyshev’s inequality, 236, 243, 358
Chi-square distribution, 244
Chinese dice game, 72
Class of states, 271
Class property, 278
Coin-tossing scheme, 36
Communicating states, 271
Complement, 3
Conditional expectation, 128
  filtration, 371
  tower property, 370
Conditional probability, 115
  basic formulas, 115–122
Contingent claim, 353
Contingent claim (see also “option”,
  “financial derivative”, 353
Convergence of distributions, 229
Convolution, 186, 197
Coordinate variable, 75
Correlation, 175
Countable additivity, 32
Countable set, 23
Coupon collecting problem, 164
Covariance, 175
Cramér, 231
Credibility of testimony, 157
D’Alembert’s argument, 27, 53
De Méré’s dice problem, 143
De Moivre–Laplace theorem, 223
De Morgan’s laws, 7
Density function, 94
Derivative security, 353
Dice patterns, 72
Difference, 8
Difference equations, 253
Discount bond, see “bond,
  zero-coupon”
Discount rate, 357
Discrete, 95
Disjoint, 10
Distribution function, 84, 95, 105
  stable, 342–344
Diversification, see portfolio
diversification
  misfortunes with lack of, 340
Dividend, see “stock dividend”
Doob, 320
Doubling the bet, 195
doubly stochastic matrix, 295
Duration of play, 288
Efficient frontier, 335, 336
Ehrenfest model, 269, 297, 312
Einstein, 127
Elementary probabilities, 85
Empty set, 2
Enron, 340
Equally likely, 21, 26, 34
Equity-type securities, 323
Ergodic theorem, 241
Errors in measurements, 172
European option price, 361
Event, 26, 34
Exchangeable events, 138
Expectation, 86, 114, 161
  addition theorem, 162, 166
  approximation of, 97
  expression by tail probabilities,
    193, 194
  multiplication theorem, 170
  of function of random variable, 88,
    96
Expected return time, 288
Exponential distribution, 101
  memoryless property, 119, 160
Factorial, 50
Favorable to, 146
Feller, 72, 112, 241
Fermat-Pascal correspondence, 29,
  143
Financial derivative, 324, 355
  equity-type, 324
  see also derivative security, 355
Financial instrument, 323
  equity-, debt-type, 323
Finite additivity, 31
First entrance time, 273
decomposition formula, 276
Fourier transform, 190
Frequency, 21, 241
Fundamental rule (of counting), 45
Gambler’s ruin problem, 253, 257
Gamma distribution, 196
Gauss–Laplace distribution, 224
Generating function, 183
  as expectation, 189
  multiplication theorem, 187, 190
  of binomial, 187
  of geometric, 188
  of negative binomial, 188
Genetical models, 150, 304, 313
Genotype, 150
Geometrical distribution, 91
Geometrical probability problems,
  98, 99
Gross return, 326
Index

Hardy–Weinberg theorem, 152
Hereditary problem, 153
Holding time, 207
Homogeneous Markov chain, see “Markov chain”
Homogeneity, 210
in space, 268
in time, 210, 263
Homogeneous chaos, 217
Identically distributed, 230
Independent events, 36, 140
Independent random variables, 139, 141
Indicator, 13, 168
Infinitely often, 258, 280
Initial distribution, 264
Insider trading, 369
Integer-valued random variable, 88
Intensity of flow, 207
Interarrival time, 167, see also “waiting time”
Intersection, 4
Joint density function, 105
Joint distribution function, 107
Joint probability distribution, 105
Joint probability formula, 121
Keynes, 118, 126, 133, 358
and short-term investors, 339
Khintchine, 236
Kolmogorov, 143
Lévy, 231, 259
Laplace, 123, see also under “De Moivre” and “Gauss”
law of succession, 127
Laplace transform, 190
Last exit time, 275
decomposition formula, 276
Law of large numbers, 235
J. Bernoulli’s, 235
strong, 240
Law of small numbers, 204
Leading to, 271
Limited liability, 325
Loan
interest, 324
principal, 324
Lognormal distribution, 346, 347
Long position, 332
Lottery problem, 163
Marginal density, 107
Marginal distribution, 105
Markov, 236, 262
Markov chain, 263
examples, 267–271
nonhomogeneous, 263, 270
of higher order, 318
positive-, null-recurrent, 295
recurrent-, nonrecurrent, 284
reverse, 318
two-state, 293
Markov property, 263
strong, 282
Markowitz, 331
Martingale, 319
discounted stock price process as, 360, 365
Matching problems, 66, 168, 176
Mathematical expectation, see “expectation”
Maximum and minimum, 145
Mean-variance optimization
definition, 331–332
effect of riskless security, 337–339
equilibrium, 339
risky assets example, 332–335
risky assets generalization, 335–337
Measurable, 25, 113
Median, 112
Moments, 172
Money market instrument, 324, 328
Montmort, 198
Multinomial coefficient, 52
Multinomial distribution, 178, 179
Multinomial theorem, 177
Multiperiod model, 326
dynamic replication, 367
European option price, 369
horizon, 326
self-financing strategy, 367
successive returns, 326
Multiperiod portfolio strategy, 369
Mutual fund, 339
Negative binomial distribution, 188
Neyman-Pearson theory, 157
Non-Markovian process, 271
Nonhomogeneous Markov chain, 263, 270
Nonmeasurable, 40
Nonrecurrent, 278, see also under “recurrent”
Normal distribution, 224
convergence theorem, 230
moment-generating function, moments, 226
positive, 244
Normal family, 227
Null-recurrent, 295
Numéraire invariance principle, 366–367
Occupancy problems, 192, see also “allocation models”
Occupation time, 288
One-period model, 326
European option price, 363
Option, 353
1-period model, 361–367
American, 353
as insurance, 354, 359
Black–Scholes formula, 361
buyer/holder of, 356
call, 353
European, 353
exercise, strike price, 353
exotic, 354
expiration/maturity date, 353
Fundamental pricing theorems, 371
multiperiod model, 367–372
payoff, 355
premium, 361
price, 356
pricing probability, 365
put, 353
standard, 354
underlying security, 353
writer/seller of, 356
Optional time, 281
Ordered $k$-tuples, 46
Pólya, 133, 231, 270
Pairwise independence, 147
Pareto, 349
Pareto distribution, 346, 349
Partition problems, 55
Pascal’s letters to Fermat, 29, 143
Pascal’s triangle, 58
Permutation formulas, 50–52
Persistent, see recurrent
Poincaré’s formula, 168
Poisson, 133
Poisson distribution, 199, 211
models for, 204–206
properties, 214–216
Poisson limit law, 202
Poisson process, 212
distribution of jumps, 217
finer properties, 244
Poisson’s theorem on sequential sampling, 133
Poker hands, 71
Portfolio allocation, 329
diversification, 330
multiperiod, 369
return, 329
risk, 329
weight, 329
Portfolio frontier, 335
Position long, 332
short, 332
Positive-recurrent, 295
Pricing probability, 365
equivalent, 365
Probability (classical definition), 24
Probability distribution, 85
Probability measure, 24
construction of, 34
Probability of absorption, 301
Probability of extinction, 307
Problem (for other listings see under key words)
of liars, 157
of points, 28, 197
of rencontre, 168
of sex, 119
Put option, 353
Put-call parity, 373
Index

Quality control, 62
Queuing process, 315–316

Random mating, 150
Random variable, 77, 113
  continuous, 95
  countable vs. density case, 96
  discrete, 95
  function of, 78
  range of, 84
  with density, 95
Random vector, 75, 105
Random walk, 250
  free, 267
  generalized, 268–269
  in higher dimensions, 270, 285
  on a circle, 294
  recurrence of, 257, 284–285
  with barriers, 268
Randomized sampling, 216
Rate of return
  see “return”, 326
Recurrent, 278, 280
  Markov chain, 284
  random walk, 258
Renewal process, 313
Repeated trials, 35
Replicating strategy, 363
Return, 325, 326
  annualization, 326, 327
  compounding effect, 327
  continuous compounding, 341
  distribution, 340
  distribution with fat tails, 341
  gross, 326
Riemann sums, 97
Risk, 328
  definition, 328
  lack of, 328
Risk-return tradeoff, 331
Risk-neutral probability, see pricing probability
Riskless security, 328
Sample function, 212
Sample point, space, 2
Sampling (with or without replacement),
  vs. allocating, 55
with ordering, 48
  without ordering, 50–52
  Sequential sampling, 50–52
  Sharpe, 339
  Sharpe ratio, 387
  Short position, 332
  Significance level, 234
  Simpson’s paradox, 148
  Size of set, 2
  St. Petersburg paradox, 111, 321
  Stable distribution, 342–344
    characteristic function, 344
    Lévy’s characterization, 345
    Stable distribution type, 343
    Stable law, see stable distribution
    Standard deviation, 172
    State of the economic world, 325
    State space, 262
    Stationary distribution, 292
    Stationary process, 139, 153, 292
    Stationary transition probabilities, 263
    Steady state, 287
    equation for, 290
    Stirling’s formula, 219, 247
    Stochastic independence, see independent events, random variables
    Stochastic matrix, 266
    Stochastic process, 129, 213
      stock price evolution as, 360
      Stochastically closed, 299
      Stock dividend, 323
      Stopping time, 281
      Strong law of large numbers, 240
      Strong Markov property, 282
      Submartingale, 357
        discounted stock price process as, 360
        expectation under, 358
      Summable, 161
      Supermartingale, 357
        discounted stock price process as, 360
        expectation under, 358
        in example of greed, 358
      Symmetric difference, 9
      Symmetric distribution, 345
Taboo probabilities, 275, 317
Tauberian theorem, 288
Time parameter, 129
Tips for counting problems, 61
Total probability formula, 122
Transient, see “nonrecurrent”
Transition matrix, 266
Transition probability, 262, 266
limit theorems for, 288, 299
Tulipmania, 356

Uniform distribution, 89, 99
Union, 4

Variance, 172
addition theorem, 173

Waiting time, 91, 101, 188
Wald’s equation, 91
Wiener process, 259

Zero-or-one law, 309
Elementary Probability Theory
With Stochastic Processes and an Introduction to Mathematical Finance
Chung, K.L.; AitSahalia, F.
2003, XIV, 404 p., Hardcover
ISBN: 978-0-387-95578-0