Preface

This book aims to construct a general framework for the analysis of a large family of random fields, also known as multiparameter processes. The need for such a development was pointed out in Doob (1990, p. 47). Referring to the theory of one-parameter stochastic processes, Doob writes:[1]

> Our definition of a stochastic process is historically conditioned and has obvious defects. In the first place there is no mathematical reason for restricting T to be a set of real numbers, and in fact interesting work has already been done in other cases. (Of course, the interpretation of t as time must then be dropped.) In the second place there is no mathematical reason for restricting the value assumed by the x_t's to be numbers.

There are a number of compelling reasons for studying random fields, one of which is that, if and when possible, multiparameter processes are a natural extension of existing one-parameter processes. More exciting still are the various interactions between the theory of multiparameter processes and other disciplines, including probability itself. For example, in this book the reader will learn of various connections to real and functional analysis, a modicum of group theory, and analytic number theory. The multiparameter processes of this book also arise in applied contexts such as mathematical statistics (Pyke 1985), statistical mechanics (Kuroda and Manaka 1998), and brain data imaging (Cao and Worsley 1999).

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[1]He is referring to a stochastic process of the form $(x_t;\ t \in T)$.

My writing philosophy has been to strike a balance between developing a reasonably general theory, while presenting applications and explicit calculations. This approach should set up the stage for further analysis and exploration of the subject, and make for a more lively approach.

This book is in two parts. Part I is about the discrete-time theory. It also contains results that allow for the transition from discrete-time processes to continuous-time processes. In particular, it develops abstract random variables, parts of the theory of Gaussian processes, and weak convergence for continuous stochastic processes. Part II contains the general theory of continuous-time processes. Special attention is paid to processes with continuous trajectories, but some discontinuous processes will also be studied. In this part I will also discuss subjects such as potential theory for several Markov processes, the Brownian sheet, and some Gaussian processes. Parts I and II are influenced by the fundamental works of Doob, Cairoli, and Walsh.

My goal has been to keep this book as self-contained as possible, in order to make it available to advanced graduate students in probability and analysis. To this I add that a more complete experience can only be gained through solving many of the problems that are scattered throughout the body of the text. At times, these in-text exercises ask the student to check some technical detail. At other times, the student is encouraged to apply a recently introduced idea in a different context. More challenging exercises are offered at the end of each chapter.

Many of the multiparameter results of this book do not seem to exist elsewhere in a pedagogic manner. There are also a number of new theorems that appear here for the first time. When introducing a better-known subject (e.g., martingales or Markov chains), I have strived to construct the most informative proofs, rather than the shortest.

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This book is dedicated to the memory of Victor Gushin, and to the recent arrival of Adrian V. Kh. Gushin.

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