Preface

This book is designed as a text for a first course on functional analysis for advanced undergraduates or for beginning graduate students. It can be used in the undergraduate curriculum for an honors seminar, or for a "capstone" course. It can also be used for self-study or independent study. The course prerequisites are few, but a certain degree of mathematical sophistication is required.

A reader must have had the equivalent of a first real analysis course, as might be taught using [25] or [109], and a first linear algebra course. Knowledge of the Lebesgue integral is not a prerequisite. Throughout the book we use elementary facts about the complex numbers; these are gathered in Appendix A. In one specific place (Section 5.3) we require a few properties of analytic functions. These are usually taught in the first half of an undergraduate complex analysis course. Because we want this book to be accessible to students who have not taken a course on complex function theory, a complete description of the needed results is given. However, we do not prove these results.

My primary goal was to write a book for students that would introduce them to the beautiful field of functional analysis. I wanted to write a succinct book that gets to interesting results in a minimal amount of time. I also wanted it to have the following features:

- It can be read by students who have had *only* first courses in linear algebra and real analysis, and it ties together material from these two courses. In particular, it can be used to introduce material to undergraduates normally first seen in graduate courses.
- Reading the book *does not* require familiarity with Lebesgue integration.

- It contains information about the historical development of the material and biographical information of key developers of the theories.
- It contains many exercises, of varying difficulty.
- It includes ideas for individual student projects and presentations.

What really makes this book different from many other excellent books on the subject are:

- The choice of topics.
- The level of the target audience.
- The ideas offered for student projects (as outlined in Chapter 6).
- The inclusion of biographical and historical information.

How to use this book

The organization of the book offers flexibility. I like to have my students present material in class. The material that they present ranges in difficulty from "short" exercises, to proofs of standard theorems, to introductions to subjects that lie outside the scope of the main body of such a course.

- Chapters 1 through 5 serve as the core of the course. The first two chapters introduce metric spaces, normed spaces, and inner product spaces and their topology. The third chapter is on Lebesgue integration, motivated by probability theory. Aside from the material on probability, the Lebesgue theory offered here is only what is deemed necessary for its use in functional analysis. Fourier analysis in Hilbert space is the subject of the fourth chapter, which draws connections between the first two chapters and the third. The final chapter of this main body of the text introduces the reader to bounded linear operators acting on Banach spaces, Banach algebras, and spectral theory. It is my opinion that every course should end with material that truly challenges the students and leaves them *asking* more questions than perhaps can be answered. The last three sections of Chapter 5, as well as several sections of Chapter 6, are written with this view in mind. I realize the time constraints placed on such a course. In an effort to abbreviate the course, some material of Chapter 3 can be safely omitted. A good course can include only an outline of Chapter 3, and enough proofs and examples to give a flavor for measure theory.
- Chapter 6 consists of seven independent sections. Each time that I have taught this course, I have had the students select topics that they will study individually and teach to the rest of the class. These sections serve as resources for these projects. Each section discusses a topic that is nonstandard in some way. For example, one section gives a proof of the classical Weierstrass approximation theorem and then gives a fairly recent (1980s) proof of Marshall Stone's generalization of Weierstrass's theorem. While there are several proofs of the Stone–Weierstrass theorem, this is the first that does not depend on the classical result. In another section of this chapter, two arguments are given that no function can be continuous at each rational number and discontinuous at each

irrational number. One is the usual Baire category argument; the other is a less well known and more elementary argument due to Volterra. Another section discusses the role of Hilbert spaces in quantum mechanics, with a focus on Heisenberg's uncertainty principle.

- Appendices A and B are very short. They contain material that most students will know before they arrive in the course. However, occasionally, a student appears who has never worked with complex numbers, seen De Morgan's Laws, etc. I find it convenient to have this material in the book. I usually spend the first day or two on this material.
- The biographies are very popular with my students. I assign each student one of these (or other) "key players" in the development of linear analysis. Then, at a subject-appropriate time in the course, I have that one student give (orally) a short biography in class. They really enjoy this aspect of the course, and some end up reading (completely due to their own enthusiasm) a book like Constance Reid's *Hilbert* [104].

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This book has evolved from notes for a course that I have taught three times over several years at Macalester. The students in those courses have motivated me with their enthusiasm for the project. I thank them for their apparent eagerness to read my sometimes not-so-polished notes, and for much help in sorting out good exercise sets. It is fair to say that if it weren't for my students, I would not have written this book.

I welcome your comments, suggestions for improvements, and indications of errors.

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