Preface to the Second Edition

Undergraduate courses in mathematics are commonly of two types. On the one hand there are courses in subjects, such as linear algebra or real analysis, with which it is considered that every student of mathematics should be acquainted. On the other hand there are courses given by lecturers in their own areas of specialization, which are intended to serve as a preparation for research. There are, I believe, several reasons why students need more than this.

First, although the vast extent of mathematics today makes it impossible for any individual to have a deep knowledge of more than a small part, it is important to have some understanding and appreciation of the work of others. Indeed the sometimes surprising interrelationships and analogies between different branches of mathematics are both the basis for many of its applications and the stimulus for further development. Secondly, different branches of mathematics appeal in different ways and require different talents. It is unlikely that all students at one university will have the same interests and aptitudes as their lecturers. Rather, they will only discover what their own interests and aptitudes are by being exposed to a broader range. Thirdly, many students of mathematics will become, not professional mathematicians, but scientists, engineers or schoolteachers. It is useful for them to have a clear understanding of the nature and extent of mathematics, and it is in the interests of mathematicians that there should be a body of people in the community who have this understanding.

The present book attempts to provide such an understanding of the nature and extent of mathematics. The connecting theme is the theory of numbers, at first sight one of the most abstruse and irrelevant branches of mathematics. Yet by exploring its many connections with other branches, we may obtain a broad picture. The topics chosen are not trivial and demand some effort on the part of the reader. As Euclid already said, there is no royal road. In general I have concentrated attention on those hard-won results which illuminate a wide area. If I am accused of picking the eyes out of some subjects, I have no defence except to say "But what beautiful eyes!"

The book is divided into two parts. Part A, which deals with elementary number theory, should be accessible to a first-year undergraduate. To provide a foundation for subsequent work, Chapter I contains the definitions and basic properties of various mathematical structures. However, the reader may simply skim through this chapter and refer back to it later as required. Chapter V, on Hadamard's determinant problem, shows that elementary number theory may have unexpected applications.

Part B, which is more advanced, is intended to provide an undergraduate with some idea of the scope of mathematics today. The chapters in this part are largely independent, except that Chapter X depends on Chapter IX and Chapter XIII on Chapter XII.

Although much of the content of the book is common to any introductory work on number theory, I wish to draw attention to the discussion here of quadratic fields and elliptic curves. These are quite special cases of algebraic number fields and algebraic curves, and it may be asked why one should restrict attention to these special cases when the general cases are now well understood and may even be developed in parallel. My answers are as follows. First, to treat the general cases in full rigour requires a commitment of time which many will be unable to afford. Secondly, these special cases are those most commonly encountered and more constructive methods are available for them than for the general cases. There is yet another reason. Sometimes in mathematics a generalization is so simple and far-reaching that the special case is more fully understood as an instance of the generalization. For the topics mentioned, however, the generalization is more complex and is, in my view, more fully understood as a development from the special case.

At the end of each chapter of the book I have added a list of selected references, which will enable readers to travel further in their own chosen directions. Since the literature is voluminous, any such selection must be somewhat arbitrary, but I hope that mine may be found interesting and useful.

The computer revolution has made possible calculations on a scale and with a speed undreamt of a century ago. One consequence has been a considerable increase in 'experimental mathematics'—the search for patterns. This book, on the other hand, is devoted to 'theoretical mathematics'—the explanation of patterns. I do not wish to conceal the fact that the former usually precedes the latter. Nor do I wish to conceal the fact that some of the results here have been proved by the greatest minds of the past only after years of labour, and that their proofs have later been improved and simplified by many other mathematicians. Once obtained, however, a good proof organizes and provides understanding for a mass of computational data. Often it also suggests further developments.

The present book may indeed be viewed as a 'treasury of proofs'. We concentrate attention on this aspect of mathematics, not only because it is a distinctive feature of the subject, but also because we consider its exposition is better suited to a book than to a blackboard or a computer screen. In keeping with this approach, the proofs themselves have been chosen with some care and I hope that a few may be of interest even to those who are no longer students. Proofs which depend on general principles have been given preference over proofs which offer no particular insight.

Mathematics is a part of civilization and an achievement in which human beings may take some pride. It is not the possession of any one national, political or religious group and any attempt to make it so is ultimately destructive. At the present time there are strong pressures to make academic studies more 'relevant'. At the same time, however, staff at some universities are assessed by 'citation counts' and people are paid for giving lectures on chaos, for example, that are demonstrably rubbish.

The theory of numbers provides ample evidence that topics pursued for their own intrinsic interest can later find significant applications. I do not contend that curiosity has been the only driving force. More mundane motives, such as ambition or the necessity of earning a living, have also played a role. It is also true that mathematics pursued for the sake of applications has been of benefit to subjects such as number theory; there is a two-way trade. However, it shows a dangerous ignorance of history and of human nature to promote utility at the expense of spirit.

This book has its origin in a course of lectures which I gave at the Victoria University of Wellington, New Zealand, in 1975. The demands of my own research have hitherto prevented me from completing it, although I have continued to collect material. If it succeeds at all in conveying some idea of the power and beauty of mathematics, the labour of writing it will have been well worthwhile.

As with a previous book, I have to thank Helge Tverberg, who has read most of the manuscript and made many useful suggestions.

The first Phalanger Press edition of this book appeared in 2002. A revised edition, which was reissued by Springer in 2006, contained a number of changes. I removed an error in the statement and proof of Proposition II.12 and filled a gap in the proof of Proposition III.12. The statements of the Weil conjectures in Chapter IX and of a result of Heath-Brown in Chapter X were modified, following comments by J.-P. Serre. I also corrected a few misprints, made many small expository changes and expanded the index.

In the present edition I have made some more expository changes and have added a few references at the end of some chapters to take account of recent developments. For more detailed information the Internet has the advantage over a book. The reader is referred to the American Mathematical Society's MathSciNet (www.ams.org/mathscinet) and to The Number Theory Web maintained by Keith Matthews (www.maths.uq.edu.au/~krm/).

I am grateful to Springer for undertaking the commercial publication of my book and hope you will be also. Many of those who have contributed to the production of this new softcover edition are unknown to me, but among those who are I wish to thank especially Alicia de los Reyes and my sons Nicholas and Philip.

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