# Contents

Preface ........................................................................................................... ix

List of Examples Analyzed ................................................................. xvii

Part I Full Factorial Designs

1 Introduction to Full Factorial Designs with Two-Level Factors .......................... 3
   1.1 Advantages of Full Factorial Designs ........................................ 3
   1.2 Standard Regression Models for Factorial Designs with Two-Level Factors ........................ 9
   1.3 Least Squares Estimation of a Regression Model ...................... 12
   1.4 Presenting a Fitted Model Graphically .................................. 19
   1.5 Four Steps for Planning a Successful Experiment ................. 23

2 Analysis of Full Factorial Experiments .............................................. 27
   2.1 Analysis Strategy Overview ................................................. 27
   2.2 Analysis of Numerical Responses with Replication ............... 29
   2.3 The Inclusion of Centerpoint Replicates ............................... 31
   2.4 Analysis of Numerical Responses Without Replication ........ 35
   2.5 Normal Plot of Effects and Other Analysis Tools .................. 45
   2.6 Diagnostics for a Fitted Model ........................................... 48
   2.7 Transformations of the Response ......................................... 53
   2.8 Analysis of Counts, Variances, and Other Statistics ............... 57
   2.9 Unequal Replication and Unequal Variance ......................... 67
   2.10 The Impact of Missing Treatment Combinations ................. 70

3 Common Randomization Restrictions ................................................. 75
   3.1 Sources of Variation and a Design’s Unit Structure ............... 75
   3.2 Treatment*Unit Interactions .............................................. 78
3.3 Blocking: Partitioning a Factorial into Smaller Experiments . . . 79
3.4 Analyzing Randomized Block Factorial Designs ................. 86
3.5 Split-Unit Designs ...................................... 97
3.6 Multiway Blocking....................................... 104

4 More Full Factorial Design Examples .............................. 115
4.1 Example 4.1: Replicated 2^3 With Subsampling Within Runs . . 115
4.2 Example 4.2: 2^9 Factorial for Peptide Research ............... 122
4.3 Example 4.3: 2^5 with Centerpoint Runs for Ceramic Strength . 134

Part II Fractional Factorial Designs

5 Fractional Factorial Designs: The Basics .......................... 145
5.1 Initial Fractional Factorial Example .......................... 145
5.2 Introduction to Regular Fractional Factorial Designs ......... 150
5.3 Basic Analysis for Regular Fractional Factorial Designs ...... 161

6 Fractional Factorial Designs for Estimating Main Effects ....... 173
6.1 Analysis of Regular Resolution III Fractional Factorial Designs 174
6.2 Some Theory Regarding Resolution III Designs ............... 190
6.3 Nonregular Orthogonal Designs of Strength 2 .......... 194
6.4 Optimal Nonorthogonal Saturated Main Effect Designs ...... 226
6.5 Supersaturated Designs .................................. 231
6.6 Conclusions ............................................. 244

7 Designs for Estimating Main Effects and Some Two-Factor Interactions .................................................. 245
7.1 Five Examples Analyzed .................................. 246
7.2 Regular Resolution IV Designs ................................ 267
7.3 Strength-3 Orthogonal Arrays ................................ 277
7.4 Nonorthogonal Resolution IV Designs ..................... 279
7.5 Summary Regarding Choice of a Design .................... 282

8 Resolution V Fractional Factorial Designs ........................ 283
8.1 Regular Resolution V 2^{k-f} Fractional Factorial Designs .... 283
8.2 Strength-4 Orthogonal Arrays .............................. 285
8.3 Three-Quarter Fraction of Regular Resolution V Designs .... 288
8.4 Smaller Nonorthogonal Resolution V Designs ............... 293
8.5 Recommendations Regarding Design Choice ............... 298
8.6 Analysis of Resolution V Experiments ...................... 299
9 Augmenting Fractional Factorial Designs .................. 317
  9.1 Follow-up Experimentation Choices .................... 317
  9.2 Confirmation Runs ..................................... 319
  9.3 Steepest Ascent Search ................................ 321
  9.4 Foldover After a Resolution III Fraction .............. 328
  9.5 Foldover and Semifolding After a Resolution IV Fraction ... 332
  9.6 Optimal Design Augmentation .......................... 338
  9.7 Adding and Dropping Factors ........................... 342

10 Fractional Factorial Designs with Randomization
  Restrictions .................................................... 343
  10.1 Randomized Block Designs for Fractional Factorials .... 343
  10.2 Split-Unit Designs for Fractional Factorials ............ 350
  10.3 Analysis of Fractional Factorials with Randomization
       Restrictions ............................................. 354
  10.4 Sequences of Fractional Factorial Designs .............. 371

11 More Fractional Factorial Design Examples ................ 375
  11.1 A Mirror-Image Foldover with Unexpected Results ....... 375
  11.2 Steepest Ascent with Constraints ........................ 382
  11.3 A Group Screening Experiment .......................... 385
  11.4 Nonorthogonal Blocking for a Fractional Factorial .... 389

Part III Additional Topics

12 Response Surface Methods and Second-Order Designs ..... 397
  12.1 The Response Surface Methodology Strategy ............. 397
  12.2 Central Composite Designs .............................. 399
  12.3 Other Composite Designs ................................. 403
  12.4 Box–Behnken Designs .................................... 407
  12.5 Analysis/Interpretation of the Fitted Second-Order Model ... 409

13 Special Topics Regarding the Design ..................... 415
  13.1 Power and the Choice of Sample Size .................... 415
  13.2 Choice of Factor Levels ................................ 420
  13.3 Tips for Studying Variation ............................ 422
  13.4 Accommodating Factors with More Levels ............... 424
  13.5 Special Requirements for Run Order and Treatment
       Combinations ............................................. 428

14 Special Topics Regarding the Analysis ................... 437
  14.1 Minimal Replication and Lenth’s Method ............... 437
  14.2 Alternatives to Lenth $t$-Tests for Unreplicated Designs ... 440
  14.3 Analyzing the Variation in Structured Samples ......... 449
14.4 Generalized Least Squares Analysis When Variances Are Unequal .............................................. 454
14.5 Mixed-Model Analysis ................................... 457
14.6 Highly Multivariate Response Data ....................... 461
14.7 Four Analysis Blunders to Avoid ........................ 466

Part IV Appendices and Tables

A Upper Percentiles of $t$ Distributions, $t_\alpha$ .................... 471
B Upper Percentiles of $F$ Distributions, $F_\alpha$ .................... 473
C Upper Percentiles for Lenth $t$ Statistics, $c_{\alpha}^{IER}$ and $c_{\alpha}^{EER}$ .... 477
D Computing Upper Percentiles for Maximum Studentized Residual ................................................... 481
E Orthogonal Blocking for Full $2^k$ Factorial Designs .......... 483
F Column Labels of Generators for Regular Fractional Factorial Designs .............................................. 485
G Tables of Minimum Aberration Regular Fractional Factorial Designs .............................................. 487
H Minimum Aberration Blocking Schemes for Fractional Factorial Designs .............................................. 497
I Alias Matrix Derivation ............................................. 511
J Distinguishing Among Fractional Factorial Designs ........ 513
References ............................................................. 517
Abbreviations and Symbols ........................................ 539
Index ................................................................. 543
A Comprehensive Guide to Factorial Two-Level Experimentation
Mee, R.
2009, XXIII, 545 p., Hardcover