Contents

Part I Gian-Carlo Rota, the Man

1 Gianco, my Brother ............................................ 3
   Ester Gasperoni Rota
   References ........................................................ 13

2 Remembering Gian-Carlo Rota .................................. 17
   Joseph J. Kohn
   References ........................................................ 21

Part II Gian-Carlo Rota, the Mathematician

3 A Glimpse of Vector Invariant Theory. The Points of View of Weyl,
   Rota, De Concini and Procesi, and Grosshans .................... 25
   Andrea Brini
     3.1 Introduction .................................................. 25
     3.2 The Characteristic Zero Approach .............................. 27
       3.2.1 Preliminaries ............................................. 27
       3.2.2 Weyl’s Theorem (The Reduction Principle) ............... 28
       3.2.3 Absolute Invariants for the Special Linear Group $SL(d)$ 29
       3.2.4 Absolute Invariants for the Symplectic Group $Sp_{2m}$ . 29
       3.2.5 Absolute Invariants for the Special Orthogonal Group
          $SO(d)$ ......................................................... 31
       3.2.6 Absolute Invariants for the Orthogonal Group $O(d)$ .... 31
     3.3 The Straightening Formulae .................................... 32
       3.3.1 Standard Young Tableaux .................................. 32
       3.3.2 Bideterminants ......................................... 33
       3.3.3 Pfaffians ............................................... 33
       3.3.4 Gramians ............................................... 34
     3.4 The E. Pascal Theorems and the Cancellation Laws .......... 35
3.4.1 The E. Pascal Theorem and the Cancellation Law for Scalar Products ................................................. 36
3.4.2 The E. Pascal Theorem and the Cancellation Law for Symplectic Products ...................................... 37
3.4.3 The E. Pascal Theorem and the Cancellation Law for Inner Products ................................................. 38
3.5 The First and the Second Fundamental Theorems ............................................................. 39
3.5.1 The First Fundamental Theorems ......................................................................................... 39
3.5.2 The Second Fundamental Theorems ....................................................................................... 40
3.6 Grosshans’s Theorem ............................................................................................................. 40

References ......................................................................................................................................... 42

4 Partitions of a Finite Partially Ordered Set .............................................................................. 45
Pietro Codara
4.1 Introduction ............................................................................................................................. 45
4.2 Background, and Preliminary Results .................................................................................... 46
4.3 Partitions as Sets of Fibres ...................................................................................................... 50
4.4 Partitions as Partially Ordered Sets of Blocks ......................................................................... 51
4.5 Partitions Induced by Quasiorders .......................................................................................... 55
4.6 Further Work .......................................................................................................................... 58

References ......................................................................................................................................... 59

5 An Algebra of Pieces of Space — Hermann Grassmann to Gian Carlo Rota ............................................. 61
Henry Crapo
5.1 Almost Ten Years Later ............................................................................................................ 61
5.2 Synthetic Projective Geometry ................................................................................................. 62
5.3 Hermann Grassmann’s Algebra ............................................................................................... 64
5.4 Extensors and Vectors .............................................................................................................. 66
5.5 Reduced Forms .......................................................................................................................... 72
5.6 Grassmann-Cayley Algebra, Peano Spaces ............................................................................. 73
5.7 Whitney Algebra ....................................................................................................................... 75
5.8 Geometric Product .................................................................................................................. 78
5.9 Regressive Product .................................................................................................................. 80
5.10 Higher Order Syzygies ........................................................................................................... 86
5.11 Balls in Boxes ......................................................................................................................... 89

References ......................................................................................................................................... 90

6 The Eleventh and Twelveth Problems of Rota’s Fubini Lectures: from Cumulants to Free Probability Theory .......................................................... 91
Elvira Di Nardo and Domenico Senato
6.1 Prologue ...................................................................................................................................... 91
6.2 The Classical Umbral Calculus .................................................................................................. 92
6.2.1 Generating Functions ........................................................................................................... 94
6.3 Sequences of Binomial Type, Bell Umbrae and Poisson Processes ...................................... 96
6.4 Cumulants .................................................................................................................................. 99
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.4.1</td>
<td>Singleton Umbra</td>
<td>100</td>
</tr>
<tr>
<td>6.4.2</td>
<td>Cumulant Umbra</td>
<td>101</td>
</tr>
<tr>
<td>6.5</td>
<td>Applications in Statistics</td>
<td>103</td>
</tr>
<tr>
<td>6.5.1</td>
<td>U-Statistics</td>
<td>104</td>
</tr>
<tr>
<td>6.5.2</td>
<td>Moments of Sampling Distributions</td>
<td>106</td>
</tr>
<tr>
<td>6.5.3</td>
<td>Products of Statistics</td>
<td>109</td>
</tr>
<tr>
<td>6.5.4</td>
<td>$k$-statistics</td>
<td>110</td>
</tr>
<tr>
<td>6.5.5</td>
<td>Fast Symbolic Computation of $k$-statistics</td>
<td>112</td>
</tr>
<tr>
<td>6.5.6</td>
<td>Sheppard’s Corrections</td>
<td>114</td>
</tr>
<tr>
<td>6.6</td>
<td>Sheffer Sequences</td>
<td>117</td>
</tr>
<tr>
<td>6.6.1</td>
<td>Abel Polynomials</td>
<td>120</td>
</tr>
<tr>
<td>6.6.2</td>
<td>Quasi-free Cumulant Umbra</td>
<td>122</td>
</tr>
<tr>
<td>6.6.3</td>
<td>Boolean Cumulants</td>
<td>123</td>
</tr>
<tr>
<td>6.7</td>
<td>Free Cumulant Theory</td>
<td>124</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>127</td>
</tr>
<tr>
<td>7</td>
<td>Two Examples of Applied Universal Algebra</td>
<td>131</td>
</tr>
<tr>
<td>7.1</td>
<td>Prologue</td>
<td>131</td>
</tr>
<tr>
<td>7.2</td>
<td>The Bohnenblust–Spitzer Identity</td>
<td>132</td>
</tr>
<tr>
<td>7.3</td>
<td>Baxter Algebras</td>
<td>133</td>
</tr>
<tr>
<td>7.4</td>
<td>The Standard Baxter Algebra</td>
<td>134</td>
</tr>
<tr>
<td>7.5</td>
<td>A Precursor Identity</td>
<td>137</td>
</tr>
<tr>
<td>7.6</td>
<td>Abstract Random Variables</td>
<td>141</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>143</td>
</tr>
<tr>
<td>8</td>
<td>On the Euler Characteristic of Finite Distributive Lattices</td>
<td>145</td>
</tr>
<tr>
<td>8.1</td>
<td>Introduction</td>
<td>145</td>
</tr>
<tr>
<td>8.2</td>
<td>Posets and Distributive Lattices</td>
<td>146</td>
</tr>
<tr>
<td>8.3</td>
<td>Euler Characteristic</td>
<td>147</td>
</tr>
<tr>
<td>8.4</td>
<td>Uniform Distributions of the Characteristic</td>
<td>151</td>
</tr>
<tr>
<td>8.4.1</td>
<td>$\chi$-uniform Lattices</td>
<td>151</td>
</tr>
<tr>
<td>8.4.2</td>
<td>Order-preserving Euler Characteristic</td>
<td>152</td>
</tr>
<tr>
<td>8.4.3</td>
<td>Rank $\chi$-uniform Lattices</td>
<td>153</td>
</tr>
<tr>
<td>8.5</td>
<td>Pseudo-planar Distributive Lattices</td>
<td>154</td>
</tr>
<tr>
<td>8.6</td>
<td>Graphs and Hypergraphs</td>
<td>158</td>
</tr>
<tr>
<td>8.7</td>
<td>Dual Gödel Lattices</td>
<td>160</td>
</tr>
<tr>
<td>8.8</td>
<td>Tree maps</td>
<td>162</td>
</tr>
<tr>
<td></td>
<td>References</td>
<td>165</td>
</tr>
<tr>
<td>9</td>
<td>Rota, Probability, Algebra and Logic</td>
<td>167</td>
</tr>
<tr>
<td>9.1</td>
<td>MV-algebraic States and De Finetti Coherence Criterion</td>
<td>167</td>
</tr>
<tr>
<td>9.2</td>
<td>MV-algebraic and $C^*$-algebraic States After Elliott Classification</td>
<td>171</td>
</tr>
<tr>
<td>9.3</td>
<td>MV-algebraic $\sigma$-states and Carathéodory Probability Theory</td>
<td>176</td>
</tr>
</tbody>
</table>
10 A Symbolic Treatment of Abel Polynomials ................................. 183
Pasquale Petrullo
10.1 Introduction .......................................................... 183
10.2 Abel polynomials, Lagrange inversion formula and sequences of binomial type .................................................. 185
10.3 Generalized Abel polynomials ......................................... 187
10.4 Cumulants and convolutions ........................................... 191
References ................................................................. 195

Part III Gian-Carlo Rota, the Philosopher

11 Ethics in Thought. Gian-Carlo Rota and Philosophy ...................... 199
Francesca Bonicalzi
11.1 Telling/Inventing the Truth ............................................ 199
11.2 The Winding Streets, and the Straight and Wide Avenue of Precision ............................................................. 202
11.3 The Food of Philosophical Thought and the Medicine of Axiomatics ............................................................. 204
11.4 The Ethics of Knowledge and the Genesis of its Disciplines ...... 206
References ................................................................. 208

12 Indiscrete Variations on Gian-Carlo Rota’s Themes ...................... 211
Carlo Cellucci
12.1 Introduction .......................................................... 211
12.2 The Existence of Mathematical Objects ............................. 211
12.2.1 Irrelevance of the Existence of Mathematical Objects ....... 211
12.2.2 Mathematical Objects as Hypotheses ........................... 213
12.2.3 Hypotheses vs. Fictions ........................................... 215
12.2.4 Existence and Identity ............................................. 215
12.2.5 The Inexhaustibility of Mathematical Objects ................ 216
12.3 Definition in Mathematics ............................................. 217
12.3.1 Definition, Description and Analysis of Concepts .......... 217
12.3.2 Definition in Mathematics and in Philosophy ............... 218
12.3.3 The Alleged Circularity of Definitions and Theorems ...... 219
12.4 The Notion of Proof .................................................. 219
12.4.1 Proof as the Opening up of Possibilities ...................... 220
12.4.2 Axiomatic Presentation and Gödel’s Incompleteness Theorems ............................................................. 221
12.4.3 Are there Definitive Proofs? ....................................... 221
12.4.4 Proof and Evidence ............................................... 222
12.5 The Relation of Philosophy of Mathematics to Mathematics .... 224
12.5.1 The Descriptive Character of the Philosophy of Mathematics ............................................................. 224
12.5.2 The Need for a Drastic Overhaul of Logic .................... 224
12.6 Rota’s Place in the Philosophy of Mathematics ......................... 225
References ................................................................. 227

13 On the Courage Needed to Do Phenomenology. Rota and Analytic Philosophy ......................................................... 229
Albino Lanciani and Claudio Majolino
13.1 Twilight of an Idol ...................................................... 230
13.2 Three Senses of Phenomenology .................................... 234
13.3 Coda ................................................................. 240
References ................................................................. 240

14 Rota’s Philosophical Insights .............................................. 241
Massimo Mugnai
14.1 Introduction ............................................................ 241
14.2 What Kind of Reductionism? ........................................ 241
14.3 Rota’s Objectivism and Husserl’s Naturalism .................... 244
14.4 Beyond Classical Logic .............................................. 245
14.5 Concluding Remarks .................................................. 248
References ................................................................. 249

15 “A Minority View”. Gian-Carlo Rota’s Phenomenological Realism  251
Fabrizio Palombi
15.1 Considerations on the Problem of Realism ....................... 252
15.2 Phenomenological Realism .......................................... 253
15.3 Fringe Phenomena and State-of-Mind .......................... 254
15.4 Psychologistic Misunderstandings ................................. 256
15.5 Descriptions, Not Prescriptions .................................... 258
References ................................................................. 259
From Combinatorics to Philosophy
The Legacy of G.-C. Rota
Damiani, E.; D'Antona, O.; Marra, V.; Palombi, F. (Eds.)
2009, XVIII, 262 p., Hardcover
ISBN: 978-0-387-88752-4