Preface

This book aims to give a user-friendly tutorial of an interdisciplinary research topic (fronts or interfaces in random media) to senior undergraduates and beginning graduate students with basic knowledge of partial differential equations (PDE) and probability. The approach taken is semiformal, using elementary methods to introduce ideas and motivate results as much as possible, then outlining how to pursue rigorous theorems, with details to be found in the references section.

Since the topic concerns both differential equations and probability, and probability is traditionally a quite technical subject with a heavy measure-theoretic component, the book strives to develop a simplistic approach so that students can grasp the essentials of fronts and random media and their applications in a self-contained tutorial.

The book introduces three fundamental PDEs (the Burgers equation, Hamilton–Jacobi equations, and reaction–diffusion equations), analysis of their formulas and front solutions, and related stochastic processes. It builds up tools gradually, so that students are brought to the frontiers of research at a steady pace.

A moderate number of exercises are provided to consolidate the concepts and ideas. The main methods are representation formulas of solutions, Laplace methods, homogenization, ergodic theory, central limit theorems, large-deviation principles, variational principles, maximum principles, and Harnack inequalities, among others. These methods are normally covered in separate books on either differential equations or probability. It is my hope that this tutorial will help to illustrate how to combine these tools in solving concrete problems.

The three basic equations go from their constant-coefficient forms, well studied in graduate textbooks, to their full stochastic glory with space–time-dependent random coefficients. However, they are all connected to Hamilton–Jacobi equations. The reaction–diffusion equations are classified. The KPP (Kolmogorov–Petrovsky–Piskunov) fronts are discussed in detail because of their connections with the Hamiltonian dynamics in classical mechanics and their elegant analysis. The recent mathematical advance in solving a long-standing turbulent combustion problem is presented for KPP fronts. The non-KPP reaction–diffusion fronts are associated with a Hamiltonian resembling that in special relativistic mechanics. The mechanical connections of reaction–diffusion fronts and KPP solution methods in the spirit of Lagrangian and Eulerian perspectives are explored.

The scope of the book goes from exact solutions of scalar deterministic PDEs (Burgers, Hamilton–Jacobi, reaction–diffusion equations) to the asymptotic solutions of their stochastic counterparts. The reader will come to appreciate new random phenomena step by step, and learn that exact solutions are harder and harder to come by, while asymptotic ones are more accessible.

The first chapter of the book discusses fronts in homogeneous media, and the second chapter is on fronts in periodic media. These two chapters serve as an introduction to differential equations, their front solutions and representation formulas, and homogenization methods.

The last three chapters introduce stochastic equations, representation formulas and asymptotics, stochastic homogenization, variational methods, and large-deviation methods for analyzing random fronts.

The first three chapters are adaptations of the author's 2000 SIAM review article with the inclusion of new results. The remaining two chapters are based on recent results on stochastic homogenization of Hamilton–Jacobi equations and KPP fronts in random media.

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