Chapter 2
Inequality and Income Poverty

2.1 Introduction

Poverty elimination is still one of the major economic policies in many countries of the world. In order to evaluate the efficacy of an antipoverty policy, it is necessary to know how much of poverty is there and observe the changes in the level of poverty over time. Poverty elimination programs also require identification of the causal factors of poverty, for example, the subgroups of population that are most afflicted by poverty. Quantification of the extent of poverty becomes necessary to address these problems. More precisely, we need an indicator of poverty that will enable us to analyze these issues.

According to Sen (1976a), poverty measurement problem involves two distinct but not unrelated exercises: (i) identification of the poor, that is, to isolate the set of poor persons from the set of nonpoor persons and (ii) to aggregate the information available on the poor into an overall indicator of poverty. That is, we need to know who are poor and how poor are the poor? While identification can be referred to as perception of poverty, the aggregation of characteristics of the poor is known as "measurement of poverty." When income is regarded as the only attribute of well-being, identification problem is solved by specifying a "poverty line," an exogenously given level of income required to maintain a subsistence standard of living. A person is identified as poor if his income does not exceed the poverty line. Thus, the poverty line is a line of demarcation that separates the set of poor persons from the set of nonpoor persons. The aggregation exercise, loosely speaking, consists of aggregating the income shortfalls of the poor from the poverty line into an overall indicator of poverty.

The index of poverty that has been used by most countries is the headcount ratio, the proportion of population with incomes not above the poverty line. This index has been criticized by Watts (1968) and Sen (1976a) on the ground that it does not consider the income distribution of the poor. For instance, consider two income distributions with the same population size and the same number of poor. Suppose that in the former, the poor have almost no income, whereas in the latter, the incomes

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of the poor are marginally below the poverty line. Evidently, poverty in the former distribution is more acute than that in the latter. But the headcount ratio will treat the two distributions as identically poor.

Another often-used index is the income gap ratio, the relative gap between the poverty line and the average income of the poor. This index may not represent the poverty status correctly. To see this, consider again two income distributions with the same population size. Assume that the first distribution has only one poor person with zero income, while in the second, there is more than one poor person with zero income, so that the two distributions have the same average income of the poor. We can definitely argue that in this case the first distribution is less poverty stricken than the second. But the income gap ratio will regard them as equally poor.

Using an axiomatic approach, Sen (1976a) suggested a more sophisticated index of poverty that avoids the above shortcomings. His path breaking contribution has motivated many researchers to focus on the issue of poverty measurement.[1] As a consequence, the literature now contains several poverty indices. In designing new poverty indices, most of the researchers have adopted Sen's axiomatic approach and proposed new poverty axioms in addition to those of Sen.

Often from policy perspective, it may be necessary to identify the subgroups of the population that are most susceptible to poverty. Subgroup decomposable poverty indices become helpful in identifying such subgroups. According to subgroup decomposability, for any partitioning of the population with respect to a homogeneous characteristic, say, age, sex, region, and race, overall poverty is given by the population share weighted average of subgroup poverty levels (Anand, 1977; Chakravarty, 1983c; Foster et al., 1984).

Now, for a set of reasonable axioms, there may be several poverty indices. Quite often there is arbitrariness in the choice of a particular index of poverty, which in turn implies arbitrariness of the conclusions based on that index. Therefore, it will be worthwhile to reduce the degree of arbitrariness by choosing all poverty indices that satisfy a set of reasonable desiderata. Thus, instead of choosing individual poverty indices, we look for a set of postulates for poverty indices that implicitly determines a family of indices. It then becomes possible to rank two income distributions unambiguously by all members of this class. Clearly, this kind of research has grown out of existence of too many poverty indices. However, in some situations, a class of indices may not be able to compare all income distributions, that is, there may not be unanimous agreement among these indices about the ranking of some income distributions. Thus, while a single poverty index completely orders all the income distributions, the ordering generated by a class of indices is partial. For some distributions, it is not possible to conclude unambiguously whether one has more or less poverty than another by all members of the class. This notion of poverty

[1] Alternatives to and variations of the Sen index have been suggested, among others, by Takayama (1979), Thon (1979), Blackorby and Donaldson (1980b), Kakwani (1980a,b), Clark et al. (1981), Chakravarty (1983a,b,c,1997a), Foster et al. (1984), and Shorrocks (1995). This literature has been surveyed by Foster (1984), Seidl (1988), Chakravarty (1990), Ravallaion (1994a), Foster and Sen (1997), Zheng (1997), Dutta (2002), and Chakravarty and Muliere (2004). Our presentation in some sections of the chapter relies, to some extent, on Chakravarty and Muliere (2004).

ordering is known as poverty-measure ordering (*see* Atkinson, 1987, 1992; Jenkins and Lambert, 1993, 1997, 1998a,b; Shorrocks, 1998; Spencer and Fisher, 1992; Zheng, 2000b).

The definition of a poverty line is crucial both for poverty indices and poverty orderings. The determination of such an income or consumption threshold on which the definition of poverty relies has been an issue of debate for quite sometime. Often the construction of a poverty line may involve a significant degree of arbitrariness. The ranking of two income distributions by a poverty index may be different for two distinct poverty lines. It will, therefore, be useful to investigate whether it is possible to rank two income distributions unanimously by a given index for all poverty lines in some reasonable interval. This area of research on partial poverty orderings arises from uncertainty about fixation of the poverty line. This second notion of ordering of distributions by a given poverty index for a range of poverty lines is referred to as poverty-line ordering (*see* Foster and Shorrocks, 1988a,b; Foster and Jin, 1998; Zheng, 2000b).

Investigation has also been made in the literature whether poverty rankings remain unaffected when all the incomes and the poverty lines are expressed in different units of measurement. Indices satisfying this condition are called unit consistent (Zheng, 2007c).

Standards of living as well as size and composition of populations are likely to change over time. Therefore, it may become necessary to reformulate public policies like expenditure on public health, public funding of education, budget allocation for removal of poverty, resource conservation, designing the social security system etc., that are affected by change in population composition and size directly and indirectly.[2] This in turn necessitates the examination of impacts of population change on poverty (Chakravarty et al., 2006).

Duration of poverty in a society is an important aspect for understanding the experience of poverty. Increased duration of poverty can have detrimental effects on society's well-being. Therefore, it becomes necessary to understand and respond to the persistence of poverty over time.

The objective of this chapter is to present an extensive and analytical discussion on income distribution-based poverty measurement problems. The poverty axioms suggested in the literature and their desirability, alternative indices of poverty and their properties, different notions of poverty ordering, the issue of poverty measurement in the presence of population growth, and the measurement of poverty over time are examined in detail.

2.2 Axioms for an Index of Income Poverty

This section presents a discussion on alternative poverty axioms and their implications. For a population of size $n \geq 1$, a typical income distribution is given by

[2] The issue of population size in evaluating welfare has been considered, among others, by Parfit (1984), Broome (1996), and Blackorby et al. (2005).

$x = (x_1, \dots, x_n)$, where x_i is the income of person i. Assuming that all income distributions are illfare-ranked, the set of income distributions in this n-person economy is D^n and the set of all possible income distributions $D = \bigcup_{n \in N} D^n$, where N is the set of positive integers. Unless specified we will define all the axioms and the indices on the domain D^n (or D). Recall that for all illfare-ranked income distributions, all increments/reductions in incomes, and transfers between two persons will be rank preserving.

The problem of identification of the poor requires the specification of an exogenously given poverty line z, an income level necessary to maintain a subsistence standard of living. This absolutist notion of poverty contrasts with the relativist view in which the poverty line is made responsive to the income distribution. For instance, a household with less than 40% of the median income may be regarded as relativist poor [see Ravallion (1994a) and Foster and Sen (1997) for further discussion][3].

We assume that the exogenously given poverty line z is positive and takes values in some subset $[z_-, z_+]$ of the real line, where $z_- > 0$ and $z_+ < \infty$ are the minimum and maximum poverty lines. For any income distribution x, person i is said to be strongly poor if $x_i \leq z$. Person i is weakly poor if the inequality $\leq$ in $x_i \leq z$ is replaced by $<$. In the literature, the former definition is more commonly used (*see* Donaldson and Weymark, 1986; Bourguignon and Fields, 1997). Person i is called nonpoor or rich if he is not poor. Assume that using the either definition of the poor, there are q poor persons in the society. For any, $x \in D^n$, let x^p be the income distribution of the poor. Since x is illfare-ranked, $x^p = (x_1, x_2, \dots, x_q)$. For any $x \in D^n$, we denote the set of poor persons in x by $z(x)$. Thus, $z(x) = \{1, 2, .., q\}$.

For a given population size n, a poverty index P is a real valued function defined on $D^n \times [z_-, z_+]$. Thus, given any income distribution, $x \in D^n$ and a poverty line $z \in [z_-, z_+]$, $P(x,z)$ determines the extent of poverty associated with x. A poverty index will be called a relative or an absolute index according as it satisfies the scale invariance or translation invariance condition stated below.

Scale Invariance: For all $x \in D^n, z \in [z_-, z_+],\ P(x,z) = P(cx, cz)$, where $c > 0$ is any scalar such that $cz \in [z_-, z_+]$.

Translation Invariance: For all $x \in D^n, z \in [z_-, z_+],\ P(x,z) = P(x + c1^n, z + c)$, where c is any scalar such that $x + c1^n \in D^n$ and $(z + c) \in [z_-, z_+]$.

Thus, a relative poverty index is invariant under equal percentage changes in all the incomes and the poverty line, whereas an absolute poverty index remains unaltered under equal absolute changes in all the incomes and the poverty line.

The following axioms have been suggested in the literature for an arbitrary poverty index P, which may be of relative or absolute variety. Unless specified, we assume that the poverty line z is given arbitrarily.

Focus Axiom: For all $x, y \in D^n$, if $z(x) = z(y)$ and $x_i = y_i$ for all $i \in z(x)$, then $P(x,z) = P(y,z)$.

[3] Discussions on problems regarding the determination of an appropriate poverty line can be found in Atkinson (1983a), Sen (1981, 1983), Paul (1989), Ravallion (1994a), Pradhan and Ravallion (2000), and Sharma (2004). For references to the earlier literature, see Atkinson (1983a) and Chakravarty (1990).

This axiom was formally proposed by Sen (1981), but it was implicitly used in Sen (1976a). It says that the poverty index should not depend on the incomes of the nonpoor persons. However, it does not demand that the poverty index is independent of the number of the nonpoor persons. Assuming that in poverty measurement, we are concerned with the insufficiency of the incomes of the poor, this axiom seems to be sensible. Chakravarty (1983a) referred to this axiom as "Independence of the Incomes of the Rich" (*see also* Clark et al., 1981). A poverty index satisfying this axiom will be called focused.

Normalization Axiom: For any $x \in D^n$ if the set $z(x)$ is empty, then $P(x,z) = 0$.

According to this axiom, if there is no poor person in the society, the value of the poverty index is zero. This is a cardinal property of the poverty index.

The next axiom will ensure that minor inaccuracy in income data and negligible imprecision of an appropriate poverty line will not give rise to a huge jump in the poverty level.

Continuity Axiom (CON): $P(x,z)$ is jointly continuous in (x,z).

Symmetry Axiom: For all $x,y \in D^n$, if y is obtained from x by a permutation of the incomes, then $P(x,z) = P(y,z)$.

The interpretation of this axiom is similar to its inequality counterpart. It enables us to define the poverty index on the ordered distributions, as we have done.

Population Replication Invariance Axiom: For all $x \in D^n, P(x,z) = P(y,z)$, where y is the l-fold replication of x, $l \geq 2$ being any integer.

This axiom parallels the Population Principle employed in the context of inequality measurement. It was introduced into the poverty measurement literature by Chakravarty (1983a) and Thon (1983a).

Assuming that the income distribution is given, consider an increase in the poverty line. In such a case, the income gaps of the poor persons from the poverty-line increase. This in turn leads to a higher level of poverty. The following axiom of Clark et al. (1981) and Chakravarty (1983a) specifies this formally.

Increasing Poverty-Line Axiom: For a given $x \in D^n$, $P(x,z)$ is increasing in z.

Weak Monotonicity Axiom: For all $x,y \in D^n$, if $x_j = y_j$ for all $j \neq i, i \in z(x)$, and $x_i > y_i$, then $P(x,z) < P(y,z)$.

This axiom of Sen (1976a) is concerned with the effect of reducing a poor person's income. Note that here the distribution y is obtained from the distribution x by reducing the income of poor person i, under the ceteris paribus assumption. The axiom demands that this income reduction has increased poverty. A stronger version of this axiom was suggested by Donaldson and Weymark (1986). It demands decreasingness of the poverty index if the income of a poor person goes up. Thus, it includes the possibility that the beneficiary of the income increase may cross the poverty line and become rich.

Strong Monotonicity Axiom: For all $x,y \in D^n$, if $x_j = y_j$ for all $j \neq i, i \in z(x)$ and $x_i < y_i$, then $P(y,z) < P(x,z)$.

Clearly, for either definition of the poor, the strong axiom implies its weak version. It follows that for the strong definition of the poor, a focused poverty index satisfying the Strong Monotonicity Axiom will achieve its lower bound if all the incomes of the poor are at the poverty line. If a focused, continuous poverty index fulfills the Strong Monotonicity Axiom, then under the strong definition of the poor, we cannot simultaneously decrease the value of the index, as demanded by monotonicity, and keep it constant, as required by continuity, when the income of a person at the poverty-line increases. This shows that under the strong definition of the poor, there is no focused poverty index that meets the Strong Monotonicity and Continuity Axioms (Donaldson and Weymark, 1986). However, under the weak definition of the poor, continuity ensures that the two versions of the monotonicity axiom are equivalent. If we adopt the strong definition of the poor, for a focused poverty index, continuity is not consistent with the weak form of the monotonicity axioms.

The third axiom proposed by Sen (1976a) is a transfer axiom, which requires poverty to increase under a transfer of income from a poor person to anyone who has a higher income. Following Donaldson and Weymark (1986), we distinguish among four transfer axioms.

Minimal Transfer Axiom: For all $x, y \in D^n$, if y is obtained from x by a regressive transfer between two poor persons such that the recipient is not becoming rich as a result of the transfer, then $P(x,z) < P(y,z)$.

This self-explanatory axiom considers a regressive transfer between two poor persons keeping the number of poor persons unchanged. Likewise, a progressive transfer between two poor persons should reduce poverty. The next axiom also keeps the set of poor persons unchanged but in this case the regressive transfer may take place from a poor person to a rich person.

Weak Transfer Axiom: For all $x, y \in D^n$, if y is obtained from x by a regressive transfer from a poor person with no one becoming rich as a result of the transfer, then $P(x,z) < P(y,z)$.

The next axiom, which has been suggested by Sen (1976a), is also stated using a regressive transfer. But it allows the possibility that the recipient, if he is poor, may cross the poverty line.

Strong Upward Transfer Axiom: For all $x, y \in D^n$, if y is obtained from x by a regressive transfer from a poor person to someone who is richer, then $P(x,z) < P(y,z)$.

In the most general case, we may consider a poor person receiving a progressive transfer crosses the poverty line.

Strong Downward Transfer Axiom: For all $x, y \in D^n$, if x is obtained from y by a progressive transfer with at least the recipient being the poor, then $P(x,z) < P(y,z)$.

The essential idea underlying the transfer axioms is that poverty increases or decreases according as the transfer is regressive or progressive. By definition, the Minimal Transfer Axiom is the weakest among the four transfer axioms. For either definition of the poor, the Strong Downward Transfer Axiom is sufficient for the Strong Upward Transfer Axiom, which in turn implies the Weak Transfer Axiom from which the Minimal Transfer Axiom follows. If the recipient of a transfer considered under the Weak Transfer Axiom is a rich person, then for a focused poverty index, the transfer has the same effect as income reduction under the Weak Monotonicity Axiom. Therefore, for a focused poverty index, the Weak Transfer Axiom is equivalent to the Minimal Transfer and Weak Monotonicity Axioms

(Zheng, 1997). Note that the Strong Upward Transfer Axiom records an increase in poverty even if a poor recipient of the transfer crosses the poverty line. This "makes poverty measurement, in an important way, independent of the number below the poverty line" (Sen, 1981, p.186, n.1). Therefore, in later works, Sen (1981, 1982) opted for the Weak Transfer Axiom. However, if we maintain Continuity and the Weak Transfer Axioms, the focused poverty index will satisfy the Strong Upward Transfer Axiom for either definition of the poor. For the weak definition of the poor, Continuity and the Strong Upward Transfer Axiom, under Focus, imply the Strong Downward Transfer Axiom (Donaldson and Weymark, 1986). Since the Weak Transfer Axiom and Continuity are quite reasonable, the use of the Strong Upward and Downward Transfers Axioms are justifiable as well (Zheng, 1997). For the strong definition of the poor, no focused poverty index can satisfy the Strong Downward Transfer Axiom.

Kakwani (1980b) noted that the Sen (1976a) index is not more sensitive to transfers at the lower end of the income profile. He suggested three sensitivity axioms, one on monotonicity and two on transfers. They are presented below formally.

Monotonicity Sensitivity Axiom: If $x', x'' \in D^n$ is obtained from $x \in D^n$ by reducing, respectively, the incomes of the poor persons i and j by the same amount, where $x_i < x_j$, then $P(x', z) - P(x, z) > P(x'', z) - P(x, z)$.

Positional Transfer Sensitivity Axiom: For all $x \in D^n$ and for any pair of poor individuals i and j, suppose that the distribution x' (respectively x'') is obtained from x by a regressive transfer of income from the ith (respectively jth) person to the $(i+l)$th (respectively $(j+l)$th) poor person where $i < j$ and $z(x) = z(x') = z(x'')$. Then $P(x', z) - P(x, z) > P(x'', z) - P(x, z)$.

Diminishing Transfer Sensitivity Axiom: For all $x \in D^n$, if y is obtained from x by a regressive transfer of income from the poor person with income x_i to the poor person with income $x_i + \hat{c}$, then for a given $\hat{c} > 0$, the magnitude of increase in poverty $P(y, z) - P(x, z)$ is higher the lower is x_i, where $z(x) = z(y)$.

Kakwani's first axiom demands that a poverty index should be more sensitive to a reduction in the income of a poor person, the poorer the person happens to be. The second axiom is the poverty counterpart to the Positional Transfers Principle considered in Chap. 1. It says that the poorer the donor of the regressive transfer, the higher is the increase in poverty, given that the number of persons between the recipient and the donor of the transfer is fixed. The third axiom is the poverty analogue to Kolm's (1976a,b) Diminishing Transfers Principle and argues that more weight should be assigned to transfers lower down the income profile. Note that the regressive transfers considered in the later two axioms do not change the set of poor persons. We can also consider a poverty version of the Shorrocks and Foster (1987) Transfer Sensitivity Axiom proposed for inequality indices. However, since the Diminishing Transfer Sensitivity Axiom is quite intuitive and easy to understand, we will regard it as sufficient for evaluation of a poverty index.

The next two axioms are concerned with partitioning of the population into subgroups and relationship of overall poverty with subgroup poverty levels.

Subgroup Consistency Axiom: For all $n, l \in N$, $x^1, x^2 \in \Gamma^l; y^1, y^2 \in \Gamma^n$, if $P(x^1, z) = P(x^2, z)$ and $P(y^1, z) < P(y^2, z)$, then $P(x^1, y^1, z) < P(x^2, y^2, z)$.

Subgroup Decomposability Axiom: For $x^i \in \Gamma^{n_i}$, $i = 1, 2, .., J$, we have

$$P(x,z) = \sum_{i=1}^{J} \frac{n_i}{n} P(x^i, z), \tag{2.1}$$

where $x = (x^1, x^2, \ldots, x^J) \in \Gamma^n$ and $\sum_{i=1}^{J} n_i = n$.

The first of these two axioms was introduced by Foster and Shorrocks (1991). It has the same intuitive appeal as the monotonicity axioms. While the latter deals with changes in individual poverty, the former is concerned with subgroup poverty. Thus, if the poverty level in a subgroup of the population reduces, given that the poverty levels in the other subgroups remain constant, it is natural to expect that global poverty will reduce.

The second axiom is quite useful for practical purposes (*see* Anand, 1977, 1983; Chakravarty, 1983c, 1990; Foster and Shorrocks, 1991; Foster et al., 1984; Kakwani, 1980a). It says that for any division of the population into nonoverlapping subgroups with respect to characteristics like region, religion etc., national poverty becomes the weighted average of subgroup poverty levels, where the weights are the population shares of different subgroups. Note that here n_i is the population size of subgroup i and $P(x^i, z)$ is its poverty level. The contribution of subgroup i to total poverty is then given by the quantity $(n_i/n)P(x^i, z)$. This is precisely the amount by which overall poverty will reduce if poverty in the ith subgroup is eliminated. This in turn shows that the percentage contribution of subgroup i to total poverty is $[\{100n_i/(nP(x,z))\}P(x^i, z)]$. This axiom, therefore, becomes helpful in identifying the subgroups that are more affected by poverty and hence in designing antipoverty policies. Clearly, according to this notion of policy, an assessment of poverty becomes dependent on the implicit valuation of the index. However, following Sen (1985a), the nonwelfarist approach to policy applications has become quite popular. Therefore, it seems worthwhile to investigate what kind of policy would be implied by the use of a particular poverty index.

Essential to the Subgroup Decomposability Axiom is independence between poverty levels of different subgroups. Sen (1992, p. 106, n.12) questioned the appropriateness of this assumption because he thought one group's poverty level may be affected by poverty characteristics of other groups. However, because of its immense popularity and usefulness, we will regard this axiom as one of the basic requirements of poverty indices. It may be mentioned that all subgroup decomposable poverty indices are subgroup consistent and under certain assumptions, all subgroup consistent indices are increasing transformations of some subgroup decomposable indices (Foster and Shorrocks, 1991).

By repeated application of the decomposability axiom, we can write the poverty index as

$$P(x,z) = \frac{1}{n} \sum_{i=1}^{n} \zeta(x_i, z), \tag{2.2}$$

where $\zeta(x_i, z) = P(x_i, z)$ is the poverty level of person i. Therefore, $\zeta(x_i, z)$ can be referred to as the individual poverty function. Note that the functional form of the individual poverty index $\zeta(x_i, z)$ does not depend on i. It may also be worthwhile to observe that the index in (2.2) is symmetric and population replication invariant.

Kundu and Smith (1983) introduced two population monotonicity axioms, one for poverty growth and the other for nonpoverty growth.

Poverty Growth Axiom: For all $n \in N, x \in D^n$, if y is obtained from x by adding a poor person to the population, then $P(x,z) < P(y,z)$.

Nonpoverty Growth Axiom: For all $n \in N, y \in D^n$, if x is obtained from y by adding a rich person to the population, then $P(x,z) < P(y,z)$.

The first (second) of these two axioms says that poverty should increase (decrease) under migration of a poor (nonpoor) person to the society. By the formulation, the latter axiom requires a focused poverty index to be a decreasing function of the nonpoor population size. That is, a focused poverty index satisfying this postulate is independent of incomes of the rich but dependent on their population size. Kundu and Smith (1983) demonstrated that for the weak definition of the poor, there is no poverty index that satisfies these two axioms and the Strong Upward Transfer Axiom simultaneously. They argued that the source of the problem here is the transfer postulate and advocated for use of some weaker form of the postulate. But we have seen how the transfer postulate can be justified by some reasonable criteria. It may be noted that these two axioms treat a poor and a nonpoor asymmetrically. More precisely, for a focused index while we do not consider the income of the nonpoor entrant, we take into account the income of the poor migrant. Zheng (1997) demonstrated that the Poverty Growth Axiom will be satisfied if the entrant's income is not higher than that of the poorest person, whereas a focused, population replication invariant poverty index satisfying the Strong Monotonicity Axiom will be affirmatively responsive to the Nonpoverty Growth Axiom. This shows that the latter of the two axioms may be regarded as a reasonable axiom for a poverty index in some specific situation. He also showed that the only subgroup decomposable poverty index that satisfies the two axioms is a linear transformation of the headcount ratio. According to Sen (1981), the problem arises because the formulation of the axioms relies on the position of the poverty line, which is not true for the transfer axiom.

In the following theorem, we show that many seemingly unrelated conditions for poverty ranking are equivalent. A variant of the theorem was stated in Chakravarty and Muliere (2004).

Theorem 2.1. *Let $x, y \in D^n$, where $z(x) = z(y) = \{1, 2, .., q\}$, be arbitrary. Then under the weak definition of the poor, the following statements are equivalent:*

(i) x^p can be obtained from y^p by a finite sequence of rank-preserving income increments of the poor and a finite sequence of rank-preserving progressive transfers among the poor or simply by rank-preserving income increments of the poor.

(ii) y^p can be obtained from x^p by a finite sequence of rank-preserving income reductions of the poor and a finite sequence of rank-preserving regressive transfers among the poor or simply by a finite sequence of rank-preserving income reductions of the poor.

(iii) *$P(x,z) < P(y,z)$ for all symmetric, focused poverty indices $P : D^n \times [z_-, z_+] \to R^1$ that satisfy the Weak Monotonicity and Weak Transfer Axioms.*
(iv) *$P(x,z) < P(y,z)$ for all focused poverty indices $P : D^n \times [z_-, z_+] \to R^1$ that are decreasing and strictly S-convex in the incomes of the poor.*
(v) *$W(x^p) > W(y^p)$, where W is any increasing, strictly S-concave social welfare function defined on the set of income distributions of the poor.*
(vi) *$\sum_{i=1}^{q} U(x_i) > \sum_{i=1}^{q} U(y_i)$ for any increasing, strictly concave individual income utility function U of the poor.*
(vii) *$\sum_{i=1}^{q} \zeta(x_i,z) < \sum_{i=1}^{q} \zeta(y_i,z)$ for all individual poverty functions ζ that are decreasing and strictly convex in the incomes of the poor.*
(viii) *x^p is generalized Lorenz better than y^p, that is, $x^p \geq_{\mathrm{GL}} y^p$.*
(ix) *x^p second-order stochastic dominates y^p.*
(x) *$\sum_{i=1}^{j} (z - x_i) \leq \sum_{i=1}^{j} (z - y_i)$ for all $1 \leq j \leq q$, with $<$ for some $j \leq q$.*
(xi) *There exists a bistochastic matrix A of order q such that $x^p \geq y^p A$.*
(xii) *There exists a finite number $T^1, T^2, \ldots, T^J$ of T-transformations such that $x^p \geq y^p T^1 \ldots . T^J$, where the order of each T^i matrix is $q \times q$.*

Proof. Equivalence between the conditions (v) and (viii) follows from the Shorrocks (1983a) theorem. Equivalence between the conditions (vi) and (viii) follows from a theorem of Marshall and Olkin (1979, p. 12). We also know that condition (ix) is equivalent to condition (viii) (*see* Chap. 1). Hence, conditions (v), (vi), (viii), and (ix) are equivalent. Condition (iv) is an alternative way of writing condition (v), whereas condition (vii) is the (equivalent) poverty analogue to condition (vi). Since for a focused, symmetric poverty index, we allow only rank-preserving transfers among the poor, the poverty index is strictly S-convex if it satisfies the Weak Transfer Axiom. If the index satisfies the Weak Monotonicity Axiom, we need its decreasingness also. Hence, condition (iii) implies condition (iv). Arguing in an analogous way, we can show that the converse is also true (see also Foster, 1984). Equivalence between conditions (i) and (viii) was established by Foster and Shorrocks (1988b, Lemma 2). Condition (ii) is an alternative way of expressing condition (i). Likewise, condition (x) expresses condition (viii) in a different but technically equivalent way. Proof of equivalence between the conditions (vi) and (xi) can be found in Marshall and Olkin (1979, p. 12). Demonstration of equivalence between conditions (xii) and (viii) can also be found in Marshall and Olkin (1979, p. 28). Hence, all the twelve conditions stated in the theorem are equivalent. □

Condition (xii) says that we postmultiply y^p by the product of a finite number (J) of T-transformation matrices and then x^p can be obtained from the resulting distribution by increasing some incomes. That is, x^p can be derived from y^p by reducing income inequality among the poor and then increasing some incomes below the poverty line. Condition (x) of the theorem says that the sum of poverty gaps $\sum_{i=1}^{j} (z - y_i)$ of the bottom j/q proportion of the poor under y is as high as the corresponding sum $\sum_{i=1}^{j} (z - x_i)$ under x and for at least one proportion, it is higher. This intuitively appealing condition is equivalent to 11 other conditions for poverty ranking. In view of equivalence of welfare dominance [condition (v)] with poverty dominance [condition (iii)], Theorem 2.1 says that we can regard Focus, Symmetry, the Weak Monotonicity, and Transfer Axioms as basic axioms for a poverty index. The other basic axioms can be Continuity, the Population Replication Invariance, Increasing Poverty Line, and Subgroup Decomposability Axioms because of their intuitive appeal. [An experimental questionnaire investigation on acceptability of different poverty axioms was made by Amiel and Cowell (1997).]

2.3 Poverty Indices

In this section, we present a discussion on alternative indices of poverty suggested in the literature. The presentation is divided into several subsections. Unless specified, the discussion of different indices relies on the assumptions that all income distributions are illfare-ranked and that there are q poor persons in the society.

2.3.1 The Classical Indices

Probably the most extensively used index of poverty is the headcount ratio, the proportion of persons that falls in poverty in the population, that is, the ratio of the total population with incomes not above the poverty line. Given that q denotes the number of poor in the society, the headcount ratio is defined as

$$P_{\mathrm{H}}(x,z) = \frac{q}{n}. \tag{2.3}$$

P_{H} possesses a joint invariance characteristic – it is a relative index as well as an absolute index. In fact, a general result along this was established by Foster and Shorrocks (1991). Their demonstration shows that under certain conditions, the only subgroup consistent poverty index that satisfies this joint invariance property and continuity in individual incomes (restricted continuity) is a continuous, increasing transformation of P_{H}. A stronger version of this result proved by Zheng (1994) also shows that the poverty indices that are both relative and absolute are related to P_{H}. P_{H} ignores actual incomes of the poor in its formulation. For instance, it regards the distributions $x = (0,0,20)$ and $y = (9,9,20)$, where the poverty line is 10, as identically poor. It, therefore, violates all versions of the monotonicity and transfer axioms. This in turn demonstrates that there is no distribution-sensitive poverty index that can be both relative and absolute. Since this index makes no distinction between a poor person and a poorer poor person, its application as a tool for the purpose of poverty alleviation purpose is not appropriate. This is because the policymakers are likely to recommend that the most pauper persons (with the highest income shortfalls from the poverty line) deserve the maximum share of a given anti-poverty budget on a priority basis. However, P_{H} is unable to identify such groups.

Another commonly used index of poverty is the income gap ratio, the average of the relative income shortfall of the poor from the poverty line. This index is formally defined as

$$P_{\mathrm{IGR}}(x,z) = \frac{\sum_{i=1}^{q}(z-x_i)}{qz}. \tag{2.4}$$

This index is also referred to as the normalized poverty gap. It is a summary indicator of poverty depths $(z-x_i)$ of different poor individuals in the society. Since qzP_{IGR} determines the aggregate income shortfall of the poor from the poverty line, from policy point of view, qzP_{IGR} gives us the total amount of money required to put all the poor persons at the poverty line. By concentrating on the average gap $(\sum_{i=1}^{q}(z-x_i)/q)$, this index ignores the distribution of income among the poor. To see this, let $x=(0,14,15,20)$ and $y=(0,0,0,20)$ be two income distributions and suppose that the poverty line is 10. Then P_{IGR} treats the two distributions as equally poor. Clearly, a sensible focused poverty index should regard y as more poverty stricken than x. The main problem is that the income gaps of the poor are aggregated linearly in P_{IGR}. This in turn implies that the index is insensitive to the redistribution of income among the poor. More precisely, it is a violator of the transfer axioms that do not modify the set of poor persons, although it meets the weak form of the monotonicity axioms. Further, it is not subgroup decomposable. The headcount ratio is, however, subgroup decomposable but may not increase if the poverty-line increases. The product $P_{\mathrm{H}}P_{\mathrm{IGR}}$ of these two indices, which is popularly known as the poverty gap ratio, can as well be an index of poverty. But it is also a violator of the transfer principles, although it is increasing in poverty line.

2.3.2 The Sen Index

We have noted that independently P_{H} and P_{IGR} are subject to many shortcomings. Sen (1976a) showed how these two indices along with $I_{\mathrm{G}}^{\mathrm{p}}$, the Gini index of the income distribution of the poor, can give an adequate picture of poverty. For a large number of poor, the Sen index is given by

$$P_{\mathrm{S}}(x,z) = P_{\mathrm{H}}[P_{\mathrm{IGR}} + (1-P_{\mathrm{IGR}})I_{\mathrm{G}}^{\mathrm{p}}]. \tag{2.5}$$

The original form of the Sen index contains an additional factor $(q/(q+1))$ in the second term of the third bracketed term. Since this additional factor can be approximated by one for a fairly large q, we will refer to the more commonly used form P_{S} as the Sen index. The presence of the Gini index in (2.5) ensures that P_{S} is sensitive to the income distribution of the poor. Under ceteris paribus assumption, an increase in the value of $I_{\mathrm{G}}^{\mathrm{p}}$ increases P_{S}. $I_{\mathrm{G}}^{\mathrm{p}}$ has not been directly incorporated in P_{S}. Sen began by assuming that the poverty index is the normalized weighted sum of the income gaps of the poor from the poverty line. Then $I_{\mathrm{G}}^{\mathrm{p}}$ dropped out as an implication of the assignment of the ith poor person's rank in the income distribution of the poor as the weight on his poverty gap $(z-x_i)$ and the normalization condition that when all the poor persons enjoy the same income, the extent of poverty is determined by the poverty gap ratio, $P_{\mathrm{H}}P_{\mathrm{IGR}}$. Assignment of ranks as weights to individual poverty gaps captures the idea that the higher is the gap, the more is the weight. The relative index P_{S} is focused, symmetric, population replication invariant, increasing in poverty line, and satisfies the weak forms of the transfer and monotonicity principles. However, it is not subgroup decomposable.

2.3.3 Some Alternatives and Variants of the Sen Index

Alternatives and variations of the Sen index have been suggested from several perspectives. For instance, use of an alternative index of inequality in P_{S} is one possibility. We can also explore the possibility of weighting the gaps in a different way.

Blackorby and Donaldson (1980b) observed that we can rewrite P_{S} as the product of the headcount ratio and proportionate shortfall of the Gini representative income of the poor $E_{\mathrm{G}}(x^p)$ from the poverty line. (See Sect. 1.6.3 for definition of the representative income and its specific forms.) We can rewrite P_{S} as

$$P_{\mathrm{S}}(x,z) = P_{\mathrm{H}}\left[1 - \frac{E_{\mathrm{G}}(x^p)}{z}\right]. \tag{2.6}$$

Recall from our discussion in Sect. 1.6.3 that indifference surfaces of E_{G} are numbered so that $E_{\mathrm{G}}(z1^q) = z$. Hence P_{S} is the product of the proportion of persons in poverty and the proportionate gap between the welfare level of the income distribution of the poor where each of them enjoys the poverty-line income and that of the actual income distribution of the poor, when welfare evaluation is done with the Gini welfare function. This shows a direct welfare interpretation of P_{S}. An ordinal transformation of the welfare function does not change the value of the poverty index (*see also* Xu and Osberg, 2002).

A natural generalization of P_{S} will be to replace $E_{\mathrm{G}}(x^p)$ with an arbitrary representative income $E(x^p)$, determined using a continuous, increasing, strictly S-concave, and homothetic social welfare function defined on the income distributions of the poor. The resulting index is the Blackorby-Donaldson relative poverty index P_{BD} (Blackorby and Donaldson, 1980b). Formally,

$$P_{\mathrm{BD}}(x,z) = P_{\mathrm{H}}\left[1 - \frac{E(x^p)}{z}\right]. \tag{2.7}$$

Note that we can rewrite $E(x^p)$ as $E(x^p) = \lambda(x^p)(1 - I_{\mathrm{AKS}}(x^p))$, the product of the Atkinson (1970)-Kom (1969)-Sen (1973) index of equity $(1 - I_{\mathrm{AKS}}(x^p))$ of the poor and their mean income $\lambda(x^p)$ under the assumption that $\lambda(x^p) > 0$ (*see* Sect. 1.6.3). This shows that the general relative index P_{BD} (hence the Sen index) is increasing in the level of inequality $I_{\mathrm{AKS}}(x^p)$ among the poor under ceteris paribus assumptions. It is also sensitive to the headcount ratio and how poor the poor are (because of its explicit dependence on the relative gap). It possesses the same welfare interpretation that we provided for P_S. Blackorby and Donaldson (1980b) discussed the relationship between the social welfare functions of the poor with those for the whole society. They concluded that the social welfare function must be completely strictly recursive in the sense that the ordering over incomes of any subset of the poor people must be separable from the income of anyone who is richer. But P_{BD} is not continuous, population replication invariant, and subgroup decomposable. However, it is symmetric, focused, and satisfies the weak versions of the monotonicity and transfer axioms, although not their strong counterparts (Chakravarty, 1983a, 1990, 1997a; Foster and Shorrocks, 1991; Zheng, 1997). To understand the reason for violation of the Strong Upward Transfer Axiom, consider an upward transfer that makes the recipient rich. This brings a decline in P_H but the change in the other component may not be so significant to indicate an unambiguous change in the product of the two terms in (2.7).

For every homothetic social welfare function of the poor, we have a corresponding poverty index of the type P_{BD}. For instance, assume that the representative income $E(x^p)$ of the poor is of the form $\sum_{i=1}^{q} x_i(q+1-i)^r/i^r$, where $r > 0$. Then the corresponding index turns out to be the one suggested by Kakwani (1980b), which is given by

$$P_K(x,z) = \frac{q}{nz\sum_{i=1}^{q} i^r} \sum_{i-1}^{q} (z-x_i)(q+1-i)^r. \tag{2.8}$$

The original form of the Sen index corresponds to the particular case $r = 1$. On the other hand, if $r = 0$, the Kakwani index P_K reduces to the form $P_H P_{IGR}$, the poverty gap ratio. The relative index P_K index was introduced with the objective that it will fulfill the Diminishing Transfer Sensitivity Axiom. Kakwani (1980b) demonstrated that for a given income distribution, a positive value of r exists for which this objective is fulfilled. But for any given r, there exists an income distribution for which P_K is not sensitive to the transfers, the way the axiom demands. However, it meets the positional version of the transfer sensitivity axiom for all $r > 1$.

Giorgi and Crescenzi (2001) suggested a variant of P_S by replacing the Gini index in (2.5) by the Bonferroni inequality index. We can rewrite it in terms of the Bonferroni representative income of the poor. The resulting index is given by

$$P_{GC}(x,z) = P_H\left(1 - \frac{1}{qz}\sum_{i=1}^{q}\frac{1}{i}\sum_{j=1}^{i} x_j\right). \tag{2.9}$$

This index has the advantage of satisfying the Positional Transfer Sensitivity Axiom. However, P_S does not fulfill this property since it involves the Gini index as its inequality component. P_K and P_{GC} behave in the same way as P_{BD} with respect to the Continuity, Monotonicity, Replication Invariance, and (Upward and Downward) Transfer Axioms.

An alternative way of employing inequality indices for construction of poverty indices was suggested by Hamada and Takayama (1977) and Takayama (1979) using censored income distributions. A censored income distribution replaces income of each nonpoor person by the poverty line z and maintains the income of a poor person at its existing level. Formally, the censored income corresponding to the income level x_i is defined as

$$x_i^* = \min\{x_i, z\}. \tag{2.10}$$

We denote the censored income distribution corresponding to x by x^*. Thus, $x^* = (x_1, x_2, \ldots\ldots x_q, z, z, .., z)$. Takayama (1979) defined the Gini index of x^* as an index of poverty. More precisely, the Takayama index is defined as

$$P_{\mathrm{T}}(x,z) = 1 - \frac{1}{n^2\lambda(x^*)}\sum_{i=1}^{n}(2(n-i)+1)x_i^*. \tag{2.11}$$

Hamada and Takayama (1977) also suggested the use of various censored income distribution-based inequality indices as poverty indices. All such indices have a clear merit – they do not ignore the existence of persons above the poverty line but ignore information on their income and count them in with the poverty line. But for either definition of the poor, these indices violate the weak form of the monotonicity axiom. To see this, suppose that all persons in the society are poor. Then multiplication of all the incomes by a positive scalar less than one keeps these indices unchanged. But the Weak Monotonicity Axiom demands increasingness of the poverty index in this situation. Under the weak definition of the poor, they also violate the strong form of the axiom (Chakravarty, 1983a, 1990).

Chakravarty (1983a, 1997a) suggested a general index by combining the Blackorby and Donaldson (1980b) and the Takayama (1979) approaches. This index is defined as the relative gap between the poverty line z and the representative income $E(x^*)$ based on the censored income distribution x^*, where E is calculated using a continuous, increasing, strictly S-concave, and homothetic social welfare function. Formally, this index is given by

$$P_{\mathrm{C}}(x,z) = 1 - \frac{E(x^*)}{z}. \tag{2.12}$$

By construction, the Chakravarty index P_{C} is focused, normalized, continuous, and symmetric. Since homotheticity of the welfare function implies that E is linear homogeneous, P_{C} is a relative index. Linear homogeneity of E along with its increasingness ensures that P_{C} is increasing in poverty line. It satisfies both forms of the monotonicity axiom and all versions of the transfer principle. If we assume that E is population replication invariant, then P_{C} is so. However, in general, it is not subgroup decomposable. Since $E(z1^n) = z, P_{\mathrm{C}}$ can be interpreted as the proportionate size of welfare loss due to existence of poverty. This loss becomes zero if there is no poor person in the society.

We can rewrite P_{C} as

$$P_{\mathrm{C}}(x,z) = 1 - \frac{\lambda(x^*)(1 - I_{\mathrm{AKS}}(x^*))}{z}. \tag{2.13}$$

This shows that we can transform the Atkinson-Kom-Sen relative inequality index of a censored income distribution $I_{\mathrm{AKS}}(x^*)$ into a poverty index in a fairly natural way. Given a poverty line, for two censored income distributions x^* and y^* with the same mean, the ranking of the distributions generated by P_C coincides with that generated by I_{AKS}. Formally, $I_{AKS}(x^*) \geq I_{AKS}(y^*) \leftrightarrow P_C(x,z) \geq P_C(y,z)$ P_C has a relationship with P_{BD} as well: if the social welfare function is completely strictly recursive, then $P_{BD}(x,z) < P_C(x,z) < (z - E(x^p)/z$. If we do welfare evaluation with the Rawlsian maximin rule $\min_i\{x_i\}$ (Rawls, 1971) and the income sum criterion $\sum_{i=1}^n x_i$, then the bounds are actually attained. However, these welfare functions are S-concave, but not strictly so. Pyatt (1987) investigated properties of P_C using affluence and basic income, and examined the implications when the society equivalent income is given by the sum of equivalent basic income and equivalent income of affluence.

Evidently, to every homothetic social welfare function, there corresponds a different poverty index in (2.12). These indices will differ in the way we aggregate the censored incomes into an indicator of welfare. For instance, suppose that welfare evaluation is done with the Gini welfare function. Then P_C turns out to be the continuous extension of the Sen index characterized by Shorrocks (1995):

$$P_{Sh}(x,z) = \frac{1}{n^2 z} \sum_{i=1}^{n} (z - x_i^*)(2(n-i)+1). \tag{2.14}$$

In addition to being population replication invariant, P_{Sh} fulfills all the postulates that are fulfilled by P_C in its general form. We can rewrite the formula for P_{Sh} using the Gini index of the censored income distribution $I_G(x^*)$ as $P_{Sh}(x,z) = P_H P_{IGR} + (1 - P_H I_{IGR}) I_G(x^*)$. This shows explicit dependence of the index on the per capita poverty gap index $P_H P_{IGR}$ and the inequality index $I_G(x^*)$. Since $I_G(x^*)$ is bounded above by one, under ceteris assumption, an increase in P_H or P_{IGR} will lead to an increase in poverty.

Earlier Thon (1979) suggested an index that has similar properties as P_{Sh}. In fact, the failure of P_S to verify the strong upward version of the transfer postulate motivated Thon to propose his index. If we employ the representative income $\sum_{i=1}^n 2(x_i^*(n+1-i))/(n(n+1))$ in (2.12), the resulting formula becomes the Thon index (Thon, 1979):

$$P_{Th}(x,z) = \frac{2}{(n+1)nz} \sum\nolimits_{i=1}^{n} (z - x_i^*)(n+1-i). \tag{2.15}$$

The difference between P_{Sh} and P_{Th} arises from the assignment of different weights on the income gap of a poor. While Thon employed the rank of a person in the total population, Shorrocks used Gini type weight in the entire population. Thus, the simple alternations in the weighting scheme in P_S makes P_{Sh} and P_{Th} to satisfy several axioms which P_S violates (*see also* Xu and Osberg, 2001).

None of the members of P_C we have discussed so far are sensitive to income transfers lower down the scale. The second Clark et al. index, which drops out as a member of P_C, when welfare evaluation is done with the symmetric mean of order $\theta < 1$, fulfills this objective (*see* Clark et al., 1981). The functional form for this index is given by

$$P_{\text{CHU}}(x,z) = \begin{cases} 1 - \frac{[1/n\sum_{i=1}^{n}(x_i^*)^{\theta}]^{1/\theta}}{z}, & \theta < 1, \theta \neq 0, \\ 1 - \frac{\prod_{i=1}^{n}(x_i^*)^{1/n}}{z}, & \theta = 0, \end{cases} \tag{2.16}$$

where $x \in D_+^n$. This index can be regarded as the poverty counterpart to the Atkinson (1970) inequality index, when applied to the censored income distribution. It is population replication invariant and retains all the properties of P_{C}. For any given income distribution, an upward income transfer will increase the value of the index by a larger amount the lower is the value of θ. For any finite value of $\theta < 1$, the social indifference curve will be strictly convex to the origin and it becomes more and more convex as the value of θ decreases. For $\theta = 0$, we get the symmetric Cobb-Douglas poverty index. As $\theta \to -\infty$, the poverty index becomes $1 - \min_i \{x_i^*/z\}$, the relative maximin poverty index. On the other hand, when $\theta \to 1$, P_{CHU} coincides with $P_{\text{H}}P_{\text{IGR}}$, which ignores many important features of a satisfactory poverty index, including redistribution of income.

From Theorem 2.1, it follows that if we adopt the weak definition of the poor, a decreasing and strictly S-convex function of the incomes of the poor can be a suitable index of poverty under appropriate formulation. Suppose we consider an illfare function H defined on the income gaps of the poor. Assume that $H((z-x_1),\ldots\ldots,(z-x_q))$ is continuous, increasing, strictly S-convex, and homothetic in its arguments. (Recall that z is given.) We now define the representative income gap g_{e} as that level of poverty gap which if suffered by each poor person will make the existing profile of gaps socially indifferent. More precisely, $H(g_{\text{e}}1^q) = H((z-x_1),\ldots\ldots,(z-x_q))$. Given assumptions about H, we can determine g_{e} uniquely. As a general relative poverty index, Chakravarty (1983b) suggested the use the following representative gap-based index:

$$P_{\text{CRG}} = P_{\text{H}} \frac{g_{\text{e}}}{z}. \tag{2.17}$$

P_{CRG} is a generalization of the first Clark et al. (1981) index, which is based on the assumption that H is given by sum of rth power ($r > 1$) of the gaps, that is, $H((z-x_1),\ldots(z-x_q)) = \sum_{i=1}^{q}(z-x_i)^r$. Then g_{e} becomes the symmetric mean of order $r > 1$ of the poverty gaps, which when substituted into (2.17) gives the first Clark et al. index. Assuming that the set of poor persons is fixed, it satisfies the diminishing sensitivity version of the transfer principle. However, like P_{S}, it is a violator of the upward strong form of the transfer axiom (*see* Thon, 1983b).

Now, consider the illfare function that uses the Sen-type weights. More precisely, H is of the form $H((z-x_1),\ldots(z-x_q)) = \sum_{i=1}^{q}(z-x_i)(q+1-i)$. Then $g_{\text{e}} = \left(2\sum_{i=1}^{q}(z-x_i)(q+1-i)/(q(q+1))\right)$, which on substitution into (2.17) yields the original form of the Sen (1976a) index. If all the poor persons have zero income, then the representative income gap g_{e} is given by z. (Assume that the income vector 01^q of the poor persons is in the domain.) Note that g_{e} is a specific representation of the illfare function H. If we assume that all the incomes are nonnegative, then the original version of the Sen index becomes the product of two ratios: the headcount ratio and the ratio between the actual illfare and the (maximal) illfare that would arise if all the poor persons are at the zero income level. This gives us an illfare interpretation of the original Sen index. We have also noted in Theorem 2.1 that, under certain assumptions, all indices of the form (2.17) can be used for poverty ranking of income distributions.

According to Vaughan (1987), poverty indices can be viewed as measuring the size of welfare loss that results from the existence of poverty. This is quite similar to the interpretation we have provided for P_{C} and P_{BD} (and hence for P_{S}). His formulation incorporates social welfare function directly into the construction of poverty indices. The Vaughan relative poverty index is defined as:

$$P_{\mathrm{VR}}(x,z) = 1 - \frac{W(x)}{W(\tilde{x})}, \tag{2.18}$$

where $\tilde{x}$ is derived from x by replacing all the incomes of the poor by the poverty line. This is a quite general index and many indices may be embedded into it. Increasingness of W will ensure nonnegativity of P_{VR}. A sufficient condition for P_{VR} to be a relative index is homogeneity of W of some arbitrary degree. Likewise, additional restrictions on W will be necessary for fulfillment of different axioms.

2.3.4 Subgroup Decomposable Poverty Indices

As we have argued in Sect. 2.2, the subgroup decomposable indices enable us to identify the more poverty stricken population subgroups on a priority basis for implementing poverty alleviation program. Equation (2.2) shows that the general form of an index satisfying the decomposability axiom is given by the symmetric average of the individual poverty functions. If we assume that the index is of relative type, then the individual poverty function must be of the form $\zeta(x_i,z) = \zeta(x_i/z,1) = h(x_i/z)$ say, where $h : R^1_+ \to R^1$. If we impose further axioms, it is possible to narrow down the functional form of the index. For instance, if we assume that the index is focused, continuous, normalized, and satisfies the weak form of the monotonicity axiom and the strong upward version of the transfer principle, then it will be of the form

$$P_{\mathrm{D}}(x,z) = \frac{1}{n}\sum\nolimits_{i=1}^{n} h\left(\frac{x_i}{z}\right), \tag{2.19}$$

where $h : R^1_+ \to R^1$ is continuous, decreasing, strictly convex. and $h(v) = 0$ for all $v \geq 1$ (*see* Foster and Shorrocks, 1991). Assume that first $q \leq n$ persons are poor.

As an illustrative example, suppose that $h(v) = -\log v$, where $v > 0$. Then the resulting subgroup decomposable index becomes the Watts index of poverty (Watts, 1968):

$$P_{\mathrm{W}}(x,z) = \frac{1}{n}\sum\nolimits_{i=1}^{q} \log\left(\frac{z}{x_i}\right). \tag{2.20}$$

Blackburn (1989) showed that we can rewrite P_{W} in terms of the Theil (1967) mean logarithmic deviation index of inequality of the poor $I_{\mathrm{TML}}(x^p) = \frac{1}{q}\sum_{i=1}^{q} \log[(\mu(x^p))/x_i]$ as

$$P_W(x,z) = P_H[I_{TML}(x^p) - \log(1 - P_{IGR})]. \tag{2.21}$$

Thus, for a given values of P_H and P_{IGR}, a reduction in the Theil inequality index on the right-hand side of (2.21) is equivalent to a reduction in the Watts index and vice versa. Zheng (1993) interpreted this index as the size of absolute welfare loss due to poverty and characterized it in such a framework using a set of axioms. Tsui (1996) noted that the change in P_W can be neatly disaggregated into growth and redistributive components. Another interesting observation is that for a given poverty line, P_W is related to the member of the Clark et al. (1981) index P_{CHU} in (2.16) for $\theta = 0$ as follows: $P_{CHU} = 1 - \exp(-P_W)$. Therefore, the two indices produce the same poverty ranking of income distributions when the poverty line is fixed.

Next, for the functional form $h(v) = 1 - v^e, 0 < e < 1,\quad P_D$ coincides with the additively decomposable index characterized by Chakravarty (1983c):

$$P_{CD}(x,z) = \frac{1}{n}\sum_{i=1}^{q}\left[1 - \left(\frac{x_i}{z}\right)^e\right]. \tag{2.22}$$

The subgroup decomposable Chakravarty index P_{CD} satisfies the diminishing form of the transfer principle and all higher order sensitivity axioms. As the value of the parameter e decreases, the index becomes more sensitive to transfers at the lower part of the profile. For $e = 1$, P_{CD} becomes the poverty gap ratio, whereas for $e = 0$, $P_{CD} = 0$. If we replace z by the mean income λ, normalize the index over [0,1] and sum over uncensored income distributions then a clear link of P_{CD} is established with the normalized Theil (1967) entropy index (Chakravarty, 1990; Zheng, 1997). In order to establish relationship between P_{CHU} and P_{CD}, note that $n[z(1-P_{CHU})]^\theta = \sum_{i=1}^{n}(x_i^*)^\theta$ and $z^e[q - nP_{CD}] = \sum_{i=1}^{q} x_i^e$. Hence, assuming that the poverty line is given, the ranking of two distributions by P_{CD} coincides with that generated by P_{CHU} for $0 < \theta < 1$. Thus, in this particular case, the two indices convey the same information in terms of ranking. However, P_{CHU} is a nondecomposable index because of the specific type of aggregation employed in it.

Finally, suppose that $h(v) = (1 - v)^\alpha$, where $\alpha > 1$, then P_D becomes the Foster et al. index (Foster et al., 1984):

$$P_{FGT}(x,z) = \frac{1}{n}\sum_{i=1}^{q}\left(\frac{z - x_i}{z}\right)^\alpha. \tag{2.23}$$

Except P_{CD}, all poverty indices proposed after Sen (1976a) and prior to P_{FGT} are not subgroup decomposable. The difference between P_S and P_{FGT} is that while the former uses the relative rank of the ith poor as the weight on his poverty gap, the latter employs the $(\alpha - 1)$th power of his normalized gap $(z - x_i)/z$ as the weight on this gap itself. The index can be rewritten as the product of the headcount ratio q/n and average of the transformed normalized gaps of the poor: $\sum_{i=1}^{q}(z - x_i/z)^\alpha/q$. Thus, it is sensitive to the proportion of population in poverty and how poor this proportion is. As $\alpha \to 0$, the index approaches P_H, whereas for $\alpha = 1$, it coincides with the poverty gap ratio $P_H P_{IGR}$ A larger value of α gives greater emphasis to the poorer of the poor in the aggregation. For $\alpha > 2$, P_α satisfies all the axioms satisfied by P_{CD}. For $\alpha = 2, P_\alpha$ can be written as

$$P_{\mathrm{FGT}}(x,z) = P_{\mathrm{H}}\left\lfloor (I_{\mathrm{IGR}})^2 + (1-P_{\mathrm{IGR}})^2 (I_{\mathrm{CV}}(x^p))^2 \right\rfloor, \tag{2.24}$$

where $I_{\mathrm{CV}}(x^p)$ is the coefficient of variation of the income distribution of the poor. This explicitly shows that for $\alpha = 2, P_{\mathrm{FGT}}$ does not exhibit transfer sensitivity. However, the formula in (2.24) recognizes its explicit relationship with an index of inequality in a positive monotonic way. As $\alpha \to \infty, P_{\mathrm{FGT}}$ approaches q_0/n, where q_0 is the number of persons with zero income, while the transformed index $(P_{\mathrm{FGT}})^{1/\alpha}$ tends to the relative maximin index of poverty. Note that we can convert P_{FGT} into the inequality index $(\sum_{i=1}^{n} |(\lambda(x) - x_i)/\lambda(x)|^\alpha)/n$ by a straightforward transformation. For $\alpha = 1$, it becomes the relative mean deviation or the Kuznets ratio and when $\alpha = 2$ the squared coefficient of variation is obtained. It becomes a transfer sensitive index of inequality in the sense of Shorrocks and Foster (1987) if α takes on values in the open interval $(2, \infty)$.[4]

Zheng (2000a) defined the measure of distribution-sensitivity for an individual poverty index $\zeta(v,z)$, for all $v < z$, as follows:

$$\mathrm{DS}_\zeta(v,z) = -\frac{\zeta_{vv}(v,z)}{\zeta_v(v,z)}, \tag{2.25}$$

where ζ_v and ζ_{vv} denote, respectively, the first and second partial derivatives of the function ζ with respect to its first argument.

The distribution-sensitivity measure $\mathrm{DS}_\zeta(v,z)$ is quite similar to the Arrow (1971)-Pratt (1964) absolute risk aversion measure $\mathrm{AP}(v) = -U''(v)/U'(v)$ for a utility function U, where v represents wealth and, U' and U'' are, respectively, the first and second derivatives of U. Taking cue from Pratt (1964), Zheng (2000b) argued that the following result concerning DS_ζ for the weak definition of the poor can be demonstrated.

Theorem 2.2. *For a given poverty line z and two individual poverty functions ζ and $\hat{\zeta}$, the following statements are equivalent:*

(a) $\mathrm{DS}_\zeta(v,z) > \mathrm{DS}_{\hat{\zeta}}(v,z)$ *for all* $v \in [0,z)$.

(b) $\zeta(v,z)$ *is a strictly convex function of* $\hat{\zeta}(v,z)$ *for* $v \in [0,z)$.

Theorem 2.2 says that ζ will indicate a higher increase in poverty than $\hat{\zeta}$ for a regressive transfer between two poor persons.

We can also interpret the distribution-sensitivity measure in terms of poverty aversion. A poverty averse policymaker will regard the reduction in social welfare of the poor less if one unit of income is taken from a poor person than from a poorer poor person. Thus, poverty aversion has essentially the same flavor as distribution-sensitivity (*see also* Seidl, 1988).

[4] Ebert (1988c) characterized indices of this type using alternative sets of axioms. See also Ebert and Moyes (2002) for discussion on P_{FGT}.

From the functional form of the second Clark et al. (1981) index, it appears that we can regard $\zeta(v,z) = \lfloor 1-(v/z)^{\theta} \rfloor /\theta$ with $\theta < 1$, as its individual poverty function. Its measure of distribution-sensitivity is $\mathrm{DS}_{\theta}(v,z) = (1-\theta)/v$. As the value of θ reduces, the index becomes more distribution-sensitive. We can interpret $(1-\theta)$ as a measure of poverty aversion: for a given income, the lower is the value of θ, the more poverty averse the index becomes. Since the Watts index corresponds to the case $\theta = 0$, the Clark et al. index with negative θ is more distribution-sensitive than the Watts index, which in turn has higher distribution-sensitivity than the Chakravarty index because of its increasing relationship with the Clark et al. index for positive θ. This provides an interesting comparison among the three indices.

Since for the Foster et al. index $\zeta(v,z) = (1-v/z)^{\alpha}$, its distribution-sensitivity measure is $DS_{\alpha}(v,z) = (\alpha-1)/(z-v)$. Thus, as the value of α increases, distribution-sensitivity increases and $(\alpha-1)$ can be regarded as an indicator of poverty aversion. It may be checked that for given (z,α,θ), neither of the Clark et al. and the Foster et al. indices is always more distribution-sensitive than the other.

2.3.5 Absolute Indices of Poverty

Absolute indices often become very useful because of their policy implications. The absolute poverty index suggested by Blackorby and Donaldson (1980b) enables us to determine the total monetary cost of poverty. It is formally defined as

$$P_{\mathrm{BDA}}(x,z) = q(z-E(x^{p})), \tag{2.26}$$

where the representative income of the poor $E(x^{p})$ is evaluated according to a continuous, increasing, strictly S-concave, and translatable social welfare function of the poor. Since E is unit translatable, if each poor in the society were given $(z-E(x^{p}))$ amount of money then the Blackorby-Donaldson absolute poverty index P_{BDA} will be zero at an aggregate cost of $q(z-E(x^{p}))$. Therefore, this index gives us the monetary cost of poverty. However, it shares all the shortcomings of its relative sister P_{BD} (*see also* Bossert, 1990b).

While P_{BDA} determines the aggregate cost of poverty, Chakravarty (1983a) suggested a per capita absolute poverty index. It is given by the difference between the poverty line and the society representative income which is calculated using a continuous, increasing, strictly S-concave, and translatable social welfare function defined on the censored income distributions. Formally,

$$P_{\mathrm{CA}}(x,z) = z-E(x^{*}). \tag{2.27}$$

Unit translatability of E shows that if each person in the censored income distribution x^{*} were given $(z-E(x^{*}))$ amount of money, then absolute poverty, as measured by P_{CA}, will be zero at an aggregate cost of $n(z-E(x^{*}))$. Thus, P_{CA} determines the per capita absolute poverty cost. This index shares all the properties of its relative counterpart P_C. We can illustrate the index using the Gini and the Kolm (1976a)-Pollak (1971) welfare functions defined on the censored income distributions. It may be worthwhile to mention here that the use of any absolute inequality index defined on the censored income distributions will not be suitable for measuring poverty because of its failure to fulfill the monotonicity axioms.

We can also define absolute indices using the illfare function of the poor. Since these functions are defined directly on income gaps, they are translation invariant. This in turn shows that any well-defined aggregation of the gaps will give us an absolute index. For instance, the index $(\sum_{i=1}^{q} \exp(r(z-x_i))/q$, where $r > 0$ is a parameter, is an absolute index that satisfies all the poverty axioms specified in condition (iii) of Theorem 2.1. We can also use the representative gap g_e calculated using a continuous, increasing, and strictly S-convex function to construct a wide class of absolute indices, namely, $(q/n)g_e$ (Chakravarty, 1983b). One member of this class is the absolute version of the Sen (1976a) index, which, as we show in Sect. 2.6, becomes quite helpful in poverty comparisons in a very general setup.

The Vaughan absolute index $P_{VA}(x,z) = W(\tilde{x}) - W(x)$, where $\tilde{x}$ is the same as in (2.18), has been proposed to determine the size of absolute welfare loss due to existence of poverty (Vaughan, 1987). Clearly, as in the relative case, we need more information on the welfare function W to examine the properties of P_{VA}.

The Zheng absolute individual poverty function $\xi(v,z) = \exp(\tilde{\omega}(z-v)) - 1$ has a constant distribution-sensitivity $(\mathrm{DS}_{\zeta}(v,z) = \tilde{\omega} > 0)$ (Zheng, 2000b). It can be verified that at a lower income level, this index is more poverty averse than the Foster et al. (1984) index but is less poverty averse than the Clark et al. (1981) index.[5]

2.4 Population Growth and Poverty

Population replication invariant poverty indices view changes in the number of poor persons in terms of changes in the fraction of population in poverty. However, a common person may not like to look at the change in such a simple way. The distinction between the two views can be explained by an illustration provided by Kanbur (2001). It was observed that in Ghana between 1987 and 1991, the head-count ratio came down by about 1% per year, while the number of the poor increased because the total population was growing by approximately 2% per year. The policy recommenders regarded the former as a measure of the success of their "structural adjustment" policies. However, the common people criticized these policies, at least partly, because they could see more poor people around.

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[5] Many of the indices discussed in this section have been applied to study the incidence of poverty in India. For a review of this literature, see Maiti (1998). See also Deaton (2008). Dominguez and Velazquez (2007) employed several indices to analyze poverty intensity in 15 countries of the European Union using European Commission Household Panel data. Bresson and Labar (2007) studied decomposition of poverty in China for the period 1990–2003. See Slesnick (1993) for a study of poverty in the USA.

Chakravarty et al. (2006) characterized the following family of poverty indices that accommodates alternative views on poverty change resulting from population growth in a common framework:

$$P_{\text{CKM}}(x,z) = \delta_1 \left[\sum_{i=1}^{n} h\left(\frac{x_i^*}{z}\right) - \delta_2 n \right], \tag{2.28}$$

where h is the same as in (2.19), and $\delta_1 > 0$ and δ_2 are constants. Consequently, P_{CKM} is symmetric, continuous, and satisfies both forms of the monotonicity axiom and the Strong Upward Transfer and the Diminishing Transfer Sensitivity Axioms. However, it is not invariant under replications of the population because of the aggregation rule and the presence of the population-size dependent term $\delta_2 n$. In fact, the latter is the major source of difference between the population replication invariant index P_{D} in (2.19) and P_{CKM}. This is because if $\delta_2 = 0$, we can convert P_{CKM} into a population replication invariant index simply by dividing by the population size n. The presence of the term $\delta_2 n$ in the latter enables us to consider alternative views on poverty change as a consequence of change in the population size n. If q is fixed, δ_2 can be interpreted as the amount by which poverty goes down when the number of rich persons increases by one. Hence, in this situation, nonnegativity of δ_2 is a reasonable assumption. However, if we allow both the numbers of poor and rich to increase simultaneously, then there is a trade off between the reduction in poverty resulting from higher number of rich and increase in poverty because of higher number of poor. As we note below, the value of δ_2 may be helpful in resolving this issue. The constant $\delta_1 > 0$ can be regarded as a scale parameter: under ceteris paribus assumption, an increase in the value of δ_1 increases poverty. Therefore, without loss of generality, we can set $\delta_1 = 1$.

For a given income distribution over a given population size, an increase in the value of δ_2 decreases P_{CKM}. If we assume that $\delta_2 = 0$, P_{CKM} is independent of the size of the nonpoor population and their income distribution. In this case, P_{CKM} can be taken as an aggregate version of P_{D}. Consequently, for $h(v) = -\log v$, $h(v) = 1 - v^e$, and $h(v) = (1-v)^{\alpha}$, P_{CKM} becomes, respectively, the aggregate form of the Watts (1968), the Chakravarty (1983c), and the Foster et al. (1984) index. Note that if $\delta_2 = 0$, an l-fold replication of the population will augment the poverty index $(l-1)$ times, which is demanded by the Replication Scaling Principle of Subramanian (2002). Subramanian also introduced a Population Growth Principle which requires that if all the poor persons in the society have the same income and a person having this identical income migrates to the society and if there is at least one nonpoor person in the society, then poverty must go up. It is easy to verify that this principle of population growth is verified by the index P_{CKM} under the assumption that $\delta_2 = 0$.

If $\delta_2 = 0$, an increase in the number of poor increases the index P_{CKM} unambiguously. Next, for $\delta_2 > 0$, the index records a reduction in poverty if the population size increases keeping the number of poor and their income distribution fixed. This therefore shows that the formulation puts the two Kundu and Smith (1983) axioms into a common framework. The major difference between the Kundu and Smith formulation and the present one, which generates P_{CKM}, is that in the former, the two population growth criteria are assumed at the outset, whereas in the latter, the two views follow separately as implications of the functional form P_{CKM}.

Now, consider the following form of the poverty index involving the number of poor and the population size:

$$P_{\mathrm{HA}}(x,z) = q - \delta_2 n. \tag{2.29}$$

Consider two distributions x^1 and x^2 over population sizes n_1 and n_2, respectively. Let q_1 and q_2 satisfying the restrictions $q_1 > q_2$ and $(q_1/n_1) = (q_2/n_2)$ be the numbers of poor persons in x^1 and x^2, respectively. These restrictions imply that $n_1 > n_2$. Then $P_{\mathrm{HA}}(x^1,z) > P_{\mathrm{HA}}(x^2,z)$ demands that $\delta_2 < ((q_1 - q_2)/(n_1 - n_2))$ which is positive. Hence, δ_2 is bounded above by a positive real number.

Next, suppose that the number of poor is the same in both the distributions but the headcount ratio in the former is higher. That is, $q_1 = q_2$ but $(q_1/n_1) > (q_2/n_2)$. This in turn gives $n_1 < n_2$. Then $P_{\mathrm{HA}}(x^1,z) > P_{\mathrm{HA}}(x^2,z)$ requires that $\delta_2(n_1 - n_2) < 0$, implying positivity of δ_2.

The following proposition of Chakravarty et al. (2006) summarizes the above observation:

Proposition 2.1. *There exists a positive value of δ_2 such that the poverty index of the form (2.29) will satisfy the following conditions simultaneously:*

(i) For a given headcount ratio, if the absolute number of poor goes down, then poverty should decline.
(ii) For a given absolute number of poor and the income distribution of the poor population, if the headcount ratio goes down, then poverty should decline.

Note that the satisfaction of condition (i) does not require positivity of δ_2. It can hold as well for negative values of δ_2. Thus, a negative value of δ_2 may be sufficient to make sure that the trade off between poverty reduction resulting from a reduction in the number of poor and poverty increase due to a reduction in the number of rich works out in favor of diminishing poverty. In other words, the effect of poverty reduction as a consequence of lower number of poor outweighs that of poverty increase following from a smaller rich population size. In such a situation, Proposition 2.1 will become an impossibility result stating that conditions (i) and (ii) of Proposition 2.1 cannot be fulfilled simultaneously [since we always need positivity of δ_2 for condition (ii) to hold]. Note also that since for simultaneous satisfaction of (i) and (ii), δ_2 should be very small, the number of poor becomes the major determinant of poverty ranking here.[6]

An alternative poverty index that fulfills these two views simultaneously is the Arriaga index $P_{\mathrm{AR}}(x,z) = q^2/n$. It was introduced by Arriaga (1970) as an urbanization index. In the Arriaga framework, the numerator of P_{AR} is the square of total

[6] The numerical illustration provided by Chakravarty et al. (2006) using data sets from South Asia and Africa confirms this.

resident population in the urban community. (If there is more than one urban community, then it will be the sum of squares of such population sizes across communities.) Since the fulfillment of the two conditions by P_{AR} does not depend on any parameter, no impossibility result emerges in the underlying framework.

Since both P_{HA} and P_{AR} are based on the number of poor persons, they fail to meet the monotonicity and transfer axioms. The following alternative to P_{HA} which meets the strong versions of the monotonicity and transfer axioms meets also the two conditions of Proposition 2.1:

$$P_{HAA}(x,z) = \frac{1}{n^{\delta_3}} \sum_{i=1}^{n} h\left(\frac{x_i^*}{z}\right), \tag{2.30}$$

where h is same as in (2.19) and $0 < \delta_3 < 1$. For a given (x,z), an increase in δ_3 will lead to a reduction in the value of P_{HAA}, which we can rewrite as $n^{(1-\delta_3)}\left[\sum_{i=1}^{n}(h(x_i^*/z)/n)\right]$. For a given headcount ratio for condition (i) to hold, under a reduction in the absolute number of poor, a corresponding proportionate contraction of the nonpoor population size is necessary. Under this contraction, a population replication invariant poverty index remains unaltered. If we replicate the population l times, the third bracketed term remains unchanged, but the multiplicative factor becomes $(nl)^{(1-\delta_3)}$, which is greater than $(n)^{(1-\delta_3)}$. Hence $P_{HAA}(x,z) < P_{HAA}(y,z)$, where y is the l-fold replication of x. Thus, condition (i) is verified. Next, when the number of poor persons and their income distribution are given, a reduction in the headcount ratio results from an increase in the number of rich persons. It is quite easy to check that the P_{HAA} decreases in such a case. The following proposition of Chakravarty et al. (2006) summarizes these observations:

Proposition 2.2. *A poverty index of the form (2.30) will satisfy the conditions (i) and (ii) stated in Proposition 2.1 simultaneously.*

However, one limitation of P_{HAA} is that it may not indicate an unambiguous direction of change in poverty if there is an increase in the number of poor.

2.5 Poverty Orderings

Our discussion in Sect. 2.3 shows the existence of a large number of poverty indices satisfying different sets of axioms. Poverty assessment of two distributions can certainly be conflicting by two different indices. As the determination of a poverty line is subjective, variation in poverty line can be identified as a major source of disagreement in ranking of distributions. While for a given poverty line, a poverty index will rank two distributions unambiguously, for two distinct poverty lines, ranking of the distributions may be different. Therefore, it will be worthwhile to investigate whether a given poverty index can order two income distributions in an unambiguous way for a range of poverty lines. This concept of poverty ordering of distributions by a given index for all poverty lines in some reasonable interval is referred to as poverty-line ordering (Zheng, 2000b).

Foster and Shorrocks (1988a,b) developed conditions under which unanimous poverty comparisons can be made by members of P_{FGT} when poverty lines are allowed to vary. To discuss the Foster-Shorrocks results, suppose that the income distributions are defined on the continuum. Let $F:[0,\infty)\to[0,1]$ be the cumulative distribution function. Then $F(v)$ is the proportion of persons with income less than or equal to v. We retain our assumptions about F made in Chap. 1. Suppose that the poverty lines are allowed to vary over the interval $(0,\infty)$. For a given $z\in(0,\infty)$ and a poverty index P, the poverty level associated with the distribution function F is denoted by $P(F,z)$. Then of two distribution functions F and G defined on the same domain $[0,\infty)$, G poverty line dominates F with respect to the index P if and only if $P(F,z)\leq P(G,z)$ for all $z\in(0,\infty)$ with $<$ for some z.

Suppose that poverty assessment is made with the counting measure, that is, the headcount ratio. Thus, $P(F,z)=F(z)$. Then G poverty line dominates F with respect to the headcount ratio if and only if $F(z)\leq G(z)$ for all $z\in(0,\infty)$ with $<$ for some z. But this is same as the condition that F first-order stochastic dominates G.

Next, assume that poverty evaluation is done with the poverty gap ratio $P(F,z)=\int_0^z((z-v)/z)\mathrm{d}F(v)=\int_0^{F(z)}(z-F^{-1}(t))\mathrm{d}t$, where F^{-1} is the inverse distribution function defined in Chap. 1. This shows that G poverty line dominates F by the poverty gap ratio if and only if F second-order stochastic dominates G, which is equivalent to the condition that $F\geq_{\text{GL}}G$.

While these two results involve two members of P_{FGT}, the following general result in terms of P_{FGT} has been demonstrated by Foster and Shorrocks (1988a) along this line.

Theorem 2.3. *For two income distribution functions F and G defined on the same domain and a nonnegative integer α, the following statements are equivalent:*

(a) $P_{\text{FGT}}(F,z)\leq P_{\text{FGT}}(G,z)$ for all $z\in(0,\infty)$, with $<$ for some z.
(b) F dominates G by the $(\alpha+1)$th degree stochastic dominance criterion.

Thus, the poverty ranking of two distributions by the member of P_{FGT} defined in (2.24) is same as third-order stochastic dominance. An implication of the P_{FGT} orderings is that if the index generates an unambiguous ranking of two distributions for $\alpha=\alpha_1$, then it is capable of ranking the two distributions in the same direction for $\alpha=\alpha_2$ if $\alpha_1\leq\alpha_2$. If the dominance relation holds for some member of the Foster et al. (1984) class, say for $\alpha=\alpha_1$, then it is not necessary to check dominance for higher values of α. The direction of dominance will not change. This means that the P_{FGT} orderings are nested. Thus, if one distribution is regarded as less poor than another by the headcount ratio, then the same will be true for the poverty gap ratio as well. While the nested characteristic of the Foster-Shorrocks result requires that α should be an integer, Tungodden (2005) extended this to the case where α can be an arbitrary nonnegative real number. In view of equivalence between stochastic dominance and welfare dominance, the P_{FGT} orderings enable us to provide welfare interpretation of the P_{FGT} family. Assuming that the poverty line is bounded above, Foster and Shorrocks (1988b) derived results analogous to Theorem 2.3.

Foster and Jin (1998) developed poverty-line ordering of distributions using indices that are based on utility gaps. Formally, a utility gap-based poverty index is defined as

$$P_{\mathrm{U}}(x,z) = \frac{\tilde{a}(z)}{n}\sum_{i=1}^{n}(U(z)-U(x_i^*)), \tag{2.31}$$

where U is the identical individual utility function and $\tilde{a}(z) > 0$ is a normalization coefficient. Thus, P_{U} aggregates the utility gaps of the poor from the poverty line. This general Dalton type index contains many well-known indices as special cases. For instance, if $U(v) = \log v, \tilde{a}(z) = 1$, then P_{U} becomes the Watts (1968) index. On the other hand, for $U(v) = \log v, \tilde{a}(z) = 1/\log z$, P_{U} coincides with the Hagenaars (1987) index. Finally, the Chakravarty (1983c) index drops out as a member of P_{U} if we assume that $U(v) = v^e$ and $\tilde{a}(z) = z^e$. For any $x \in D_+^n$, we denote the utility distribution of x by $U^x = (U(x_1), U(x_2), \ldots, U(x_n))$.

The following theorem of Foster and Jin (1998) provides a characterization of the utility gap -based indices in terms of the poverty-line ordering.

Theorem 2.4. *Let U be continuous and increasing, and $x, y \in D_+^n$ be arbitrary. Then the following conditions are equivalent:*

(a) $P_{\mathrm{U}}(x,z) \leq P_{\mathrm{U}}(y,z)$ for all $z \in (0,\infty)$ with $<$ for some z.
(b) $U^x \geq_{\mathrm{GL}} U^y$, that is, U^x generalized Lorenz dominates U^y.

Theorem 2.4 says that if the distribution y poverty line dominates the distribution x, then the utility distribution of x will be generalized Lorenz superior to the utility distribution of y. The converse is true as well. Since the utility gap-based Dalton type poverty index and the generalized Lorenz curve remain invariant under replications of the population, we can use Theorem 2.4 for cross-population poverty comparisons of income distributions.

An alternative direction of research on poverty ranking involves identification of a family of poverty indices that will rank different distributions unambiguously when the poverty line is given. This is referred to as poverty-measure ordering. Given the poverty line, we need to specify a set of axioms such that all the poverty indices fulfilling these axioms will rank the distributions in the same direction. That is, we need to check whether, for a given poverty line, it is possible to compare two distributions unanimously by all members belonging to the class of indices satisfying these axioms.

Atkinson (1987) developed conditions on poverty-measure ordering for subgroup decomposable indices with a common arbitrary poverty line. He considered the range $[z_-, z_+]$ for the poverty lines, where, as before, $z_- > 0$ and $z_+ < \infty$ are the minimum and maximum poverty lines. The poverty line arbitrarily varies within this range. Instead of considering a single poverty index, he focused attention on a given class of poverty indices. The two theorems presented below summarize the poverty-measure orderings developed by Atkinson (1987).

For presenting the theorems, we follow Zheng (2001) and consider poverty indices that are additively separable, so that

$$P(F,z) = \int_0^\infty \zeta(v,z)\mathrm{d}F(v), \tag{2.32}$$

where the individual poverty function $\zeta(v,z)$ is zero if $v \geq z$, it is positive otherwise.

Theorem 2.5. *Consider two income distribution functions F and G defined on the same domain and assume the weak definition of the poor. Then (i) the necessary and sufficient condition for $P(F,z) < P(G,z)$ to hold for all individual poverty functions that are continuous on $[0,\infty)$ and decreasing in the incomes of the poor, and a given poverty line $z \in [z_-, z_+]$ is that F first-order stochastic dominates G over $[0,z]$, and (ii) the necessary and sufficient condition for $P(F,z) < P(G,z)$ to hold for all individual poverty functions that are continuous on $[0,\infty)$ and decreasing in the incomes of the poor, and all poverty lines $z \in [z_-, z_+]$ is that F first-order stochastic dominates G over $[0, z_-]$ and F weakly first-order stochastic dominates G over $[z_-, z_+]$.*

Theorem 2.6. *Consider two income distribution functions F and G defined on the same domain and assume the weak definition of the poor. Then the necessary and sufficient condition for $P(F,z) < P(G,z)$ to hold for all individual poverty functions that are continuous on $[0,\infty)$, decreasing and strictly convex in the incomes of the poor, and a given poverty line $z \in [z_-, z_+]$ is that F second-order stochastic dominates G over $[0,z]$, (ii) the necessary and sufficient condition for $P(F,z) < P(G,z)$ to hold for all individual poverty functions that are continuous on $[0,\infty)$, decreasing and strictly convex in the incomes of the poor, and all poverty lines $z \in [z_-, z_+]$ is that F second-order stochastic dominates G over $[0, z_-]$ and F weakly second-order stochastic dominates G over $[z_-, z_+]$.*

These two theorems have very strong implications. If a dominance relation holds, then no individual poverty index with the relevant properties needs to be consulted in ordering the distributions under considerations. The dominance conditions are quite easy to implement and our discussion in Chap. 1 shows that they have very nice welfare theoretic interpretations (*see also* Howes, 1993). Zheng (1999) extended Theorem 2.6 to the case of third-order dominance. Clearly, the poverty index in this case is required to be diminishing transfer sensitive.

Spencer and Fisher (1992), Jenkins and Lambert (1997, 1998a,b), and Shorrocks (1998) derived conditions for ranking one distribution as having more poverty than another in terms of dominance condition that relies on the poverty gap profile. The aggregate normalized poverty gap of the cumulative population proportion i/n for the distribution x is $\sum_{j=1}^{i}(z - x_j^*)/nz$, where $1 \leq i \leq n$. Given the poverty line z, this is the ordinate $\mathrm{PG}(x,z,i/n)$ of the poverty gap profile at the cumulative population proportion i/n. The poverty gap profile $\mathrm{PG}(x,z,t)$ of x, where $0 \leq t \leq 1$, is completed by setting $\mathrm{PG}(x,z,0) = 0$ and by defining

$$\mathrm{PG}\left(x,z,\frac{i+\tau}{n}\right) = (1-\tau)\mathrm{PG}\left(\mathrm{x},\mathrm{z},\frac{\mathrm{i}}{\mathrm{n}}\right) + \tau\mathrm{PG}\left(\mathrm{x},\mathrm{z},\frac{\mathrm{i}+1}{\mathrm{n}}\right), \tag{2.33}$$

for all $0 \leq \tau \leq 1$ and $1 \leq i \leq (n-1)$. This curve is nondecreasing and concave (*see* Fig. 2.1). The 45° line represents the situation where all the persons have zero

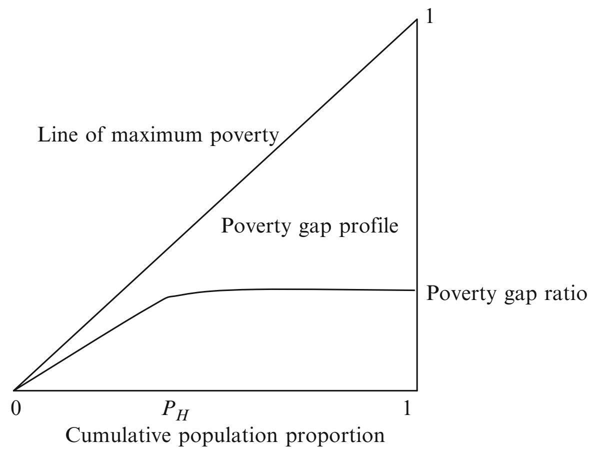

Fig. 2.1 Poverty gap profile

income. This is the line of maximum poverty. The vertical distance between the 45° line and the poverty gap profile is the generalized Lorenz curve of the normalized distribution x^*/z. Shorrocks (1998) has shown that P_{Sh}, the continuous version of the Sen index P_{S}, can be expressed as the area under the poverty gap profile expressed as a fraction of the area under the 45° line. The headcount ratio, which represents the poverty intensity, is the population proportion at which the curve becomes flat (given that there is no person with exactly the poverty-line income). The poverty gap ratio, the relative gap between the poverty line and the mean of the censored income distribution, is the maximum height of the curve. The curvature of the curve between the origin and the headcount ratio is an indicator of inequality among the poor because in this position the curve is not flat. Since the curve depicts these three important components of poverty, Jenkins and Lambert (1997, 1998a,b) renamed the curve as the TIP (three Is of poverty) curve and analyzed it in greater details. We may also refer to the curve as the illfare curve of the society. Spencer and Fisher (1992) called the absolute poverty gap profile $z\mathrm{PG}(x,z,t)$ absolute rotated Lorenz curve of the censored income distribution x^*.

Given two income distributions $x,y \in D^n$, we say that y poverty gap profile dominates x, which we denote by $y \geq_{\mathrm{PG}} x$, if for a given poverty line z, $\mathrm{PG}(x,z,t) \leq \mathrm{PG}(\mathrm{y,z,t})$ for all $0 \leq t \leq 1$ with $<$ for some t. The following theorem of Spencer and Fisher (1992), Jenkins and Lambert (1997, 1998a,b), and Shorrocks (1998) gives an implication of poverty gap profile dominance in terms of illfare indices.

Theorem 2.7. *Let $x,y \in D^n$ be arbitrary. The poverty line $z \in [z_-, z_+]$ is assumed to be given. Then the following conditions are equivalent:*

(i) $y \geq_{\mathrm{PG}} x$.
(ii) $H((1-x_1^*/z),(1-x_2^*/z),\ldots,(1-x_n^*/z)) < H((1-y_1^*/z),(1-y_2^*/z),\ldots,(1-y_n^*/z))$ *for all illfare indices* $H:[0,1]^n \to R^1$ *that are increasing and strictly S-convex in the relative poverty gaps* $(1-x_i^*/z)$.

The theorem shows an interesting application of the illfare functions we have considered in Sect. 2.3. Clearly, it can be regarded as an extension of the equivalence between conditions (iv) and (x) of Theorem 2.1 to the censored income distributions (*see also* Foster and Sen, 1997, pp. 192–193).[7] This equivalent condition can be used to check poverty ranking of distributions for a large class of poverty indices. Spencer and Fisher (1992) referred to the illfare functions as the aggregate hardship functions. Jenkins and Lambert (1998b) also demonstrated equivalence between censored generalized Lorenz dominance and TIP curve dominance. Finally, it may be worthwhile to mention that Atkinson (1992) and Jenkins and Lambert (1993) studied poverty-measure orderings when the poverty line is adjusted for differences in family composition. Contributions along this line have also been made by Chambaz and Maurin (1998) and Zoli and Lambert (2005).

As Zheng (2000b) noted all the poverty indices that are covered by Theorem 2.6 can be expressed in terms of their distribution-sensitivity measures. He considered the poverty-measure ordering based on the class of minimum distribution-sensitive indices. The reason for concentrating on such a class is that minimum distribution-sensitive indices may be able to increase the completeness or power of poverty ordering beyond second-degree dominance (*see also* Zheng, 1999). He restricted attention on the set of all subgroup decomposable focused poverty indices $\mathrm{SP}(\hat{\zeta}(v,z)) = \{P(x,z) = (1/n)\sum_{i=1}^{n} \zeta(x_i,z) | \mathrm{DS}_{\zeta}(v,z) \geq \mathrm{DS}_{\hat{\zeta}}(v,z)$ for all $v \in [0,z)\}$, where, as before, the individual poverty function $\zeta(v,z) > 0$ on $[0,z)$, $\zeta(v,z) = 0$ for $v \geq z$. Further, ζ is decreasing, strictly convex in the incomes of poor and continuous on $[0,\infty)$. If $\hat{\zeta}(v,z) = (z-v)$, then $\mathrm{SP}(\hat{\zeta}(v,z))$ and the family of distribution-sensitive poverty indices considered by Atkinson (1987) coincide. Zheng (2000b) then showed that for two income distributions x and y over a given population and a given poverty line z, y has at least as high poverty level as x for all poverty indices belonging to this particular set if and only if $\sum_{i=1}^{j} \hat{\zeta}(y_i,z) \geq \sum_{i=1}^{j} \hat{\zeta}(x_i,z)$ for all $1 \leq j \leq n$. Clearly, this result can be regarded as a generalization of the Spencer and Fisher (1992), Jenkins and Lambert (1997, 1998a,b), and Shorrocks (1998) theorem because in the latter, we set $\hat{\zeta}(x_i,z) = (1 - x_i^*/z)$. He also showed how to compare poverty orderings with different $\hat{\zeta}$ functions. It is shown that the necessary and sufficient condition for the set of distributions ordered by $\mathrm{SP}(\hat{\zeta}_1(\mathrm{v,z}))$ be included in the set ordered by $\mathrm{SP}(\hat{\zeta}_2(\mathrm{v,z}))$ is that $\hat{\zeta}_2$ is a convex function of $\hat{\zeta}_1$. An implication of this result is that if $\hat{\zeta}_2$ is an increasing and convex function of $\hat{\zeta}_1$, then the poverty ordering by $\mathrm{SP}(\hat{\zeta}_2(v,z))$ will have at least as much power as that by $\mathrm{SP}(\hat{\zeta}_1(v,z))$. Recall that for the second Clark et al. (1981), the Watts (1968), the Chakravarty (1983c), the Foster et al. (1984), and the constant distribution-sensitivity indices, the individual poverty functions are given, respectively, by $(1-(v/z)^{\theta}/\theta$, where $\theta < 1$; $\log(z/v)$; $(1-(v/z)^{\theta})$, where $0 < \theta < 1$; $(1-(v/z))^{\alpha}$, where $\alpha > 1$; and $\exp(\tilde{\omega}(z-v)) - 1$ with $\tilde{\omega} > 0$. It then turns

[7] In fact, Shorrocks (1998) referred to the poverty gap profile as deprivation profile. He looked at the issue from a quite general perspective and used the curve for ranking societies in terms of bads such as unemployment duration and wage discrimination, in addition to poverty. See also Xu and Osberg (1998).

out that the poverty ordering by SP($\hat{\zeta}(v,z)$) with $\hat{\zeta}$ being anyone of these indices has more power than the Atkinson second-degree stochastic dominance criterion. (This is because for a given $z > 0$, the individual poverty functions $(1-(v/z)^{\theta})/\theta$, where $\theta < 1$, and $(\exp(\tilde{\omega}(z-v))-1)$ are increasing and strictly convex functions of $(z-v)$.) The poverty ordering associated with the Clark et al. index for $\theta < 0$ has more power than that with the Watts index, which in turn has more power than that with the Chakravarty index. The power of the poverty ordering induced by the Foster et al. index increases with the value of α. The power of poverty ordering with the constant distribution-sensitivity index is not comparable with that of any other index.

2.6 Unit Consistent Poverty Indices

The unit consistency axiom, introduced by Zheng (2007c), demands that the poverty rankings of income distributions remain unaltered if all the incomes and the poverty line are expressed in different units of measurement. To illustrate this, suppose that when incomes and poverty lines are expressed in euros, region I is regarded as poorer than region II. It is reasonable to expect that the regional poverty ranking does not change if incomes and poverty lines are expressed in terms of one thousand euros. A unit consistent poverty index will achieve this objective. Thus, unit consistency ensures that measurement of incomes and poverty lines in different units will not lead to contradictory conclusions.

A poverty index $P : D_+ \times [z_-, z_+]$ is said to be unit consistent if for $x, y \in D_+$ and two given poverty lines $z_1, z_2 \in [z_-, z_+]$, $P(x,z_1) < P(y,z_2)$ implies $P(cx,cz_1) < P(cy,cz_2)$, where $c > 0$ is any scalar such that $cz_1, cz_2 \in [z_-, z_+]$.

Since for a relative poverty index P, $P(cx,cz) = P(x,z)$, all relative poverty indices are unit consistent. But the converse is not true. However, people may not always like to view income shortfalls from the poverty line in relative terms. Sometimes they may like to look at poverty in terms of absolute shortfalls and, as argued earlier, in this case, we concentrate on absolute indices. Therefore, it might be worthwhile to look for absolute indices that satisfy unit consistency.

In order to identify unit consistent absolute poverty indices, we consider some specific type of indices. Following Zheng (2007c), we say that a poverty index P is semi-individualistic if for any $(x,z) \in D_+ \times [z_-, z_+]$, $P(x,z) = (1/n)\sum_{i=1}^{q} \zeta(x_i,n,q,i,z)$, where the nonnegative semi-individualistic poverty function $\zeta(x_i,n,q,i,z)$ is invertible, twice differentiable, decreasing in $x_i < z$, also $\lim_{v \to z-} \zeta(v,n,i,z) = 0$ and $\zeta(v,n,q,i,z) = 0$ if $v \geq z$. Because of dependence of ζ on n,q, and i in addition to v and z, it is referred to as semi-individualistic, instead of individualistic in which case dependence is assumed only on (v,z). Thus, a semi-individualistic poverty function for person i does not change in response to a change in another person's income as long as i's rank in the distribution and the number of poor persons remain unaltered.

Zheng (2007c) showed that an absolute semi-individualistic poverty index is unit consistent if and only if it is of the form

$$P_{\mathrm{ZA}}(x,z) = \frac{1}{n}\sum_{i=1}^{q} \tilde{f}(n,q,i)(z-x_i)^{\bar{f}(n,q)}, \tag{2.34}$$

for some positive functions $\tilde{f}(n,q,i)$ and $\bar{f}(n,q)$.

If $\bar{f}(n,q)=1$ and $\tilde{f}(n,q,i)=2(q+1-i)/(q+1)$, then P_{ZA} becomes the absolute form of the Sen (1976a) index, which we have discussed in Sect. 2.3.5. On the other hand, if $\tilde{f}(n,q,i)=1$ and $c(n,q)=\alpha$, then P_{ZA} coincides with the absolute version of the Foster et al. (1984) index. Finally, for $\bar{f}(n,q)=1$ and $\tilde{f}(n,q,i)=(2(n-i)+1)/n$, P_{ZA} will be the absolute variant of P_{Sh}, the continuous form of the Sen index characterized by Shorrocks (1995), which is a member of the general Chakravarty (1983a) index.

Next, we consider poverty indices within a more specified framework to study implications of unit consistency. Recall that the Dalton (1920) index of inequality is based on utility ratio. Chakravarty (1983c) and Hagenaars (1987) applied the Dalton index to the measurement of poverty. For all $x \in D_+$ and $z \in [z_-, z_+]$, a generalized Dalton-Chakravarty-Hagenaars poverty index $P_{\mathrm{DCH}}(x,z)$ is defined as

$$P_{\mathrm{DCH}}(x,z) = \frac{1}{n}\sum_{i=1}^{q} \frac{(\tilde{\phi}(z)-\tilde{\phi}(x_i))}{\tilde{\varphi}(z)}, \tag{2.35}$$

where the real valued functions $\tilde{\phi}$ and $\tilde{\varphi}$ are assumed to be twice differentiable. If we assume that $\tilde{\phi}$ and $\tilde{\varphi}$ are identical then P_{DCH} reduces to the general form of the Hagenaars index.

By construction, P_{DCH} is subgroup decomposable and semi-individualistic. The set of all poverty indices defined by (2.35) is a proper subset of the set of all subgroup decomposable indices and the latter set is a proper subset of the set of semi-individualistic indices. Therefore, the framework that defines P_{DCH} is narrower than the framework for semi-individualistic indices.

It may be noted that for all subgroup decomposable unit consistent poverty indices, the individual poverty function will be homogeneous of some arbitrary degree in its arguments (Zheng, 2007c). However, this does not identify any specific form of the index. If we restrict our attention to P_{DCH} in (2.35), then it is possible to isolate some functional forms uniquely. Zheng (2007c) showed that a generalized Dalton-Chakravarty-Hagenaars poverty index $P_{\mathrm{DCH}}(x,z)$ satisfies unit consistency if and only if it is of the form

$$P_{\mathrm{DCH}}(x,z) = \begin{cases} \frac{1}{r_1}\frac{1}{nz^{r_2}}\sum_{i=1}^{q}[z^{r_1}-x_i^{r_1}], r_1 \neq 0, \\ \frac{1}{nz^{r_2}}\sum_{i=1}^{q}\log\left(\frac{z}{x_i}\right), r_1 = 0, \end{cases} \tag{2.36}$$

where r_1 and r_2 are constants.

The logarithmic member of this family is a parametric extension of the Watts (1968) index, while the other member is a two parameter generalization of the second Clark et al. (1981) and the Chakravarty (1983c) indices. The entire family may be regarded as the poverty counterpart to the generalized entropy family of inequality indices. Note that the family contains the absolute poverty gap ratio $(\sum_{i=1}^{q}(z-x_i))/n$ as a member. In fact, the only member of the P_{DCH} in (2.36) that satisfies absolute scale invariance and unit consistency simultaneously is a positive multiple of this index.

2.7 Measuring Chronic Poverty

Our analysis so far has ignored one important aspect of poverty, its duration. Looking at poverty trends using a particular index of poverty does not tell us whether individuals are persistently poor or they have been able to move out of poverty. The duration aspect of poverty deserves attention for several reasons. For instance, longer duration in poverty may lead to worse health status for individuals, particularly, for children and aged persons. Long exposure to poverty has quite important implications for future strategies of individuals or households.

A distinction has been made between transitory and chronic poverty in the context of intertemporal poverty measurement. The former is a consequence of a short-term fall in the level of living of an individual, while the latter arises from low long-term well-being of the person. For instance, a person in the short-term transition period between two jobs may be in poverty over the unemployment duration period. This can be referred to as transitory poverty. In contrast, chronic poverty deals with the prolonged concept of poverty.[8]

Identification of chronically poor persons and aggregation of their characteristics require panel data on income. Broadly, two different approaches for identifying chronically poor persons have been suggested in the literature (*see* Yaqub, 2000a,b; McKay and Lawson, 2003). The first is the components approach that separates the permanent component of income from its transitory component and identifies a person as chronically poor if his permanent income falls below the poverty line (*see* Jalan and Ravallion, 1998; Calvo and Dercon, 2007). This approach is not sensitive to the time for which a person remains in poverty (Foster, 2008).

The second approach, which is referred to as the spells approach, identifies a person as chronically poor in terms of the number of spells of poverty he experiences. That is, this approach depends on a duration threshold as well as an income threshold. More precisely, a person is identified as chronically poor by the spells approach if he is in income poverty for a fraction of the total duration not less than the duration threshold. For instance, if we have data for 8 years and the duration threshold

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[8] For further discussions along this line, see Rodgers and Rodgers (2006), Ravallion (1988), Jalan and Ravallion (1998, 2000), Hulme and Shepherd (2003a,b), McKay and Lawson (2003), Clark and Hulme (2005), Hulme and McKay (2005), Duclos et al. (2006), and Kurosaki (2006).

is .5, then we say that a person is chronically poor if his incomes are not above the poverty line for at least 4 years.[9]

Axiomatic approaches to the measurement of chronic poverty have been suggested, among others, by Hoy and Zheng (2006), Calvo and Dercon (2007), Bossert et al. (2008), and Foster (2008). In this section, we follow the axioms proposed by Foster (2008) and consider the subgroup decomposable chronic poverty index.

Suppose there are observations on incomes of a set $\{1,2,..,n\}$ of individuals over the periods $\{1,2,..,K\}$. Let $x_{ij} \geq 0$ be the income of person $i \in \{1,2,..,n\}$ in period $j \in \{1,2,..,K\}$. The row vector $x_{i.}$ represents the incomes of person i in different periods. The $n \times K$ matrix X shows the incomes of different persons in K periods. Thus, the jth column of X shows the income distribution among n persons in period j. We denote the duration threshold by $z_K > 0$ and, as before, the income threshold, the poverty line, is denoted by $z > 0$. It is assumed that the incomes have been properly deflated to take into account the intertemporal variations so that the same poverty line can be used to identify the poor in each period. Person i becomes chronically poor if he remains in poverty for $K_i \geq Kz_K$ periods. That is, for person i, the inequality $x_{ij} \leq z$ is satisfied for K_i values of j, where $(K_i/K) \geq z_K$. Let $CP(z,z_K)$ be the set of chronically poor persons.

To illustrate the issue numerically, suppose that $K = 4, n = 4, z_K = .6$, and $z = 9$. The income distributions of the four persons in different periods are given, respectively, by $x_{1.} = (7,4,8,15), x_{2.} = (3,8,3,4), x_{3.} = (9,10,3,20)$, and $x_{4.} = (4,25,5,6)$. Note that person 1 has incomes below the threshold z in three periods and hence $K_1 = 3$. Likewise $K_2 = 4$ and $K_4 = 3$. However, person 3 is not chronically poor because $K_3 = 2$. The headcount ratio, $P_{\mathrm{H}}(X,z,z_K)$, for this example is then given by $3/4$ since there are three persons who are chronically poor.[10]

Now, suppose we reduce person 1's income in period 4 to 8. This increases K_1 to 4. Clearly, the headcount ratio remains unchanged although we have increased the time duration of poverty for person 1. In order to make some adjusted form of the headcount ratio sensitive to time monotonicity, let us consider another counting index, the average of the fractional durations K_i/K of the chronically poor. We denote this index by $P_{\mathrm{AD}}(X,z,z_K)$. It is the fraction of the total time period for which the average chronically poor person remains in poverty. For the original example we have considered, this index becomes $2.5/3$. If we multiply the two indices, we get the duration adjusted headcount ratio $P_{\mathrm{HAD}}(X,z,z_K) = P_{\mathrm{H}}(X,z,z_K)P_{\mathrm{AD}}(X,z,z_K)$ that becomes sensitive to the changes in the duration of a person in poverty (Foster, 2008). For our example, the value of this adjusted index is $10/16$. This value is the total number of periods, 10, for which all the chronically poor persons experience poverty as a fraction of the total number of periods $16 (= Kn)$ for the entire population.

[9] See Bane and Ellwood (1986), Gaiha (1989, 1992), Gaiha and Deolikar (1993), Morduch (1994), Baluch and Masset (2003), and Carter and Barrett (2004) for discussion on duration issues.

[10] For applications of the headcount/headcount ratio, see The Chronic Poverty Report (2004–2005), Gaiha and Deolikar (1993), and Mehta and Shah (2003).

Since the h function in (2.19) captures the depth of poverty in an analytical way, taking cue from the above discussion, we can say that the subgroup decomposable chronic poverty index is given by

$$P_{\mathrm{CP}}(X,z,z_K) = \frac{1}{Kn} \sum_{i \in \mathrm{CP}(z,z_K)} \sum_{j=1}^{K} h\left(\frac{x_{ij}^*}{z}\right), \tag{2.37}$$

where the real valued function h is the same as in (2.19). This index is the sum of transformed censored income shortfalls $h(x_{ij}^*/z)$ of the chronically poor persons divided by the maximum value this sum can take. If we assume that $z_K = 0$, then P_{CP} takes into account all the poverty spells of all persons. In contrast, for a positive given value of z_K, it considers the spells of only chronically poor persons, as determined by z and z_K. Consequently, the difference $P_{\mathrm{CP}}(X,z,z_K) - P_{\mathrm{CP}}(X,z,0)$ is based on spells of those who are not chronically poor. Therefore, this difference is an indicator of transitory poverty. Thus, the subgroup decomposable transitory poverty index is given by

$$P_{\mathrm{TP}}(X,z,z_K) = P_{\mathrm{CP}}(X,z,z_K) - P_{\mathrm{CP}}(X,z,0). \tag{2.38}$$

Clearly, we can have chronic poverty variants of the Watts (1968), Chakravarty (1983c), and Foster et al. (1984) indices for appropriate specifications of the functions h. Thus, the functional form for the Watts chronic poverty index will be $1/Kn \sum_{i \in \mathrm{CP}(z,z_K)} \sum_{j=1}^{K} \log(z/x_{ij}^*)$. The corresponding functional forms for the Chakravarty and the Foster et al. indices are given, respectively, by $1/Kn \sum_{i \in \mathrm{CP}(z,z_K)} \sum_{j=1}^{K} (1-(x_{ij}^*/z)^e)$ and $1/Kn \sum_{i \in \mathrm{CP}(z,z_K)} \sum_{j=1}^{K} (1-x_{ij}^*/z)^{\alpha}$. This Foster et al. form was suggested and analyzed by Foster (2008). Each of these indices can be used to generate the corresponding transitory poverty index.

The general index P_{CP}, in addition to satisfying the standard income-based axioms for a specific time period, satisfies Time Anonymity, Time Focus, Time Monotonicity, and Chronic Poverty Transfer Axioms introduced by Foster (2008). The first axiom says that if there is a permutation of incomes across time, poverty does not change. That is, if $Y = X\Pi$ for a $K \times K$ permutation matrix Π, then $P(X,z,z_K) = P(Y,z,z_K)$. According to the second axiom, an increase in the nonpoor income of a chronically poor person does not alter the level of poverty. That is, if there is some period j' and person i' who is chronically poor in Y and if $x_{ij} > y_{ij} > z$ for $(i,j) = (i',j')$ and $x_{ij} = y_{ij}$ for all $(i,j) \neq (i',j')$, then $P(X,z,z_K) = P(Y,z,z_K)$. The third axiom demands that if a chronically poor person is out of poverty in a spell and if because of reduction in income, the person becomes poor in that spell, then poverty should go up. Technically, if there is some period j' and a person i' who is chronically poor in X and $y_{ij} \leq z < x_{ij}$ for $(i,j) = (i',j')$ and $x_{ij} = y_{ij}$ for all $(i,j) \neq (i',j')$, then $P(X,z,z_K) < P(Y,z,z_K)$. Finally, let Y_{CP}^* be the censored submatrix of Y representing the y_{ij} values of the chronically poor persons, that is, the (i,j)th entry of Y_{CP}^* is $\min\{z, y_{ij}\}$, where person i is chronically poor and let $X_{\mathrm{CP}}^* = AY_{\mathrm{CP}}^*$ for some nonpermutation bistochastic matrix A of order q, the number of chronically poor persons. Then the fourth axiom says that $P(X_{\mathrm{CP}}^*, z, z_K) \leq P(Y_{\mathrm{CP}}^*, z, z_K)$. In words, if there is a redistribution of income among the chronically poor persons, then chronic poverty does not increase. [This formulation, which is based on Kolm (1977), Tsui (2002), and Bourguignon and Chakravarty (2003), is slightly different from that of Foster (2008).] The general index also fulfills the Normalization and Nondecreasingness in Duration Threshold Axioms. According to the first of these two axioms, $P(X,z,z_K)=0$ if the set $\mathrm{CP}(z,z_K)$ is empty. That, is, the value of the chronic poverty index is zero, if nobody is chronically poor in the society. The second axiom says that chronic poverty does not decrease if there is an increase in the duration threshold.

Let us now illustrate the index $1/Kn\sum_{i\in\mathrm{CP(z,z_K)}}\sum_{j=1}^{K}(1-(x_{ij}^{*}/z)^{e})$ using the numerical example considered above when $e=0.5$. Since the calculation is based on censored incomes, we first determine the censored distributions corresponding to x_i's. These distributions are: $x_{1\cdot}^{*}=(7,4,8,9), x_{2.}^{*}=(3,8,3,4), x_{3.}^{*}=(9,9,3,9)$, and $x_{4.}^{*}=(4,9,5,6)$. Let ρ_i be the intertemporal poverty profile of person i. That is, the jth entry of ρ_i is $\rho_{ij}=(1-(x_{ij}^{*}/z)^{0.5})$, the level of person i's poverty in period j. For instance, $\rho_{11}=(1-(x_{11}^{*}/z)^{0.5})=(1-(7/9)^{0.5})=0.118$. The ρ-vectors for the chronically poor persons 1, 2, and 4 become $\rho_1=(0.118,0.333,0.057,0), \rho_2=(0.423,0.057,0.423,0.333)$, and $\rho_4=(0.333,0,0.255,0.183)$, respectively. Now, the level of chronic poverty for this example is calculated by taking the sum of these 12 ρ_{ij} values, which is 2.515, and then dividing the sum by 16. Thus, the required poverty level is $(2.515/16)=0.157$. To calculate the transitory poverty, we also have to consider $\rho_3=(0,0,0.423,0)$, the poverty profile of person 3 who is not chronically poor. The total poverty level for the example will be $((2.515+0.423)/16)=0.184$. Hence, the level of transitory poverty is $(0.184-0.157)=0.027$.

Bossert et al. (2008) argued that the length of poverty spells is an important component of intertemporal poverty analysis. For instance, consider the following two per period individual poverty profiles: $\rho_1=(1/8,0,1/3,2/5)$ and $\rho_2=(1/8,1/3,2/5,0)$. The third entry in the vector ρ_1 gives the level of poverty experienced by person 1 in period 3, where the poverty level is calculated using a given poverty index. Similarly, other entries can be explained. They argued that since in the first profile the person experiences a break from poverty rather than being in poverty for three consecutive periods, the first profile should depict less poverty than the second one. Further, their formulation does not rely on the assumption that the poverty line is fixed over time. They also developed an axiomatic characterization of an intertemporal poverty index that takes this into account. Note that the general poverty index in (2.37) regards the two profiles as identically poor. The functional form of the Bossert et al. (2008) index is given by

$$P_{\mathrm{BCD}}(\rho_1,\rho_2,..,\rho_n)=\frac{1}{nK}\sum_{i=1}^{n}\sum_{j=1}^{K}r^{d^{j}(\rho_i)-1}\rho_{ij}, \tag{2.39}$$

where $d^{j}(\rho_i)$ is the maximum number of consecutive periods including j with positive (zero) per period poverty values in ρ_i and $r\geq 1$ is a parameter. The value of the poverty index P_{BCD} increases as r increases.

To illustrate the formula, let $r=2$. For example, we have considered here, $d^1(\rho_1)=d^2(\rho_1)=1, d^3(\rho_1)=d^4(\rho_1)=2$ and $d^1(\rho_2)=d^2(\rho_2)=d^3(\rho_2)=3$,

$d^4(\rho_2) = 1$. For person 1, the length of the first poverty spell is one and hence $d^1(\rho_1) = 1$. This is followed by a nonpoverty spell of length one, which gives $d^2(\rho_1) = 1$. For the next two periods, he is in poverty and hence $d^3(\rho_1) = d^4(\rho_1) = 2$. A similar explanation holds for $d^j(\rho_2)$ values. It is easy to see that the individual poverty function $\sum_{j=1}^{K} r^{d^j(\rho_i)-1}\rho_{ij}/K$ is higher for person 2. The value of the aggregate poverty index in (2.39) turns out to be 0.628.

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