

*I bring here all: what have I lived thru,
 And that what keeps my soul alive,
 My rectitude and aspirations,
 And what have seen my own eyes.*
 – Boris Pasternak, *The Waves*

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