In Chapter 1 we began our discussion of macroeconomic theory with a view of nominal wages and prices as fully flexible. This approach ensures that markets are always in equilibrium, in the sense that there is continual balance between the quantities demanded and the quantities supplied. The classical model was the dominant macroeconomic theory until the Great Depression in the 1930s. The prolonged unemployment, however, in the United Kingdom and the United States during the 1930s prompted John Maynard Keynes to significantly depart from the classical assumption of perfectly flexible prices and develop models based on the assumption that there are constraints on the flexibility of some prices.

The crucial assumption in the Keynesian models is that some prices are sticky — i.e., do not adjust promptly to ensure continual balance between the quantities supplied and demanded. Hence, unlike the classical model, some markets do not always clear and output and employment typically end up below the optimal amounts. Although Keynes’s analysis and some subsequent treatments — see, for example, Don Patinkin (1965, Chapter 13) and Barro and Herschel Grossman (1976) — focused
on sticky money wages, the price level is sometimes assumed to be perfectly flexible (leading to the so-called complete Keynesian model), but is more often treated also as sticky. Here, we follow Sargent (1979, Chapter 2) and develop the complete Keynesian model.

2.1 The Keynesian Consumption Function

The key assumption in Keynesian analysis is that sticky money wages result in excess supply in the labor market. This has important consequences for employment and consumption. For example, producers do not produce more than is demanded, suggesting that employment is restricted to the minimum necessary to produce the given level of output. Also, consumers find that their income is constrained to be less than it would have been in the absence of excess supply in the output market.

The implication of this is that people’s consumption depends on their exogenously given level of real income. Formally, the consumption function now takes the form

\[ C = C(Y - T, R - \pi^e), \]

relating real consumption spending, \( C \), to real disposable income, \( Y - T \) (where \( T \) is defined as real tax collections net of transfers), and to the real rate of interest, \( R - \pi^e \). It is assumed that \( 0 < C_1 < 1 \) and \( C_2 < 0 \), where \( C_1 \) is the marginal propensity to consume out of real disposable income and \( C_2 \) is the interest sensitivity of consumption demand. The above function is known as the Keynesian consumption function.

Another argument that has been put forward in deriving the Keynesian consumption function concerns the loan market. In particular, it is argued that many people are liquidity constrained, in the sense that they would like to borrow at market rates, but face higher borrowing costs, because they have poor collateral. In this case, people will change their consumption almost one-to-one with changes in their income. Hence, this point of view can also explain why consumption is a function of current income — see Barro (1997, Chapter 20) for a more detailed discussion.

2.2 The Complete Keynesian Model

The Keynesian model’s assumption of excess supply in the output market means that employment is determined by labor demand, which defines the short side of the market. Therefore, removing the labor supply
2.2. The Complete Keynesian Model

In order to arrive at the Keynesian model, the following changes must be made in the classical model of Chapter 1:

1. The money wage rate must be exogenous, and
2. Consumption must be a function of real disposable income.

These changes lead to the following system, which constitutes the complete Keynesian model:

\[
\frac{w}{P} = F_L; \quad (2.1)
\]
\[
Y = F(K, L); \quad (2.2)
\]
\[
C = C(Y - T, R - \pi_e); \quad (2.3)
\]
\[
I = I \left( q(K, L, R - \pi_e, \delta) \right); \quad (2.4)
\]
\[
Y = C + I + G; \quad (2.5)
\]
\[
\frac{M}{P} = \Phi(Y, R), \quad (2.6)
\]

where \(w\) denotes the exogenous money wage, determined outside the system. In fact, we treat \(w\) as a predetermined variable, not as one that is strictly exogenous, and we assume that it changes through time — that is, \(d\bar{w}/dt\) can be nonzero. Notice that the other assumptions underlying these equations are all as they were in the classical model. Thus, the Keynesian model consists of the above six equations in the six endogenous variables:

\[L, Y, C, I, R, \text{ and } P.\]

The exogenous variables are:

\[M, G, T, K, \pi_e, \delta, \text{ and } \bar{w},\]

and the parameters of the model are:

\[F_K, F_{KK}, F_L, F_{LL}, F_{KL}, L', C_1, C_2, I',\]
\[q_L, q_K, q_{R-\pi_e}, \Phi_Y, \Phi_R, \text{ and } \delta.\]

Clearly, the Keynesian model utilizes the same theory of aggregate demand as the classical model [i.e., equations (2.3)-(2.6) are the same in the two models] but a different theory of aggregate supply. To investigate the implications of the Keynesian version of the aggregate demand, \(L = L(w/P)\), and the implicit labor-market-clearing condition \((L^D = L^S = L)\), making the money wage rate exogenous, and making consumption also a function of real disposable income are the essential changes that must be made in the classical model of Chapter 1 in order to arrive at the Keynesian model.
supply function, we totally differentiate the above six equations (assuming that $dK = d\delta = 0$) to obtain the following linear system in the differentials of the six variables:

$$\frac{d\bar{w}}{\bar{w}} - \frac{dP}{P} = \frac{F_{LL}}{F_L}dL;$$

(2.7)

$$dY = F_LdL;$$

(2.8)

$$dC = C_1dY - C_1dT + C_2(dR - d\pi^e);$$

(2.9)

$$dI = I'q_LdL + I'q_{R-\pi^e}(dR - d\pi^e);$$

(2.10)

$$dY = dC + dI + dG;$$

(2.11)

$$\frac{dM}{P} - \frac{M dP}{P} = \Phi_YdY + \Phi_RdR.$$

(2.12)

Notice that this system, unlike the classical system, is not block recursive, in the sense that it is impossible to find an independent subset of equations that determine a subset of variables. That means, of course, that output is not determined solely on the basis of aggregate supply considerations, as it is in the classical model. In other words, the Keynesian model does not dichotomize.

In what follows, we shall generate the basic Keynesian results in the context of three different versions of the Keynesian model. In particular, the Keynesian-cross, the IS-LM, and the AD-AS Keynesian models are developed.

### 2.3 The Keynesian-Cross Model

An extreme version of the Keynesian model is the Keynesian-cross. In addition to assuming that there is perpetual excess supply in the goods market, the Keynesian-cross model also assumes that the nominal interest rate is fixed. This allows it to ignore the money market and focus exclusively on the goods market to determine the level of output, which is demand determined.

We can summarize the main aspects of this simple Keynesian model using equations (2.8), (2.9), (2.10), and (2.11) to obtain the total differential of the reduced form of $Y$ (assuming that $dR = d\pi^e = 0$)

$$\left(1 - C_1 - I'\frac{q_L}{F_L}\right)dY = dG - C_1dT.$$  

(2.13)

Assuming that the marginal propensity to save out of disposable income, $1 - C_1$, exceeds the marginal propensity to invest out of income,
\[ I'q_L/F_L, \text{ the coefficient on } dY \text{ in } (2.13) \text{ is positive.} \]

Then the reduced form partial derivatives of \( Y \) with respect to \( G \) and \( T \) are given by

\[
\frac{\partial Y}{\partial G} = \frac{1}{1 - C_1 - I'q_L/F_L};
\]

\[
\frac{\partial Y}{\partial T} = -\frac{C_1}{1 - C_1 - I'q_L/F_L},
\]

where the expression for \( \frac{\partial Y}{\partial G} \) is the government purchases multiplier — the amount output changes in response to a unit change in government purchases. The expression for \( \frac{\partial Y}{\partial T} \) is the tax multiplier — the amount output changes in response to a unit change in taxes. Notice that if investment does not respond to changes in income, that is if \( I' \) equals zero, the above expressions reduce to the standard simple Keynesian multiplier formulas.

Finally, if \( I' = 0 \), the effect on output of a change in \( G \) matched by an equal change in \( T \) is given by

\[
\frac{\partial Y}{\partial G} \bigg|_{dG=dT} = 1,
\]

which is the so-called balanced budget multiplier.

### 2.4 The IS-LM Model

The Keynesian-cross model shows how to determine the level of output for a given interest rate. The assumption, however, that the interest rate is given means that the analysis is seriously incomplete. Therefore, we now want to go further to determine simultaneously the interest rate and the level of output. To carry this analysis we use John Hick’s (1937) IS-LM curve apparatus. That is, we collapse equations (2.7), (2.8), (2.9), (2.10), (2.11), and (2.12) into a system of two equations in \( dY \) and \( dR \), this being accomplished by eliminating the other endogenous variables by substitution.

---

1. To see that \( I'q_L/F_L \) is the marginal propensity to invest out of income, differentiate the investment schedule partially with respect to \( Y \) to obtain

\[
\frac{\partial I}{\partial Y} = \frac{\partial I}{\partial q} \frac{\partial q}{\partial L} \frac{\partial L}{\partial Y} = I'q_L/F_L.
\]
First we obtain the total differential of the IS curve, the locus of the combinations of $R$ and $Y$ that satisfy (2.5), the aggregate demand-aggregate supply equality. Substituting (2.8), (2.9), and (2.10) into (2.11) and rearranging yields the total differential of the IS curve

$$(1 - C_1 - I'q_L/F_L)\, dY = -C_1dT + dG + (C_2 + I'q_{R-\pi_e})(dR - d\pi_e).$$

(2.14)

The slope of the IS curve in the $R-Y$ plane is thus given by

$$\frac{dR}{dY} = \frac{1 - C_1 - I'q_L/F_L}{C_2 + I'q_{R-\pi_e}},$$

which is negative, since $C_2 + I'q_{R-\pi_e} < 0$ and $1 - C_1$ has been assumed to be greater than $I'q_L/F_L$. Notice that the smaller the government purchases multiplier and the smaller the sensitivity of aggregate demand to the interest rate, $C_2 + I'q_{R-\pi_e}$, the steeper the IS curve.

To determine how the IS curve shifts when the exogenous variables, $T, G,$ and $\pi_e$, change, we can use (2.14) to determine the horizontal shift in the IS curve by evaluating the partial derivatives of $Y$ with respect to each exogenous variable, $dR$ being set equal to zero. Alternatively, we can use (2.14) to determine the vertical shift in the IS curve by evaluating the partial derivatives of $R$ with respect to each exogenous variable, $dY$ being set equal to zero. So we have:

$$\frac{\partial Y}{\partial T} = \frac{-C_1}{1 - C_1 - I'q_L/F_L} < 0;$$

$$\frac{\partial Y}{\partial G} = \frac{1}{1 - C_1 - I'q_L/F_L} > 0;$$

$$\frac{\partial R}{\partial \pi_e} = 1.$$

An increase in government purchases or a decrease in taxes will shift the IS curve out to the right, the extent of the shift depending on the size of the relevant (Keynesian-cross model) multiplier. Also, when the expected inflation rate changes, the IS curve shifts upward by the amount of the increase in $\pi_e$.

The IS curve does not determine either $R$ or $Y$. It only provides the combinations of nominal interest rates and income (output) that
clear the goods market. To determine the equilibrium of the economy, we need another relationship between these two variables, to which we now turn.

By using (2.7) and (2.8) to eliminate \( \frac{dP}{P} \) from (2.12) yields the total differential of the LM curve, a schedule that shows all combinations of interest rates and levels of income that clear the market for money balances

\[
\left( \frac{F_{LL} M}{P} \Phi - \Phi Y \right) dY = -\frac{dM}{P} + \frac{M d\bar{w}}{\bar{w}} + \Phi dR. \tag{2.15}
\]

The slope of the LM curve is

\[
dR \frac{dY}{dY} = \frac{1}{\Phi R} \left( \frac{F_{LL} M}{P^2} \Phi - \Phi Y \right) > 0.
\]

Notice that the smaller the interest sensitivity and the larger the income sensitivity of the demand for money, the steeper the LM curve. In fact, as \( \Phi_R \to 0 \), the LM curve approaches a vertical position while as \( \Phi_R \to -\infty \), as is supposed in the case of the liquidity trap, the LM curve approaches a horizontal position.

To determine how the LM curve shifts when the exogenous variables, \( M \) and \( \bar{w} \) change, we use equation (2.15) to evaluate the partial derivatives of \( R \) with respect to each of the exogenous variables, \( dY \) being set equal to zero. Thus,

\[
\frac{\partial R}{\partial M} = \frac{1}{\Phi R P};
\]

\[
\frac{\partial R}{\partial \bar{w}} = -\frac{M}{\Phi R P \bar{w}}.
\]

The expression \( \partial R/\partial M \) is zero when \( \Phi_R \to -\infty \) and negative when \( \Phi_R > -\infty \). Also, the expression \( \partial R/\partial \bar{w} \) is zero when \( \Phi_R \to -\infty \) and positive as long as \( \Phi_R > -\infty \). Hence, the LM curve shifts down and to the right when the nominal money supply rises or the money wage falls.

We now have all the components of the IS-LM model. Given that the two equations of this model are (2.14) and (2.15) we can solve the system analytically to analyze the (short run) effects of policy changes and other events on national income. Alternatively, using our knowledge of how changes in the various exogenous variables of the model shift
the IS and LM curves, we can make use of a graphical device — see, for example, Barro (1997, Chapter 20).

Substituting (2.15) into (2.14) to eliminate \( dR \) yields the total differential of the reduced form of \( Y \)

\[
HdY = -C_1dT + dG - (C_2 + I'_q R - \pi^e) d\pi^e
\]

\[
+ \frac{C_2 + I'_q R - \pi^e}{\Phi_R} \left( \frac{dM}{P} - \frac{M}{P} \frac{d\bar{w}}{\bar{w}} \right),
\]

where the coefficient on \( dY, H \), is given by

\[
H = 1 - C_1 - I'_q L - \frac{C_2 + I'_q R - \pi^e}{\Phi_R} \left( \frac{F_{LL}}{F_L^2} M - \Phi_Y \right).
\]

Under the assumption that \( 1 - C_1 \) exceeds \( I'_q L / L \), \( H \) is positive and the reduced form partial derivatives of \( Y \) with respect to the exogenous variables of the model are given by

\[
\frac{\partial Y}{\partial T} = -\frac{C_1}{H} \leq 0; \quad \frac{\partial Y}{\partial G} = \frac{1}{H} \geq 0;
\]

\[
\frac{\partial Y}{\partial \pi^e} = -\frac{C_2 + I'_q R - \pi^e}{H} \geq 0; \quad \frac{\partial Y}{\partial M} = \frac{C_2 + I'_q R - \pi^e}{\Phi_R PH} \geq 0;
\]

\[
\frac{\partial Y}{\partial \bar{w}} = -\frac{(C_2 + I'_q R - \pi^e) M}{\Phi_R P \bar{w} H} \leq 0.
\]

Thus, except in limiting cases, increases in \( G, \pi^e, \) and \( M \) and decreases in \( T \) and \( \bar{w} \) will in general increase the level of real income. Therefore, money is not neutral in this model.

Notice that if money demand is insensitive to the interest rate \( (\Phi_R \rightarrow 0 \) and the LM curve is vertical), \( H \rightarrow \infty \) and the effect on output from a disturbance that shifts the IS curve is nil, that is, \( \partial Y/\partial T, \partial Y/\partial G, \) and \( \partial Y/\partial \pi^e \) all approach zero. Under those circumstances, a fiscal expansion raises the interest rate and crowds out interest sensitive private spending. However, any shift in the (vertical) LM curve has a maximal effect on the level of income.

On the other hand, in the liquidity trap \( (\Phi_R \rightarrow -\infty \) and the LM curve is horizontal), monetary policy has no impact on the equilibrium of the economy, since \( \partial Y/\partial M \) and \( \partial Y/\partial \bar{w} \) both approach zero. Fiscal policy, however, has its full multiplier effect on the level of income,
since $\partial Y/\partial T$ and $\partial Y/\partial G$ reduce to the tax multiplier and government purchases multiplier, respectively, of the Keynesian cross model.

Finally, if the interest rate has a negligible effect on aggregate demand ($C_2 = I' = 0$), the IS curve is vertical and changes in the money supply and the money wage have no effect on output, that is, $\partial Y/\partial M = \partial Y/\partial \pi = 0$. On the other hand, if aggregate demand is extremely sensitive to the interest rate, the IS curve is very flat and shifts in the LM have a large effect on output.

### 2.5 The Keynesian AD-AS Model

In the previous section we solved the Keynesian model [equations (2.7), (2.8), (2.9), (2.10), (2.11), and (2.12)] by collapsing it into two equations in a pair of variables, $dR$ and $dY$. We can also solve the same model by collapsing it into two equations in another pair of variables, $dP$ and $dY$, thereby obtaining the Keynesian version of the aggregate demand (AD)-aggregate supply (AS) model.

Solving (2.7) and (2.8) for $dY$ yields the total differential of the aggregate supply function in the $P - Y$ plane

$$dY = \frac{F_L^2}{F_{LL}} \frac{d\bar{w}}{w} - \frac{F_L^2}{F_{LL}} \frac{dP}{P}. \tag{2.16}$$

Since $F_{LL} < 0$, equation (2.16) implies that aggregate supply increases in response to an increase in the price level or (for a given price level) a decline in the money wage. The slope of the aggregate supply schedule is

$$\frac{dP}{dY} = -\frac{PF_{LL}}{F_L^2} > 0.$$  

This expression equals zero if the marginal product of labor is constant — that is, if $F_{LL} = 0$ — as it happens, for example, when capital and labor are combined in fixed proportions.

The total differential of the aggregate demand curve in the $P - Y$ plane, a schedule that, in the present context, represents all those combinations of $P$ and $Y$ that satisfy the demands for goods and assets, comes from equations (2.9), (2.10), (2.11), and (2.12) and is given by

$$\tilde{H}dY = -C_1dT + dG - (C_2 + I'q_{R-\pi_e})d\pi_e + \frac{C_2 + I'q_{R-\pi_e}}{\Phi_R} \frac{dM}{P} - \frac{C_2 + I'q_{R-\pi_e}}{\Phi_R} \frac{M}{P^2} dP, \tag{2.17}$$

where $T$, $G$, and $\pi_e$ denote the tax, government spending, and expected inflation, respectively.
where the coefficient on \(dY\), \(\hat{H}\), is

\[
\hat{H} = 1 - C_1 - I' q_L F_L + \frac{C_2 + I' q_R - \pi^e}{\Phi_R} \Phi_Y.
\]

The slope of the aggregate demand schedule is thus given (substituting back for \(\hat{H}\)) by

\[
\frac{dP}{dY} = - \left[ \left( 1 - C_1 - I' q_L F_L \right) \Phi_R + \left( C_2 + I' q_R - \pi^e \right) \Phi_Y \right] \frac{P^2}{(C_2 + I' q_R - \pi^e) M},
\]

which, under the assumption that \(1 - C_1 > I' q_L / F_L\), is negative.

Notice that the AD curve is flatter the smaller the interest sensitivity of the demand for money, \(\Phi_R\), the smaller the income sensitivity of money demand, \(\Phi_Y\), and the larger the interest sensitivity of aggregate demand, \(C_2 + I' q_R - \pi^e\). Also, the larger the marginal propensity to consume out of disposable income, \(C_1\), (or, equivalently, the smaller the marginal propensity to save out of disposable income, \(1 - C_1\)), and the larger the sensitivity of investment demand to income, that is, the larger \(I' q_L / F_L\), the flatter the AD curve. It is also interesting to note that as \(C_2 + I' q_R - \pi^e \to 0\) or \(\Phi_R \to -\infty\), the aggregate demand curve becomes vertical in the \(P - Y\) plane.

To determine how the aggregate demand curve shifts when the exogenous variables, \(T, G, \pi^e\), and \(M\) change, we use equation (2.17) to evaluate the partial derivatives of \(Y\) with respect to each exogenous variable, \(dP\) being set equal to zero. Letting \(\hat{H}\) stand for the coefficient on \(dY\) in equation (2.17) we obtain

\[
\frac{\partial Y}{\partial T} = - \frac{C_1}{\hat{H}} < 0; \quad \frac{\partial Y}{\partial G} = \frac{1}{\hat{H}} > 0;
\]

\[
\frac{\partial Y}{\partial \pi^e} = - \frac{C_2 + I' q_R - \pi^e}{\hat{H}} \frac{\hat{H}}{\hat{H} \Phi_R P} > 0; \quad \frac{\partial Y}{\partial M} = \frac{C_2 + I' q_R - \pi^e}{\hat{H} \Phi_R P} > 0.
\]

Thus, increases in \(G, \pi^e\), and \(M\) and decreases in \(T\) will in general shift the aggregate demand curve outward and/or upward in the \(P - Y\) plane.

We now have all the components of the Keynesian AD-AS model. It consists of equations (2.16) and (2.17), which we can solve to analyze the effects of policy actions on national income — alternatively, we can make use of a graphical device as, for example, in Barro (1997,
Chapter 20). In particular, substituting (2.16) into (2.17) to eliminate $dP/P$ yields the total differential of the reduced form of $Y$

$$\tilde{H}dY = -C_1dT + dG - (C_2 + I'q_{R-\pi^e})d\pi^e$$

$$+ \frac{C_2 + I'q_{R-\pi^e}}{\Phi_R} \left( \frac{dM}{P} - \frac{M}{P} \frac{d\bar{w}}{\bar{w}} \right),$$

where the coefficient on $dY$, now $\tilde{H}$, is given by

$$\tilde{H} = 1 - C_1 - I'q_L/F_L - \frac{C_2 + I'q_{R-\pi^e}}{\Phi_R} \left( \frac{F_{LL} M}{F_L^2} - \Phi \right).$$

Again, under the assumption that $1 - C_1$ exceeds $I'q_L/F_L$, $\tilde{H}$ is positive and the reduced form partial derivatives of $Y$ with respect to the exogenous variables are:

$$\frac{\partial Y}{\partial T} = -\frac{C_1}{\tilde{H}} \leq 0; \quad \frac{\partial Y}{\partial G} = \frac{1}{\tilde{H}} \geq 0;$$

$$\frac{\partial Y}{\partial \pi^e} = -\frac{C_2 + I'q_{R-\pi^e}}{\tilde{H}} \geq 0; \quad \frac{\partial Y}{\partial M} = \frac{C_2 + I'q_{R-\pi^e}}{\Phi_R P \tilde{H}} \geq 0;$$

$$\frac{\partial Y}{\partial \bar{w}} = -\frac{(C_2 + I'q_{R-\pi^e}) M}{\Phi_R P \bar{w} \tilde{H}} \leq 0.$$  

Hence, this model produces the same qualitative results as the IS-LM model. The reader should also notice that the AD-AS Keynesian model does not dichotomize as the classical model does, as presented in Chapter 1.

### 2.6 Conclusion

In this chapter, we have summarized a great deal of traditional Keynesian macroeconomic theory. We have seen that if there are constraints on the flexibility of some prices, then the financial market and the money demand function play a crucial role in determining the effects not only of monetary policy, but also of fiscal policy. In fact, the relationship between the demand for money and the level of real income and the nominal rate of interest is of crucial importance in these Keynesian models.
In particular, with sticky prices knowledge of the various functions and of the values of their parameters is particularly useful in evaluating the effects of policy actions on the macroeconomy. In the case, for example, of the money demand function, if the interest elasticity of the demand for money balances is high, then fluctuations in the level of income are not likely to be caused by variations in the money supply. If it is low, then exactly the converse is true.

Of course, the theories of macroeconomic behavior that we have so far discussed in Chapters 1 and 2 are static in specification. As Bennett McCallum (1989, p. 77-78) puts it

“one way in which these models are static is that they treat the economy’s capital stock — its collection of productive machines, plants, highways, and so on — as fixed in quantity. As a result of that simplification, the models are not well designed for the analysis of policy actions or other events that would tend to induce substantial changes in the stock of capital within the relevant time frame.”

Although this weakness of the classical and Keynesian models can be remedied, the current fashion is to explore short-run and long-run phenomena in the context of dynamic analyses. Models of this type have displaced the IS-LM and AD-AS frameworks in mainstream macroeconomic theory and dominate current research in almost all areas in economics. In the light of these developments, we now turn to these models.
The Demand for Money
Theoretical and Empirical Approaches
Serletis, A.
2007, XXIV, 382 p., Hardcover