Inventory management is among the most important topics in operations management and operations research. It is perhaps also the earliest, as evidenced by the economic order quantity (EOQ) formula, which dates back to Ford W. Harris (1913). Inventory management is concerned with matching supply with demand. The problem is to find the amount to be produced or purchased in order to maximize the total expected profit or minimize the total expected cost.

In the two decades following Harris, several variations of the formula appeared, mostly in trade journals written by and for inventory managers. These were first collected in a full-length book on inventory management by Raymond (1931). Erlenkotter (1989,1990) has provided a fascinating account of this early history of inventory management.

Another classic formula in the inventory literature is the newsvendor formula. It can be attributed to Edgeworth (1888), although it was not until the work of Arrow, Harris and Marschak (1951) and Dvoretzky, Kiefer and Wolfowitz (1953) that a systematic study of inventory models, incorporating uncertainty as well as dynamics, began. These early papers were followed by the well-known treatise by Arrow, Karlin and Scarf (1958) as well as the seminal work of Scarf (1960). These works established the optimality of base-stock and \((s,S)\) policies, the most well-known policies in dynamic stochastic inventory modeling.

An important assumption in this vast literature has been that the demands in different periods are independent and identically distributed (i.i.d.). In real life, demands may depend on environmental considerations or the states of the world, such as the weather, the state of the economy, etc. Moreover, these states of the world are represented by stochastic processes – exogenous or controlled.

In this book, we are concerned with inventory models where these world states are modeled by Markov processes. We then show that we obtain the optimality of \((s,S)\)-type policies, or base-stock policies (i.e., \(s = S\)) when there are no fixed ordering costs with the provision that the policy parameters \(s\) and \(S\) depend on the current state of the Markov process representing the environment. Models allowing backorders when the entire demand cannot be filled from the available inventory, as well
as those when the current demand is lost, are considered. As for the cost criterion, we treat both the minimization of the expected total discounted cost and the long-run average cost. The average cost criterion is mathematically more difficult than the discounted cost criterion. Finally, we generalize the usual assumptions on holding and shortage costs and on demands that are made in the literature.

Our research on Markovian demand inventory models was carried out over a period of ten years, beginning in the early nineties. The research was supported by the National Science and Engineering Research Council of Canada, the Manufacturing Research Corporation of Ontario (a provincial Center of Excellence), the Laboratory for Manufacturing Research and The Canadian Centre for Marketing Information Technologies at the University of Toronto, and the University of Texas at Dallas. The first and second authors gratefully acknowledge the support from Alexander v. Humboldt Foundation and IBM Corporation, respectively.

This research has appeared in several journals, and it is now the subject of this book.

The models and the results presented in this book could be used in the analysis of inventory models with forecast updates. This topic is covered in the book by Sethi et al. (2005a).

Mathematical tools employed in this book involve dynamic programming and stochastic processes. This book is written for students, researchers, and practitioners in the areas of operations management and industrial engineering. It can also be used by those working in the areas of operations research and applied mathematics.

We are grateful to Alain Bensoussan, Ganesh Janakiraman, Ernst Presman, Selda Taskin, Yusen Xia, Xiangtong Qi, Hanqin Zhang, Jun Zhang, Qing Zhang, Arnab Bisi, and Maqbool Dada for their careful reading of parts of the manuscript. In addition, we express our appreciation to Barbara Gordon for her assistance in the preparation of the manuscript.

San Mateo, CA, USA, September, 2009
Washington D.C., USA, September, 2009
Richardson, TX, USA, September, 2009
Columbia, MO, USA, September, 2009

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