The term *hidden Markov model* (HMM) is more familiar in the speech signal processing community and communication systems but recently it is gaining acceptance in finance, economics and management science. The term HMM is frequently restricted to models with states and measurements in a discrete set and in discrete time. However, there is no reason why these restrictions cannot be relaxed, and so one can extend the modelling in continuous time and include observations with continuous range. The theory of HMM deals with *estimation*, which involves signal filtering, model parameter identification, state estimation, signal smoothing, and signal prediction; and *control*, which refers to selecting actions which effect the signal-generating system in such a way as to achieve certain control objectives. In the HMM implementation, reference probability methods are employed. This is a set of procedures designed in the reformulation of the original estimation and control task in a fictitious world so that well-known results for identically and independent distributed random variables can be applied. Then the results are reinterpreted back to the real world with the original probability measure.

To get a better understanding of an HMM, consider a message sequence \( X_k(k = 1, 2, \ldots) \) consisting of 0’s and 1’s depicted in Figure 1. Then, possibly the binary signal \( X \) (a Markov chain) is transmitted on a noisy communications channel such as a radio channel and the additive noise is illustrated in Figure 2. When the signal is detected at the receiver, we obtain some resultant \( Y_k \). What we get therefore after combining Figures 1 and 2 is a binary Markov chain *hidden* in noise given in Figure 3.

Essentially, the goal is to develop optimal estimation algorithms for HMMs to filter out the random noise in the best possible way. HMM filtering theory, therefore, discusses the optimal recursive estimation of a noisy signal given a sequence of observations. In electrical engineering for example, one is interested to determine the charge \( Q(t) \), at time \( t \) at a fixed point in an electric circuit. However, due to error in the measurement of \( Q(s), (s < t) \) one cannot really measure \( Q(s) \), but rather just a noisy version of it. The objective is to
“filter” the noise out of our observations. In a similar manner, we might ask whether financial data, interest rates, asset price processes, exchange rates, commodity prices, etc. contain information about latent variables. If so, how might their behaviour in general and in particular their dynamics be estimated?

The use of HMMs is also motivated by significant empirical evidence from the literature that favours and endorses Markov-switching models in the study of many macroeconomic variables. This provides more flexibility to financial models and incorporates stochastic volatility in a simple way. Earlier development in this area during the late 80’s within the time series context was pioneered by James Hamilton, amongst others, in a work that proposed to have the unobserved regime follow a Markov process. Indeed, examples of
many models in which the shift of regimes is governed by a discrete or continuous time Markov chain abound in finance and economics in the areas of business cycles, stock prices, foreign exchange, interest rates and option valuation. The rationale behind the regime-switching framework is that the market may switch from time to time between, say, a “quiet” (stable low volatility) state and a “turbulent” (unstable high volatility) state. In general the Markov chain states can refer to any number of conceivable “state of the economy”.

Within the HMM set-up and related modelling structures, this monograph offers a collection of papers dealing with the theory and empirical investigations probing the particular aspects of dynamic financial and economic modelling outlined above. The main themes in this collection include pricing, risk management, model calibration and parameter estimation.

This volume opens with two papers devoted to term structure of interest rates. In ‘An exact solution of the term structure of interest rate under regime-switching risk’, Shu Wu and Yong Zeng derive a closed-form solution to the term structure under an essentially affine-type model using log-linear approximation. It is shown that the market price of regime-switching risk affects the long-end of the yield curve and hence this is a significant component of the term premium for long-term bonds. Then Robert Elliott and Craig Wilson in ‘The term structure of interest rates in a hidden Markov setting’, develop an interest rate model whereby the stochastic nature of volatility and mean reversion is introduced in a simple and tractable way. Zero-coupon bond price is calculated. Empirical work using non-linear regression model illustrates that a 3-state Markov chain is able to explain considerably the dynamics of the yield rate data.

The theme of HMM regime-switching-based models continue with Tak Kuen Siu’s paper that demonstrates the interplay of methodologies in finance and actuarial science to successfully price insurance products with recent innovations. In ‘On Fair valuation of participating life insurance policies with regime switching’, he employs a regime-switching Esscher transform to value insurance policies with embedded exotic features. The valuation is performed within the basic geometric Brownian motion model but whose drift and volatility parameters are modulated by a hidden Markov model. Under the same market framework, Robert Elliott and Anatoliy Swishchuk investigate the valuation of options and variance swaps.

Two contributions then tackle the measurement and management of financial risks. In ‘Smoothed parameter estimation for a hidden Markov model of credit quality’, Malgorzata Korolkiewicz and Robert Elliott propose a model for the evolution of companies’ credit rating using a hidden Markov chain in discrete time. Smooth estimates for the state of the Markov chain and auxiliary parameters are also obtained. Kim Bong Siu and Hailang Yang, in ‘Expected shortfall under a model with market and credit risks’, present an integrated model that handle both credit and market risks. Two approaches in calcu-
lating VaR and ES are given: recursive equations and Monte Carlo methods. A weak Markov chain model is also outlined in their attempt to take into account the dependency of risks.

This is followed by papers on the filtering of HMM via change of probability measures. The development of general filters for the state, occupation time and total number of jumps of a weak Markov chain is examined by Shangzhen Luo and Allanus Tsoi in their article ‘Filtering of hidden weak Markov chain-discrete range observations’. Weak Markov chains may be suitable in modelling financial and economic processes that exhibit some form of memory. The study of future demands and inventory level via a discrete HMM in discrete time is the focus of Lakhdar Aggoun’s paper ‘Filtering of a partially observed inventory system’. The recursive estimation of the joint distribution of the level of stock and actual demand together with the re-estimation of model parameters is highlighted.

The monograph culminates with two papers that explore a thought-provoking hypothesis and challenging questions in economics. Emilio Russo, Fabio Spagnolo and Rogemar S Mamon, in ‘An empirical investigation of the unbiased forward exchange rate hypothesis in a regime-switching market’, use a Markov chain to describe structural change brought about by the intervention of central banks and other changes in the monetary policies as well as test the validity of the unbiased forward exchange rate hypothesis using US dollar/UK sterling pound exchange rate data. Abdul Abiad put forward the use of Markov regime-switching model to identify and characterise currency crisis periods. In ‘Early warning systems (EWS) for currency crises: A regime-switching approach’, he provides empirical support that a regime-switching model outperforms standard EWS in signalling crises and reducing false alarms. Country-by-country analyses of data for the period 1972-1999 from five Asian countries (Indonesia, Korea, Malaysia, the Philippines and Thailand), all of which experienced currency crises, were conducted.

We hope that this monograph will provide more insights to the financial research community and open avenues for more interesting problems. Specifically, it is our hope that this volume will raise more stimulating questions for further discussions in our concerted effort to build dynamic models that could incorporate the important stylised features of a financial market and capture better the significant factors of an economy.

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