Abstract  This chapter was prepared primarily for "non power" engineers to introduce reactive power and voltage control concepts, and to identify several issues that remain as research challenges in this area. It begins with basic definitions and information on reactive power, and then focuses on problems that have been known for many years and have surfaced recently in power system operations. These problems are described in the framework of traditional security analysis used in control centers throughout the world. The research challenges identify areas where collaboration between power systems engineers and applied mathematicians could yield major advances in power system reliability.

Keywords: Voltage collapse, Maximum power transfer, Var reserves, reactive power, voltage control, security analysis, power transfer capability.

1. Introduction

There has been a very large effort to understand reactive power and voltage control issues in power systems in the last 15 years [1, 2, 3, 4, 5]. While this work has produced a significant volume of results that explain various phenomena and propose potential solutions to problems, much of it has not been taken to the stage of useful application software and hardware in actual power systems. In addition, there remain serious issues of how traditional controls impact the interaction between the flow of real and reactive power on a system level when an objective might be to maximize real power transfer capabilities. There has been a strong temptation to try and separate the voltage/MVar problems with
the angle/MW problems. While this separation is valuable because it allows decoupling that might be useful in some computational algorithms and in understanding fundamental phenomena, this separation cannot be made when systems are stressed to their physical limitations. On the contrary, the interaction between real and reactive power (and voltage magnitudes and angles) becomes very complex under these extreme conditions [6]. This chapter begins with an introduction to reactive power and how it relates to voltage control. This is followed by several examples that illustrate problems which remain in the operations environment of electric power systems.

2. Reactive Power

Reactive power is a quantity that has become fundamental to the understanding and analysis of AC electric power systems. It is rarely mentioned in physics textbooks and is often clouded in mystery because of its "imaginary" status in the mathematics that electrical engineers use to describe certain phenomenon in AC circuits. The fundamental definition of reactive power can be explained by first looking at the relationship between a sinusoidal voltage and current waveforms of the same frequency (Figure 2.1). Reactive power has its origin in the phase shift between these two waveforms. When a device consumes real power such that the voltage and current waveforms are in phase with each other, the device consumes zero reactive power. When the current defined "into" a device lags the voltage, it consumes reactive power (this is the case in Figure 2.1). The amount of reactive power consumed by the device depends on the phase shift between the voltage and current.

![Figure 2.1. Voltage and current waveforms.](Image)

To formally see how this works, consider the instantaneous power (simple product of voltage times current) consumed by a device as shown in Figure 2.2. This power has two interesting characteristics. First, it is double the frequency of the original voltages and currents. Second, it is not symmetric above and below the horizontal axis. This power can
be decomposed into two waveforms one that is always positive plus one that is symmetric about the horizontal axis as shown in Figure 2.3.

\[ P(1 - \cos(2\omega t)) : P = 0.275 \text{ Per Unit Watts} \]
\[ -Q \sin(2\omega t) : Q = 0.205 \text{ Per Unit Vars} \] (2.1)

This decomposition yields the following two terms which formally define real \( (P) \) and reactive \( (Q) \) power, where the units of \( Q \) are Volt-Amperes-Reactive (Vars) to distinguish this component from \( P \):

\[ S = VI^* = P + jQ \] (2.2)

where \( V \) and \( I \) are the complex “Root Mean Square” phasor representations of the voltage and current, and * denotes conjugation. From this relationship, it is clear that reactive power \( (Q) \) will have a serious impact on both the voltages in a network and the currents.

### 3. Reactive Power in Operations

Reactive power affects power system operation in numerous ways:

1. Loads consume reactive power, so this must be provided by some source.
The delivery system (transmission lines and transformers) consumes reactive power, so this must be provided by some source (even if the loads do not consume reactive power). Note however that all transmission lines do provide some reactive power from their shunt line charging which offsets their consumption of reactive power in their series line losses.

The flow of reactive power from the supplies to the sinks causes additional heating of the lines and voltage drops in the network.

The generation of reactive power can limit the generation of real power.

So, one primary dilemma with reactive power is that a sufficient quantity of it is needed to provide the loads and losses in the network, but having too much reactive power flowing around in the network causes excess heating and undesirable voltage drops. The normal answer to this dilemma is to provide reactive power sources exactly at the location where the reactive power is consumed. And, since strictly speaking it does not take any "fuel" to provide reactive power, it should be possible to distribute reactive power sources (such as capacitors) all around the network to avoid the problem of heating the conductors and causing voltage drops. Unfortunately, this is not practical in the extreme since there are literally millions of lines and loads connected to the grid and so this would require millions of reactive power sources - all controlled to provide exactly the right amount of reactive power at the right time - every second of every day. The best we can do in most cases is work with some type of aggregation of load (say at the feeder leaving a substation) and at terminals of major lines and transformers. This also brings up the issue of the difference between power factor control (trying to exactly provide the right amount of reactive power needed to equal that which is consumed) and voltage control (trying to keep voltage levels at exactly the right level no matter how much reactive power it takes).

Reactive power is both the problem and the solution to network voltage control. The reactance of lines creates a voltage drop which must be compensated for whether the actual line flow is transferring real or reactive power (or both). The reactance also consumes reactive power which must be provided from some source. Serving reactive power to loads is especially difficult because the reactive power must flow from the source to the load - thereby increasing reactive losses - which in turn requires more reactive power from the source. The response of reactive power sources varies from milliseconds to seconds. The use of synchronous machines is the most common source for reactive power
Reactive Power and Voltage Control Issues in Electric Power Systems

(and also voltage control). The excitation system which provides the dc to the field winding of the machine adjusts the reactive power output (or input) to maintain the desired voltage set point. But, generators also have capability curves which bound the combination of real and reactive power output. The phenomena which bound the output vary from heating of the stator to heating of the rotor, stability, and other physical constraints. In some regions, this curve is simply the bound on the square root of the sum of the squares of $P$ and $Q$ (called “apparent power”).

There is a basic concept that is taught in power system analysis that provides an interesting fact about transmission lines and their loading level. The Surge Impedance Load (SIL) of a lossless transmission line is the amount of load delivered to a pure resistance equal to the characteristic impedance (square root of $L/C$ where $L$ and $C$ are the incremental distributed series inductance and shunt capacitance). A line with 1.0 SIL loading will have a flat voltage profile (same voltage from sending to receiving end), and the same current all along the line. The voltage and current will be in phase along the entire line. The reactive power into the line from the shunt capacitance charging is exactly equal to the reactive power consumed by the series inductance losses. Approximate values of 1.0 SIL are given in Table 2.1.

<table>
<thead>
<tr>
<th>Voltage (kV)</th>
<th>Load (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>69</td>
<td>10</td>
</tr>
<tr>
<td>138</td>
<td>50</td>
</tr>
<tr>
<td>230</td>
<td>150</td>
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<tr>
<td>345</td>
<td>400</td>
</tr>
<tr>
<td>500</td>
<td>1000</td>
</tr>
<tr>
<td>765</td>
<td>2000</td>
</tr>
</tbody>
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In some sense, 1.0 SIL is the “ideal” loading for a transmission line. In very early work on the analysis of transmission line loading capabilities, St. Clair created a composite curve which provided the maximum loading for transmission lines in terms of SIL and line length in miles [7].

The bounds on loading in the St. Clair curve consider three phenomena – thermal, voltage, and stability. For short lines, the loading is constrained by thermal limits. For medium length lines the loading is constrained by voltage drop. For long lines the loading is limited by steady-state stability (or maximum power transfer). While these limitations are all well known, the composite plot of these constraints for any line length of any voltage is interesting and provides a good physical feeling for line loadability and the possible sources of limitation.
The voltage collapse phenomenon remains a major issue for power system networks. In the words of Carson Taylor [8], “A power system at a given operating state and subject to a given disturbance undergoes voltage collapse if post-disturbance equilibrium voltages are below acceptable limits. Voltage collapse may be total (blackout) or partial.” Similarly, Prabha Kundur says [9], “Voltage stability is the ability of a power system to maintain steady acceptable voltages at all buses in the system under normal operating conditions and after being subjected to a disturbance. A system enters a state of voltage instability when a disturbance, increase in load demand, or change in system condition causes a progressive and uncontrollable drop in voltage.” Prabha Kundur goes on to say, “The main factor causing instability is the inability of the power system to meet the demand for reactive power. The heart of the problem is usually the voltage drop that occurs when active power and reactive power flow through the inductive reactance associated with the transmission network.” He also states, “Voltage collapse is the process by which the sequence of events accompanying voltage instability leads to a low unacceptable voltage profile in a significant portion of the power system.” Finally, he indicates, “A criterion for voltage stability: inject Vars at bus \( k \) and the voltage at bus \( k \) goes up (V-Q sensitivity is positive).” These collective statements indicate how widely different the interpretation of the issue of “voltage collapse” can be.

Walter Lachs [10] provided a possible scenario for voltage collapse (given in [9]):

1. Generating units near load centers out of service. Heavily loaded lines and low Var reserves.

2. Loss of a heavily loaded line. Increases other line loadings and Var losses - voltage reduction.

3. Load consumption would temporarily lower to stabilize. AVRss would act to restore generator voltages, but increased Var flow would lower voltages elsewhere.

4. The ULTCs (Under Load Tap Changers) at load centers would increase distribution voltages and so the load would go back up, and EHV voltages would go back down.

5. Generators would hit Var limits.

Over the years, a fairly common interpretation of voltage collapse is: “Static” voltage collapse has become synonymous with maximum power transfer and the ability to solve a load flow problem. “Dynamic” voltage collapse involves detailed load models, control and other dynamics.
4. **A Fundamental Illustration**

A very simple illustration reveals several very fundamental issues with reactive power and voltage control as they relate to power system security analysis. Consider a case where two areas with generation are interconnected through multiple transmission lines as shown in Figure 2.4. For simplicity, the lines do not have resistance or shunt capacitive charging elements.

This “Case 1” shows that both areas have adequate voltage control (evident by the voltage levels of 1.0 p.u.). The West area is providing 3,000 MW to the East area through 6 identical transmission lines (500 MW per line). The East generator is providing 150 MVar into the lines and the West generator is also providing 150 MVar into the lines to collectively provide the 300 MVar of reactive power losses. The East generator has a MVar limit of 1,200. The West generator has unlimited Var capability.

![Figure 2.4. Illustration Case 1 (all lines in).](image)

Figure 2.5 shows the situation when one of the transmission lines is removed (perhaps by relay action). The 3,000 MW transfer is now divided equally as 600 MW per line and the voltage at both ends is acceptable (voltage levels of 1.0 p.u.) and the East generator Var supply requirement is still below the 1,200 limit. The reactive power losses in the 5 remaining lines now totals 362 MVar (half from each generator).

Figure 2.6 shows the result of losing 2 lines, which causes the East generator to hit its reactive power supply limit. However, the East voltage has not dropped below 1.0, so this is a critical point where the
Voltage is 100% of rated voltage (362 MVARs required by lines).

Figure 2.5. Illustration Case 2 (one line out).

reactive power supply available in the East is exhausted, but the voltage is still normal. The reactive power losses in the lines are now 453 MVar.

Figure 2.6. Illustration Case 3 (two lines out).

Figure 2.7 shows the result of losing 3 lines. The East voltage is now starting to drop (down to 0.99 p.u.), and the line reactive power losses have increased to 611 MVar.

Figure 2.8 shows the case for four lines out which loads the remaining two lines to 1,500 MW each with an accompanying reactive power total loss of 957 MVar. The East generator bus voltage drops to 0.97 p.u. (still above acceptable levels). One important point here is that with
four lines out, the system still seems acceptable in terms of voltage levels (remaining line loadings are not enforced here).

Figure 2.9 shows that when a fifth line is removed, the system can no longer transfer 3,000 MW and the computer solution fails to converge to a solution.

While in the real network, the loads might change under this condition, the computer solution attempts to enforce the 3,000 MW transfer. Some would consider this a voltage collapse, although examination of the “rotor angles” of the East generator indicates that this bus likely
Figure 2.9. Illustration Case 6 (five lines out).

has an angle difference which could be considered an "angle stability" problem or a "maximum power transfer" problem. Whatever it is called, this illustrates the importance of reactive power in the cases where lines are lost.

The remaining figures indicate what would happen if unlimited reactive power supply was available in the East.

Figure 2.10. Illustration Case 7 (two lines out - unlimited Vars).
Figure 2.11. Illustration Case 8 (three lines out - unlimited Vars).

Figure 2.12. Illustration Case 9 (four lines out - unlimited Vars).

Figure 2.13. Illustration Case 10 (five lines out - unlimited Vars).
While line loading may be a thermal problem, there is no "voltage collapse" or "power transfer" problem in the cases where 5 lines are lost. In fact, the absolute maximum power transfer that can be achieved with full voltage control at both ends is shown in Figure 2.14 to be 4,900 MW.

5. Challenges in Voltage Control and Related Security

There are many challenges to voltage control and security in power systems. A few of these are indicated below:

1 Determining AVR set points and supplementary input signals: The Automatic Voltage Regulator set points for voltage control are normally not currently determined by any global system strategy for some optimal performance. Supplementary signals to enhance stability during disturbances are also based primarily on local considerations. There is a need to formulate a global strategy for voltage control and reactive power dispatch for both static and dynamic situations.

2 Modeling what really happens when excitation systems hit limits: The mathematical models for excitation limiters and associated protective relaying need to be incorporated in security analysis simulations. In many cases the excitation limit controls have several steps which yield different results in terms of what is controlled at what time.

3 Optimal placement and control of Static Var Compensators (SVC) and other Var sources: There remains a major challenge to deter-
mine the best locations and controls for reactive power devices to provide maximum benefit to the system as a whole during normal operation and contingencies.

4 The constraints of “acceptable” margins: Several components of security analysis are based on maintaining a reasonable distance to limits. While several margin concepts have been proposed over the years, these margins have not been properly incorporated into analysis tools that determine secure operation. This is particularly difficult for the margins to voltage collapse and transient stability.

5 Determination of “acceptable” Var margins: It is clear that in some cases, even an infinite amount of reactive power may not be enough support to accommodate certain levels of power transfer. Determining the acceptable size, location, and response characteristics of Var support is a major challenge in this area.

6 New security concepts: A challenging and useful margin would be to compute the minimum number of things that can be lost without resulting in cascading failure. This departs from the traditional N-1 criteria and characterizes a security margin in terms of number of contingencies required to create a blackout.

7 Reactive power computation in fast contingency analysis: Traditional linear contingency analysis completely ignores voltage and reactive power issues. There is an open challenge to rapidly compute the impact of contingencies on voltage and the margin to voltage collapse.

6. Conclusions

The purpose of this chapter was to provide background information on the mathematical challenges associated with voltage control and reactive power supplies. Several fundamental properties of reactive power and the consequences of shortages in reactive power reserves have been discussed. Open research issues associated with voltage control and reactive power support have also been discussed. These research issues are especially difficult because of the complex interaction between real and reactive power from an engineering point of view, and from an economic point of view. The power engineering community and industry at large could benefit greatly from the contributions that the mathematical community might make in these areas.
References


