# Contents

Preface v
Illustrations xiv

## I. Question and Method 1
1. Methodological Approaches to the History of Science 1
2. Categories for the Analysis of Culturally Shaped Conceptual Developments 8
3. An unusual Pair: Negative Numbers and Infinitely Small Quantities 10

## II. Paths Toward Algebraization – Development to the Eighteenth Century. The Number Field 15
1. An Overview of the History of Key Fundamental Concepts 15
   1.1. The Concept of Number 16
   1.2. The Concept of Variable 19
   1.3. The Concept of Function 20
   1.4. The Concept of Limit 22
   1.5. Continuity 25
   1.6. Convergence 27
   1.7. The Integral 30
2. The Development of Negative Numbers 32
   2.1. Introduction 32
   2.2. An Overview of the Early History of Negative Numbers From Antiquity to the Middle Ages 35
      European Mathematics in the Middle Ages 39
   2.3. The Onset of Early Modern Times. The First “Ruptures” in Cardano’s Works 40
   2.4. Further Developments in Algebra: From Viète to Descartes 45
VIII Contents

2.5. The Controversy Between Arnauld and Prestet 49
   A New Type of Textbook 49
   Antoine Arnauld 50
   Jean Prestet 52
   The Controversy 54
   The Debate’s Effects on Their Textbook Reedications 57

2.6. An Insertion: Brief Comparison of the Institutions for
   Mathematical Teaching in France, Germany, and England 61
   Universities and Faculties of Arts and Philosophy 61
   The Status of Mathematics in Various Systems of
   National Education 64
   New Approaches in the Eighteenth Century 66

2.7. First Foundational Reflections on Generalization 67

2.8. Extension of the Concept Field to 1730/40 73
   2.8.1. France 73
   2.8.2. Developments in England and Scotland 88
   2.8.3. The Beginnings in Germany 95

2.9. The Onset of an Epistemological Rupture 99
   2.9.1. Fontenelle: Separation of Quantity from Quality 99
   2.9.2. Clairaut: Reinterpreting the Negative as Positive 102
   2.9.3. D’Alembert: The Generality of Algebra—An
   Inconvénient 104

2.10. Aspects of the Crisis to 1800 114
   2.10.1. Stagnant Waters in the French University
   Context 114
   2.10.2. The Military Schools as Multipliers 121
   2.10.3. Violent Reaction in England and Scotland 126
   2.10.4. The Concept of Oppositeness in Germany 132

2.11. Looking Back 149

III. Paths toward Algebraization—The Field of Limits: The
    Development of Infinitely Small Quantities 151
    1. Introduction 151
    2. From Antiquity to Modern Times 153
       Concepts of the Greek Philosophers 154
    3. Early Modern Times 157
### Contents

4. The Founders of Infinitesimal Calculus 161
6. The Concept of Infinitely Small Quantities Emerges 186
7. Consolidating the Concept of Infinitely Small Quantities 192
8. The Elaboration of the Concept of Limit 206
   8.1. Limits as MacLaurin’s Answer to Berkeley 206
   8.2. Reception in the *Encyclopédie* and Its Dissemination 209
   8.3. A Muddling of Uses in French University Textbooks 213
   8.4. First Explications of the Limit Approach 220
   8.5. Expansion of the Limit Approach and Beginnings of Its Algebraization 228
9. Operationalizations of the Concept of Continuity 238
10. A Survey 255

### IV. Culmination of Algebraization and *Retour du Refoulé* 257
1. The Number Field: Additional Approaches Toward Algebraization in Europe and Countercurrents 257
   1.1. Euler: The Basis of Mathematics is Numbers, not Quantities 257
   1.2. Condillac: Genetic Reconstruction of the Extension of the Number Field 260
   1.3. Buée: Application of Algebra as a Language 265
   1.4. Fundamentalist Countercurrents 268
       Klostermann: Elementary Geometry versus Algebra 276
2. The Limit Field: Dominance of the Analytic Method in France After 1789 279
   2.1. Apotheosis of the Analytic Method 279
   2.2. Euler’s Reception 283
   2.3. Algebraic Approaches at the *École Polytechnique* 286
   3.1. The Original Conception 295
   3.2. Changes in the Structure of Organization 296
   3.3. The First Crisis: Pressure from the *Corps du Génie* and the Artillery 298
## Contents

3.4. The New Teaching Concept of 1800 302
3.5. Modernization of the *Corps du Génie* and Extension of the School in Metz 303
3.6. The Crisis at the École Polytechnique in 1810/11 306

### V. Le Retour du Refoulé: From the Perspective of Mathematical Concepts 309

1. The Role of Lazare Carnot and His Conceptions 309
   1.1. Structures and Personalities 309
   1.2. A Short Biography 312
   1.3. The Development of Carnot’s Ideas on Foundations: *Analyse* and *Synthèse* 318
   1.4. The Change in Carnot’s Ideas 325
   1.5. Relations Between Mechanics and the Foundations of Mathematics 330
   1.6. Infinitesimal Calculus: Carnot’s Shift from the Concept of Limit to the Infinitely Small 334
     1.6.1. The Memoir for the Berlin Academy 334
     1.6.2. The 1797 Version 346
     1.6.3. The 1813 Version 348
   1.7. Substituting Negative Numbers with Geometric Terms 353
     1.7.1. Carnot’s Writings on Negative Quantities: An Overview 353
     1.7.2. Carnot’s Basic Concepts and Achievements 354
     1.7.3. The Development of Carnot’s Concept of Negative Quantities 355
     1.7.4. Later Work: *L’analyse: La Science de la Compensation des Erreurs* 361

2. Carnot’s Impact: Rejecting the Algebraization Program 365
   2.1. Initial Adoption 365
   2.2. Limits and Negative Numbers in the Lectures at the École Normale 366
   2.3. Dissemination of the Algebraic Conception of Analysis 369
   2.4. Analysis Concepts at the École Polytechnique to 1811 371
     2.4.1. The Reorganization About 1799 371
     2.4.2. Lacroix: Propagator of the *Méthode des Limites* 372
2.4.3. Further Concept Development: Lacroix, Garnier, and Ampère 380
2.4.4. Prony: An Engineer as a Worker on Foundations 390
2.4.5. Lagrange’s Conversion to the Infiniment Pétits? 394
2.5. The Impact of the Return to the Infiniment Petits 398
  2.5.1. Impact Inside the École: The “Dualism” Compromise 398
  2.5.2. The Impact Outside the École 402
  2.5.3. The First Overt Criticism Back at the École: Poinsot in 1815 408
3. Retour to Synthèse for the Negative Numbers 410
  3.1. Lacroix Propagates the Absurdité of the Negative 410
    3.1.1. ...in Algebra 411
    3.1.2. ...in the Application of Algebra to Geometry 416
  3.2. The Criticisms of Gergonne and Ampère 420
  3.3. A Further Look at England 426

VI. Cauchy’s Compromise Concept 427
  1. Cauchy: Engineer, Scientist, and Politically Active Catholic 427
  2. Conflicting Reception of Cauchy in the History of Mathematics 431
  3. Methodological Approaches to Analyzing Cauchy’s Work 433
  4. Effects of the Context 436
  5. The Context of Cauchy’s Scientific Context: “I’m Far from Believing Myself Infallible” 441
  6. Cauchy’s Basic Concepts 445
    6.1. The Number Concept 446
    6.2. The Variable 450
    6.3. The Function Concept 451
    6.4. The Limit and the Infiniment Petit 452
    6.5. Continuity 457
    6.6. Convergence 466
    6.7. Introduction of the Definite Integral 477
    6.8. Some Final Comments 480
VII. Development of Pure Mathematics in Prussia/Germany 481

1. Summary and Transition: Change of Paradigm 481

2. The Context of Pure Mathematics: The University Model in the Protestant Neohumanist System 483

3. Negative Numbers: Advances in Algebra in Germany 486
   3.1. Algebraization and Initial Reactions to Carnot 486
       H. D. Wilckens (1800) 486
       F.G. Busse (1798, 1801, 1804) 488
       J.F. Fries (1810) 493
   3.2. The Conceptual–Structural Approach of W.A. Förstemann 495
       On Förstemann’s Biography 496
       His 1817 Work 498
   3.3. Reception Between Refutation and Adaptation 505
       M. Metternich 505
       J.P.W. Stein 510
       W. A. Diesterweg and His Students 516
       Martin Ohm 521
   3.4. The Continued Dominance of the Quantity Concept 525

4. The “Berlin Discussion” of Continuity 534

5. The Advance of Pure Mathematics 540
   5.1. Summary of Dirksen’s Work 541
       5.1.1. Number 542
       5.1.2. Series 544
       5.1.3. Theory of Functions 548
   5.2. Dirichlet’s Work on Rigor 558
   5.3. The Reception of Pure Mathematics in Textbook Practice 561

VIII. Conflicts Between Confinement to Geometry and Algebraization in France 567

1. Keeping Up the Confinement of Negative Quantities to Geometry 567

2. Last Culmination Points of the *Infiniment Petits* 574
   2.1. Poisson’s Universalization of the *Infiniment Petits* 574
   2.2. The Apotheosis of the Dualist Compromise: Duhamel 587
       2.2.1. The Principle of Substitution for *Infiniment Petits* 590
Contents XIII

2.3. The End of the Classical *Infiniment Petits* 598

IX. Summary and Outlook 601
1. Principle of Permanence and Theory of Forms 601
2. On the New Rigor in Analysis 606
3. On the Rise of Modern *Infiniment Petits* 609
4. Some Closing Remarks 616

Appendix 619
A. The Berlin Contest of 1784 Reassessed 619
B. Carnot’s Definitions of *Quantités Infiniment Petites* 620
   Part I 620
   Part II 621
   Part III 622
   Part IV 623
C. Calendar of Cauchy’s Traceable Correspondence 624

References 631
Sources 631
Publications 632

Index of Names 671
Conflicts Between Generalization, Rigor, and Intuition
Number Concepts Underlying the Development of Analysis in 17th-19th Century France and Germany
Schubring, G.
2005, XIV, 678 p. 22 illus., Hardcover